Beyond Balanced Growth: On the Analysis of Growth Trajectories

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Abstract

The mathematical theory of economic growth is complicated enough but at the same time it is very restrictive. In the paper we want to demonstrate that in many well-known models the analysis of a growth path and its neighborhood may be simplified by the use of slightly more sophisticated mathematical tools. Moreover, these tools can be applied to models with a less restrictive, more flexible structure. Technically, the tools are closely related to the ones which usually are applied. They are based on linear approximations and involve the computations of eigenvalues.

The setup of the paper is as follows: First we develop a proper technical definition of balanced growth and show how standard models of growth theory match into this framework. Second we describe two mathematical tools designed to analyse particular classes of models of balanced growth.[1] Finally we demonstrate the application of these tools to well-known models of endogenous growth.

Growth theory provides an increasing variety of mathematical models which describe forces driving economic growth and focus on some kind of dynamic equilibrium. There have been substantial improvements in the understanding of what an equilibrium is and how it evolves over time. Changes over time considered in models of economic growth are due to savings and investment in capital, accumulation of human capital, innovation in products and processes, creative destruction to name the most important ones. Individual and collective incentives, market structures as well as forms and degree of competition influence scope and direction of these changes. Beyond short run flucuations we observe regularities in the long run dynamics. They still are assumed to justify the assumption of balanced growth in the long run.

In new growth theories the more elaborate microeconomic foundations of short run market reaktions and allokation processes on the one hand and the long run trends on the other hand are interconnected. However, should the model be tractable there have to be limitations to the motion close to the path of balanced growth. If the model can be expressed in ratios of variables the path of balanced growth reduces to a rest point. The analysis of the dynamics

[1]Here I refer to two papers which are recent work by Charles Pugh of the Department of Mathematics at the University of California, Berkeley, and myself.

in the neighborhood of this point is standard. Obviously, this trick places severe constraints on the degree of freedom in modelling economic dynamics off the path of balanced growth. For this and possibly for other reasons as well Robert Solow[2] judges upon the state of the art: "There is a dangerous lack of robustness in the assumptions that, so far, underlie every version of the theory. ... a model of genuine endogenous growth seems to be achievable only if everything in the model turns out just so." (p. viii). And later on in his book he votes "for less focus on steady state paths and more on other kinds of equilibrium trajectories, even if that means dependence on simulations" (p. 182). The goal of this paper is to present mathematical tools which can be applied to generalized models and even to growth paths which show patterns of non-balanced growth.

The first tool exploits the fact that apart from the sustainability problem and from overall optimality economists are interested in the near future of the growth path. This is not only a question of time passing bye it is a question of stocks of resources and knowledge, quantities and diversity of commodities produced, which altogether should not be to different from today. The more we look into the future the less reliable are the quantifications. However, severe changes to be expected in the far future due to current activities - e.g. sustainability problems - should not be neglected in the analysis. A tool which meets these requirements is the compactification of the growth path mapping the far future to a finite interval. Under fairly weak and economically reasonable conditions a compactification is possible. In mathematical terms it requires normal hyperbolicity of the growth path to carry over to infinity.

The second tool is based on the theory of so called 'slowly varying systems' developed in electrical engineering. The idea is to derive conditions under which the linearization in directions perpendicular to the growth path provides sufficient information on the qualitative behavior of the system. The conclusion is that the information is sufficient if the system does not change to fast.

Both tools work in a framework more general than that of balanced economic growth! Sufficient conditions for the applicability are much more generic. No ratios of variables in the neighborhood of a growth path are involved. Nor is the condition of constant growth rates necessary for the analysis. They may therefore serve as a step towards a more robust and generic theory of endogenous economic growth.

[2]Robert M. Solow (2000) *Growth Theory*, 2. Edition, Oxford University Press.