

Financial Fragility, Fluctuations and Growth

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Preliminary Draft

1 Introduction

The aim of this paper is to analyze how growth and cycles - in a market consisting of potentially innovative firms - are affected by the financial sector in presence of capital market imperfections. Since it is well known that the non-validity of the Modigliani-Miller theorem implies that the financial situation of the firm affects its investment decisions, it is straightforward the effect on investment in R&D too (Brown, 1997) that - since the earlier works on new growth theory - is supposed to be a fundamental variable in order to understand the ultimate causes of growth (Cameron, 1998).

In macroeconomic models with financial constraints (see, for instance, Greenwald and Stiglitz, 1993; Bernanke, Gertler and Gilchrist, 1998; Kiyotaki and Moore, 1997) firms' supply decisions depend upon the degree of financial robustness/fragility, which is identified and measured in different ways. In the theoretical framework put forward by Greenwald and Stiglitz (GS hereafter), for example, financial fragility is due to the presence of bankruptcy risk: the higher the firm's financial fragility - i.e. the lower its net worth - the higher the risk of bankruptcy and the lower employment and output.

This theoretical setup, therefore, explicitly allows for bankruptcy. GS analyze a market characterized by a constant level of technology and implicitly assume that the number of firms is constant. The second assumption means that in case of bankruptcy the defaulted firm which leaves the market is replaced by a newly born firm with the same features. In this case therefore,

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GS allow for a very peculiar entry-exit process: the turnover of firms, in fact, is constrained by construction to yield a constant number of existing firms. Of course, this one-to-one replacement assumption is unduly restrictive and unrealistic. It is a well known empirical regularity, in fact, that the entry-exit process continuously modifies the total number of agents operating in a market, plays an important role in the evolution of the distribution of firms and affects macroeconomic variables.

In this paper we abandon the one-to-one replacement assumption allowing for an unconstrained turnover of firms and introduce investments in $R\&D$ in the decision problem of the firm. In particular we exploit the presence of bankruptcy risk in a theoretical framework *à la* Greenwald-Stiglitz (Delli Gatti, Gallegati and Palestrini, 2000) to model an endogenous flow of exiting firms. As to the flow of entrants, we assume that it is affected by current profitability of the market and the technological difference between the most innovative firm and the average technological level. This entry-exit pattern of industrial dynamics will affect the evolution of the distribution of firms according to the degree of financial robustness/fragility and technological level which in turn will affect aggregate outcomes. In our opinion, in fact, firms' heterogeneity plays a crucial role in determining the macroeconomic performance.

The evolution and persistence of heterogeneity over time and its role in business fluctuation is the core issue of a model of production and capital accumulation with imperfect capital markets presented in Delli Gatti, Gallegati and Palestrini (2000) (DGP hereafter). In this model, firms' heterogeneity is due to differences in the degree of financial robustness captured by the equity ratio, i.e. the ratio of net worth or equity base to the capital stock. Firms characterized by high (low) equity ratio, are financially robust (fragile).

The first though some form of imperfection of the market. In what follow we reason along the line of Greenwald-Stiglitz (1993) that explain how the presence of asymmetric information in the stock market can cause the non-validity of the Modigliani-Miller theorem.

The specific contribution of the present paper consists in showing how – introducing investments in $R\&D$ – the (complex) dynamics already present in DGP are affected by the entry of new potentially innovative firms and the endogenously generated process of firms' exit through bankruptcy. In this new framework, the structural characteristics of the economy (such as the distribution of firms by the degree of technology) and the aggregate variables (the capital stock, the equity base and aggregate output) can be interpreted as the outcome of a dynamic process which involves persistent financial heterogeneity and firms' turnover (birth and death of industrial units and changes in size and financial fragility of surviving units).

The paper is organized as follows. In section 2 we present the building blocks of the model. In section 3 we describe the entry-exit process implemented, and in section 4 the results of the simulations are presented. Section 5 concludes.

2 The model

2.1 The essential dynamics of the model

Our aim in this section is to derive the dynamical equations that describe the behavior of the firm. The key variables are: invested capital K_{it} , the equity base A_{it} , the debt L_{it} and the technology ϕ_{it} . Due to the balance relationship we can write:

$$K_{it} = L_{it} + A_{it}$$

To better understand we divide the dynamic in two moments. In the first one (that we label the beginning of period t) the firm decide the level of investment and expenses in $R\&D$, in the second one (the end of period t) the production is realized and sold according to the decision taken at the beginning of the period so that the profit is realized.

The beginning of period t . At the beginning of a period the firm knows the levels of the last period variables: K_{it-1} , A_{it-1} , L_{it-1} and ϕ_{it-1} . Now they have to decide their investment level and the expense in research and development. To describe briefly what is going on, we suppose for the moment that we know the choice of the firm (I_{it}^* and $R\&D_{it}^*$) and delay the detailed explanations in the following subsections. Once these variables are known we identify the new balance sheet variables with a star superscript.

I_{it}^* and $R\&D_{it}^*$ are financed by debt. So we have the following equation for debt:

$$L_{it}^* = L_{it-1} + I_{it}^* + (R\&D)_{it}^* \quad (1)$$

Equity base is not changed after these decisions, so we have

$$A_{it}^* = A_{it-1}$$

Since the balance sheet relation must be verified again we have

$$K_{it}^* = L_{it}^* + A_{it}^* = K_{it-1} + I_{it}^* + (R\&D)_{it}^* \quad (2)$$

The end of period t . Once the optimal level of investments and R&D are determined we can compute the other relevant quantities: production and consequently profit π_{it} . We want now to explain what we mean by production. The output of our firm is represented basically by two products. The first one is the traditional physical output (Y_{it}) and the second one is the technological progress (pt_{it}). Substantially in our firms we have two department: the factory where physical output is obtained and the research center where technological progress is obtained as a product. Each of the two department has its own production function that is an increasing function of the resources devoted to it. The total resources available to the firm are represented by total invested capital. Denoting with K^Y the resources devoted to the production department and K^ϕ those devoted to the research activities we have

$$K_{it}^* = K_{it}^{*Y} + K_{it}^{*\phi}$$

from equation (2) we can say that

$$K_{it}^{*\phi} = (R\&D)_{it}^*$$

so it will be

$$K_{it}^{*Y} = K_{it-1} + I_{it}^*$$

Finally the firm output is determined by the following production function:

$$Y_{it} = \phi_{it} K_{it}^{*Y}$$

and

$$pt_{it} = pt(K_{it}^{*\phi}) \tag{3}$$

The physical output has a very simple linear production function. We will specify in section 2.3 the analytical expression for the technological progress.

The real profit or loss (π) are related to the variables determined at the beginning of the period and to the relative price (u_{it}) known at the end of the period. As we will specify later u_{it} is a idiosyncratic stochastic variable (Greenwald and Stiglitz, 1993).

$$\pi_{it} = \pi(u_{it}, K_{it}^{*Y}, K_{it}^{*\phi}, L_{it}^*)$$

We are now ready to close the dynamics of the model determining the period t variables. Technology an equity base have a straight derivation:

$$\phi_{it} = \phi_{it-1} + pt_{it-1} \tag{4}$$

$$A_{it} = A_{it-1} + \pi_{it} \quad (5)$$

Production activity depreciate capital. We denote by δ the depreciation rate.

$$K_{it} = K_{it}^* - \delta_\phi K_{it}^{*\phi} - \delta_Y K_{it}^{*Y}$$

Expenses in research and development are not represented by durable goods so we pose $\delta_\phi = 1$. Instead we treat physical capital in a standard way ($0 < \delta_Y < 1$). Finally the firm increases the level of its capital for two reasons. First, it devotes a share ϵ of the economic return to improve capital. Second, the firm doesn't let the capital depreciate so it makes a further capital improving amounting to $\delta_Y K_{it}^{*Y}$. So at the end of the period we have a second wave of investment:

$$I_{it} = \delta_Y K_{it}^{*Y} + \epsilon\pi$$

The expression for capital at the end of the period is thus

$$K_{it} = K_{it}^* - K_{it}^{*\phi} - \delta_Y K_{it}^{*Y} + I_{it}$$

and after substitution

$$K_{it} = K_{it}^{*Y} + \epsilon\pi_{it} \quad (6)$$

finally, the debt level will be

$$L_{it} = K_{it} - A_{it} \quad (7)$$

So far we describe the dynamic framework of the model deferring the determination of the investment level and of the R&D and the related technological progress *pt*. Next subsections are devoted respectively to the determination of Investment level and the technological progress.

2.2 The determination of investments

Real profit, π_{it} , for the firm is the difference between real revenue¹ ($u_{it}Y_{it}$) and real cost. The latter are given by three components: the burden of debt $r_{it}L_{it}$, the depreciation cost for capital and the adjustment cost for capital² $\frac{\gamma}{2} \frac{(K_{it}^Y - K_{it-1})^2}{K_{it-1}}$

¹Nominal revenues are $P_{it}Y_{it}$. P_{it} is the price at which the firm sell its output, it's assumed to be a random variable with mean equal to the general level of price P_t . Dividing by P_t we obtain the real revenue $u_{it}Y_{it}$ with $u_{it} = P_{it}/P_t$.

²all the variables are in real term.

$$\pi_{it} = u_{it}Y_{it} - r_{it}L_{it} - (\delta_Y K_{it}^Y + K_{it}^\phi) - \frac{\gamma (K_{it}^Y - K_{it-1})^2}{2 K_{it-1}} \quad (8)$$

Expected real profits therefore is:

$$E(\pi_{it}) = Y_{it} - r_{it}L_{it} - (\delta_Y K_{it}^Y + K_{it}^\phi) - \frac{\gamma (K_{it}^Y - K_{it-1})^2}{2 K_{it-1}} \quad (9)$$

Using equation (1) and remembering that

$$I_{it}^* = K_{it}^Y - K_{it-1} \quad (10)$$

we can write

$$E(\pi_{it}) = Y_{it} - r_{it} (L_{it-1} + K_{it}^Y - K_{it-1} - R\&D_{it}) - (\delta_Y K_{it}^Y + K_{it}^\phi) - \frac{\gamma (K_{it}^Y - K_{it-1})^2}{2 K_{it-1}} \quad (11)$$

In their decision, firms account for the bankruptcy cost. Several papers in the literature point out the existence and relevance for economic activity of this kind of costs. So, to arrive to the objective function we have to add to expected real profit the bankruptcy cost. They are given by

$$F_{it} = CB_{it}Pr(u < \bar{u})$$

with

$$CB_{it} = \phi_{it}K_{it}^Y cb = \phi_{it}K_{it}^Y (\alpha_1 - \alpha_2 a_{it})$$

bankruptcy condition is

$$cf_{it} + A_{it-1} < 0$$

where cf_{it} differs from the profit because the depreciation is not considered. From this inequality we can write

$$u_{it} \leq \frac{1}{\phi_{it}K_{it}^Y} \left(rK_{it}^Y - rA_{it-1} + (1-r)R\&D_{it} + \frac{\gamma (K_{it}^Y - K_{it-1})^2}{2 K_{it-1}} \right)$$

As we mentioned above, u_{it} is a random variable. We suppose for simplicity that it is uniformly distributed between 0 and 2 so we have

$$Pr(u_{it} \leq \bar{u}) \leq \frac{1}{2\phi_{it}K_{it}^Y} \left(rK_{it}^Y - rA_{it-1} + (1-r)R\&D_{it} + \frac{\gamma (K_{it}^Y - K_{it-1})^2}{2 K_{it-1}} \right)$$

The bankruptcy cost is

$$F_{it} = \frac{cb}{2} \left(rK_{it}^Y - rA_{it-1} + (1-r)R\&D_{it} + \frac{\gamma(K_{it}^Y - K_{it-1})^2}{2K_{it-1}} \right)$$

So each firm maximizes

$$E(\pi_{it}) - F_{it} \quad (12)$$

Maximizing (12) with respect to K_{it}^Y one has:

$$\frac{K_{it}^{*Y} - K_{it-1}}{K_{it-1}} = \frac{1}{\gamma_{it}} (\phi_{it} - r_{it} - \delta_Y) \quad (13)$$

where $\gamma_{it} = \gamma(1 + cb/2)$ and $r_{it} = r(1 + cb/2)$.

Using equation (8) we can write

$$I_{it}^* = \frac{K_{it-1}}{\gamma_{it}} (\phi_{it} - r_{it} - \delta_Y) \quad (14)$$

Last equation is significant: To make investments the firms compare the revenue and cost of the investment. Indeed ϕ_{it} is the revenue for an additional unit of capital and $r_{it} + \delta_Y$ is the cost of the same unit of capital if the firm borrow to buy it. It is convenient to make investments only if revenue is grater than cost. In the opposite case the firm will realize a loss from the investment so it doesn't invest. So we use the following investment rule:

$$\begin{aligned} I_{it}^* &= 0 && \text{if } r_{it} + \delta_Y > \phi_{it} \\ I_{it}^* &= \frac{K_{it-1}}{\gamma_{it}} (\phi_{it} - (r_{it} + \delta_Y)) && \text{if } r_{it} + \delta_Y \leq \phi_{it} \end{aligned} \quad (15)$$

Depreciation cost are given in our model to the obsolescence of capital due to technological progress. So we suppose it is proportional to the capital productivity:

$$\delta_Y = (\phi_{it} - \bar{\phi}_0)$$

where $\bar{\phi}_0$ is a constant. The investment level is

$$I_{it}^* = \frac{K_{it-1}}{\gamma_{it}} (\bar{\phi}_0 - r_{it})$$

An observation is in order here: the variables

$$\gamma_{it} = \gamma \left(1 + \frac{\alpha_1 - \alpha_2 a_{it-1}}{2} \right)$$

$$r_{it} = r \left(1 + \frac{\alpha_1 - \alpha_2 a_{it-1}}{2} \right)$$

depend negatively on the equity ratio. So, the improving of a firm financial position reduces both the interest rate and the adjustment cost for capital making the investments of these firms higher.

2.3 The determination of technological progress

Once we describe the dynamic of investment we have to model the technological progress according to equation (4). First of all we know from equation 3 that pt_{it} is a function of $R\&D$.

We let the level of $R\&D$ depend upon tree factors: the dimension of the firm represented by its invested capital (K), its financial soundness (a) and its propensity (β_{it}) to invest in $R\&D$:

$$R\&D_{it} = cK_{it-1}(a_{it-1})^d + \beta_{it}$$

with

$$\beta_{it} = e [\max \phi_{t-1} - \phi_{it-1}]$$

where c, d and e are constants. So $R\&D$ expenses increase with the size of the firms, with its financial soundness and with its gap with the more efficient firm.

We suppose that an innovation increases the technological variable by a fixed amount $\Delta\phi$. Research ad Development has an uncertain result. In other words, $R\&D$ expenses can produce an innovation with a certain probability or may not. In this way the appearance of an innovation is a random variable ω which takes two values: 1 if the innovation appear and 0 if not. We suppose that the probability to have an innovation depends on $R\&D$ expenses according to an exponential distribution function:

$$Pr(\omega_{it} = 1) = 1 - \exp(-gR\&D_{it})$$

where g is a constant. Making substitution we can write:

$$Pr(\omega_{it} = 1) = 1 - \exp [hK_{it-1}(a_{it-1})^d + (\max \phi_{t-1} - \phi_{it-1})]$$

where $h = cg$

In the end we can write equation (3) as

$$pt_{it} = \omega_{it}\Delta\phi$$

and equation (4) as

$$\phi_{it+1} = \phi_{it} + \omega_{it}\Delta\phi$$

2.4 The complete dynamic

We are mainly interested in the dynamic of the aggregate output. Our model is thus composed by the following equations:

$$Y_{it+1} = \phi_{it+1} K_{it+1}^{*Y}$$

$$\phi_{it+1} = \phi_{it} + p t_{it}$$

$$K_{it+1}^{*Y} = K_{it} + I_{it+1}^*$$

$$L_{it+1}^* = L_{it+1} + I_{it+1}^* + R\&D_{it+1}^*$$

$$A_{it+1}^* = A_{it}$$

$$I_{it+1}^* = f(K_{it})$$

$$R\&D_{it+1}^* = c(K_{it})^{(1-d)} (A_{it})^d + e [\max \phi_t - \phi_{it}]$$

$$A_{it} = A_{it-1} + \pi_{it}$$

$$K_{it} = K_{it}^{*Y} + \epsilon \pi_{it}$$

$$K_{it}^{*Y} = K_{it-1} + I_{it}^*$$

$$L_{it} = K_{it} - A_{it}$$

$$\pi_{it} = u_{it} Y_{it} - r_{it} L_{it}^* - (\delta_Y K_{it}^{*Y} + K_{it}^{*\phi}) - \frac{\gamma (K_{it}^{*Y} - K_{it-1})^2}{2 K_{it-1}}$$

Substitution leave us with the following system

$$Y_{it+1} = \phi_{it+1} K_{it+1}^{*Y}$$

$$\phi_{it+1} = \phi_{it} + p t_{it}$$

$$K_{it+1}^{*Y} = K_{it}^{*Y} + \epsilon \pi_{it} + f(K_{it}^{*Y}, \epsilon \pi_{it})$$

$$L_{it+1}^* = K_{it}^{*Y} + \epsilon \pi_{it} - A_{it} + f(K_{it}^{*Y}, \epsilon \pi_{it}) + c[f(K_{it}^{*Y} + \epsilon \pi_{it})]^{(1-d)} (A_{it})^d + e [\max \phi_t - \phi_{it}]$$

$$A_{it+1}^* = A_{it}$$

$$\pi_{it} = u_{it} \phi_{it} K_{it}^{*Y} - r_{it} L_{it}^* - (\delta_Y K_{it}^{*Y} + K_{it}^{*\phi}) - \frac{\gamma (K_{it}^{*Y} - K_{it-1})^2}{2 K_{it-1}}$$

So, knowing all the variables in t and $t-1$ we determine the profit of period t and consequently all the values of period $t+1$. Unfortunately these equation are too complicate to proceed analytically, so we use simulations.

3 The entry process

Our model differs from traditional ones because it take into account seriously the fluxes of firms out and into the market. A recent OECD report emphasizes how the turnover of firm assume relevant volume and it should be considered to better understand real phenomena (see also Caves, 1998).

From what has been written above we can say that a firm exits the market when it goes bankrupt. In other words firms exit the market when their equity base becomes negative because of adverse shocks.

We model the entry flow in a stochastic way that takes into account economic consideration. Indeed, the entry decision depends on the possibility to survive of the firms. So they are more inclined to enter the market when the profitability of the market is high. We use as a proxy of the profitability two indicators: the first one is the difference between the maximum and the mean technology ($\max \phi - \bar{\phi}$). This is because a situation where firms have all the same technology is similar to a perfect competitive market where no profit is realized. The possibility to have a technology higher than the mean open the possibility to realize profit. The second one is the difference between the mean return on investment ($\bar{\pi}_{it}/\bar{K}_{it}$) and the interest rate r . We collect these two term in the variable λ'_t

$$\lambda'_t = \left(\frac{\bar{\pi}_{it}}{\bar{K}_{it}} - r \right) + (\max \phi - \bar{\phi})$$

So a high level of this variable signals a high convenience to enter the market.

As we say above, our entry mechanism is stochastic: we suppose that the probability the firms enter the market is positively related to λ'_t . In particular we assume that the number of new entry N^{en} has a Poisson distribution with arrival rate proportional to λ'_t :

$$Pr(N^{en}; \lambda_t) = \frac{\lambda_t^{N^{en}}}{N^{en}!} e^{-\lambda_t}$$

where $\lambda_t = l\lambda'_t$ and l is a constant.

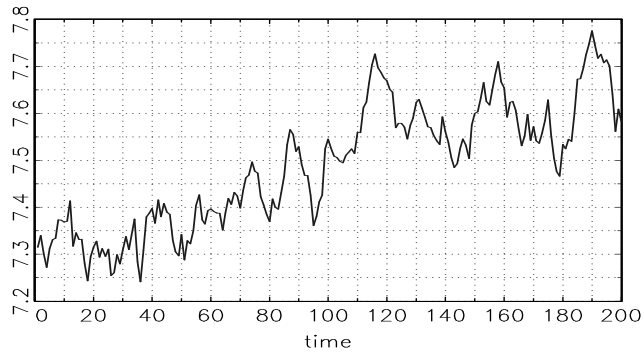
4 Simulations

As we mentioned above, we proceed with simulations using the SWARM simulation tool. Here we report some graphics that describe the characteristics of our model. First of all it is evident how the model display fluctuating

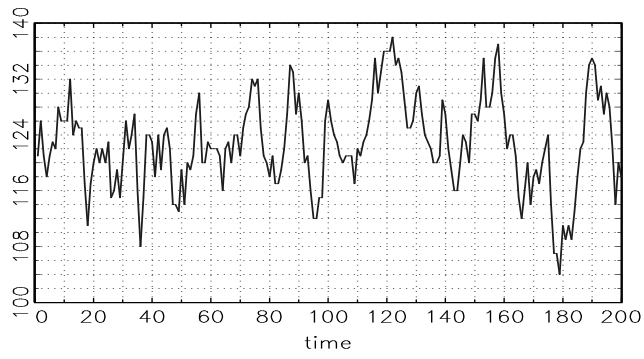
growth. The upward trend of the aggregate output is lead by the increase in the productivity of capital while fluctuations are mainly driven by the fluctuations in the number of firms (i.e. by the entry and exit process).

Technological progress seems to proceed by jumps. Period of stasis of different length are interrupted by period of innovations. When a technological gap is present firms devote more resources to *R&D* activities. This give to each firm a higher probability to innovate so innovations concentrate in a short interval of time. The process seems to be dissipative i.e. the innovation following the first main innovation are more and more weak and the process goes in a new period of stasis. Probably it is because *R&D* increase the financial fragility of the economy. Firms are forced to adequate to the increase in the technology because the use of inefficient technologies lowers the profit and make it difficult to survive. In a sense this process seems to be similar to the Darwinian theory of evolution: an improve in the technology force the firms to mutate to survive. The market makes the selection excluding the most weak (leveraged) of them. So a technological innovation is the base for the growth of aggregate output, but to obtain really this advantage it is necessary that new and more technologically advanced firms replace the older and less technologically advanced that seems to have problem to adequate.

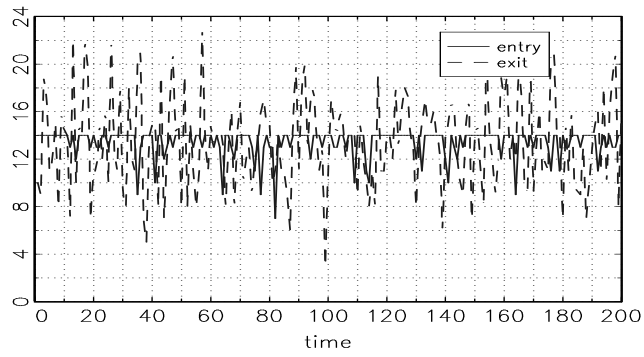
Aggregate output (log)



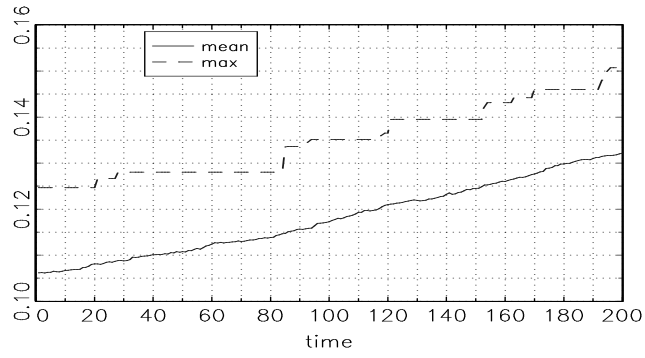
number of firms



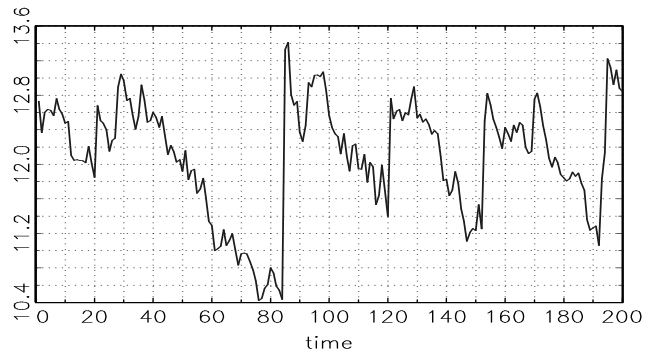
number of entry and exit



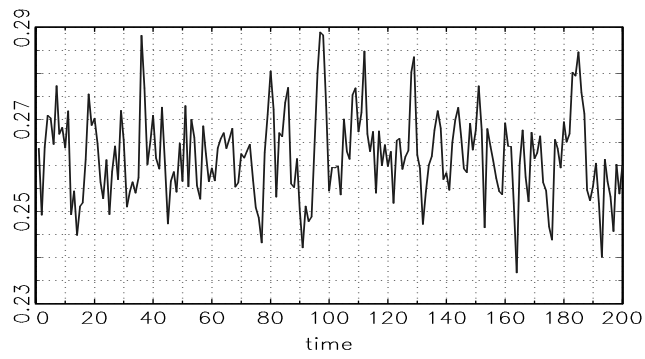
technology



R&D expenses (mean)



mean equity ratio



5 Conclusions

In this paper we analyze the problem of economic growth and fluctuations from a different point of view respect to the traditional approach.

Modelling the economic system using a population of interacting heterogeneous agents opens a new possibility which combines the traditional approaches. Small idiosyncratic shocks continuously hit the economy, but it is the propagation mechanism which modifies in time. Consequently, a shock can have irrelevant effects if the state of the beaten agent and links with the others are such that they prevent, or at least dampen, its propagation. In other circumstances the system may be in a state of “fragility” and the same shock can be amplified involving a great number of agents. Furthermore, since both cycles and growth are affected by the financial position of firms, the resulting complex dynamics shows an high degree of interrelation between the two time series components.

Our benchmark model (Greenwald and Stiglitz , 1993) is particularly suitable to our goal because it presents some of the characteristics we need: The presence of idiosyncratic price shocks maintains some heterogeneity of the firms’ financial position. We enlarge the basic model in two ways. First we consider the entry and exit of firms. Secondly, we model the investment in *R&D*.

References

- [1] Bernanke B., M. Gertler, S. Gilchrist (1998), The Financial Accelerator in Quantitative Business Cycle Framework, NBER Working Paper 6455.
- [2] Bernanke, B, and Gertler, M. (1989), Agency Costs, net Worth and Business Fluctuations, *American Economic Review*, vol . 79, pp. 14-31.
- [3] Brown, W. (1997), *R&D Intensity and Finance: Are Innovative Firms Financially Constrained?*, Discussion Paper n. 271, London School of Economics.
- [4] Caballero R., M. Hammour (1994), The Cleansing Effect of Recessions, *American Economic Review*, 64, 1350-68.
- [5] Caballero R.J., (1998), *Innovation and Growth: a survey of the empirical evidence*, Nuffield College, Oxford.

- [6] Caballero R.J., E.M. Engel , J.C. Haltiwanger (1997), Aggregate Employment Dynamics: Building From Microeconomic Evidence, *American Economic Review*, 87, pp 115-37.
- [7] Campbell J.R. (1997), Entry, Exit, Embodied Technology, and Business Cycle, NBER working paper 5955. [9] Caves R.E. (1998), Industrial Organization and New Findings on the Turnover and the Mobility of Firms, *Journal of Economic Literature*, 36, 1947-82.
- [8] Caves, R. E. (1998), Industrial Organization and New Findings on the Turnover and Mobility of Firms, *Journal of Economic Literature*, Vol. 36, pp. 1947-1982.
- [9] Cooley, T. and V. Quadrini (1998), Financial Markets and Firm Dynamics, mimeo. [11] Cooley, T. and V. Quadrini (1999), Monetary Policy and the Financial Decisions of Firms, mimeo.
- [10] Davis, S. T., J. C. Haltiwanger (1996), Driving Forces and Employment Fluctuations: New Evidence and Alternative Fluctuations, NBER working paper 5775
- [11] Davis, S. T., J. C. Haltiwanger, and S. Schuh (1996). Job Creation and Destruction. Cambridge, Mass.: MIT Press. [14] Day, R. (1994), *Complex Economic Dynamics*, Cambridge (Mass.), MIT Press. [15] Delli Gatti, D., Gallegati, M. and Minsky, H. (1996), Financial Institutions, Economic Policy and the Dynamic Behavior of the Economy, in Helmsstadter, E. and Perlman, M. (eds), *Behavioral Norms, Technological Progress and Economic Dynamics*, Ann Arbor, The University of Michigan Press.
- [12] Delli Gatti, D., Gallegati M., Palestrini, A. (2000), Agents Heterogeneity, Aggregation and Economic Fluctuations, in Delli Gatti, D., Gallegati, M. and Kirman, A.P., eds., *Interaction and Market Structure*, Springer, pp 133-149.
- [13] Greenwald, B.C., Stiglitz, J.E. (1990), Macroeconomic Models with Equity and Credit Rationing, in Hubbard, R. G. (ed.), *Financial Markets and Financial Crises*, Chicago, Chicago University Press.
- [14] Greenwald, B.C., Stiglitz, J.E. (1993), Financial Market Imperfections and Business Cycles, *Quarterly Journal of Economics*, vol . 108, pp. 77-114.

- [15] Kiyotaki, N. and J. Moore (1997), 93Credit Cycles, *Journal of Political Economy*.