GROWTH, AUTONOMOUS DEMAND AND TECHNICAL PROGRESS IN A SIMPLIFIED FIXED CAPITAL SYSTEM

Graham White

SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

UNIVERSITY OF SYDNEY

(Provisional Draft)

GROWTH, AUTONOMOUS DEMAND AND TECHNICAL PROGRESS IN A SIMPLIFIED FIXED CAPITAL SYSTEM

I Introduction

The present paper is intended as a contribution to recent debate on the nature of a demand-constrained growth, specifically debate over the contours of a Keynesian view of long-run growth and its synthesis with a classical-Sraffian explanation of value and distribution. In doing so, the intention is to bring together some disparate elements of Sraffian inspired research of the last few decades.

An interesting and important question running through recent literature on alternative, non-marginalist approaches to explaining growth, is the precise detail of an alternative which would encompass a coherent long-run version of Keynes's principle of effective demand and be consistent with a classical-Sraffian perspective on value and distribution (*cf*. Trezzini, 1995, Serano, 1995, Park, 2000). Debate over this question has focused particularly on the nature of autonomous demands and their relation with both steady-state and non-steady-state growth paths, an integral part of that relation being the so-called "Sraffian supermultiplier". For the most part however, this debate has taken place in the context of aggregative models, while explicit consideration of sectoral independencies has remained largely implicit and submerged in assumptions about relative prices and distribution.

The aim of this paper is to shed further light on the idea of demand-led growth and in particular the debate referred to above, by exploring the relation between growth and autonomous demands in a fixed capital model proper – viz., where used fixed capital is treated as both output and input. As is well known, this treatment of fixed capital allows for a richer consideration of technical progress. In turn this allows for a coherent interpretation of "autonomous" investment demand (which is commonly identified with technical change), and a clarification of the sense in which one can talk of "non-capacity creating" autonomous demands.

The intention of the paper is also to bring together the debate over growth in a Sraffa-Keynes framework and the arguments presented by others (e.g. Caminati, 1986) regarding the demand effects of technical change. This will allow some assessment of the extent to which recent literature about autonomous demand and long-run growth can be thought of as applicable to growth with technical progress.

The paper considers a simplified two sector model producing a pure consumption good and a machine with variable efficiency and in the process attempts a clarification of the "Sraffian supermultiplier" and autonomous investment demand. A "fixed-price" model is considered whereby relative prices and the real wage are held at their long-period equilibrium levels, so that disequilibrium in the model is limited to quantities. Thus there are no "cross-dual" dynamics, but only "dual" dynamics which are limited to the interaction of output and demand.

However, these dynamics are nonetheless complicated by the treatment of fixed capital as a joint product, since depreciation allowances impact on net profit and hence on capitalists' consumption. The dynamics of quantities are also complicated by the fact that both investment decisions and depreciation allowances are influenced by the age composition of the capital stock in each sector.

Section II developes the model by representing the dynamics of quantities in terms of first order difference equations in growth rates of demand, investment growth rates, utilization rates and the relative size of the two sectors. Section III makes use of the structure of the model in attempting a brief clarification of the notion of exogenous versus endogenous growth. In effect, the model allows as it were for alternative "closures": a feature which may assist in clarifying different positions in recent literature on demand-constrained long-run growth.

Section IV takes up the question of what is meant by autonomous investment demand, and specifically the concept of non-capacity creating autonomous investment. A simple case of labour-augmenting technical progress is considered and the conditions required for this to have a impact on investment demand are derived.

Section V provides a discussion of the stability of the equilibrium growth path. This is done by considering stability in the vicinity of equilibrium. Section VI provides some brief concluding remarks.

II A two-sector fixed capital model

Production

Consider a model with two sectors, one producing a pure consumption good, the other producing a machine. Both commodities require as inputs a quantity of labour and machines. Other than a quantity of used machines in each sector at the end of each production period there is no joint production. Machines have a maximum (technical) life of two periods, and have a zero scrap value. It is assumed that disposal of two-year old machines is costless.

It is further assumed that machines have a variable efficiency. This is reflected in the requirement of greater quantities of labour per unit of output with the use of older machines as compared with new machines. More precisely, with a unit labour requirement of l^0_{it} with the use of new (zero-years old) machines in the production of output in sector i in period t, then the corresponding labour requirement with (one-year) old machines is given by

$$l^1_{it} = l^0_{it}.(1+\alpha) \qquad \text{…..(1)}$$

where α is positive and less than unity. The assumption of variable efficiency warrants a distinction between the components of total output corresponding to machines of different ages, if only because variable efficiency may entail different desired rates of utilization of different aged machines. To allow for this possibility, output in sector *i* during period *t* is represented as follows:

$$Y_{it} = Y_{it}^{0} + Y_{it}^{1} \qquad i = 1,2 \qquad \text{.....(2)}$$

where Y_{it}^{0} and Y_{it}^{1} refer to outputs of commodity *i* on new and old machines respectively. Utilization rates of new and one-year old-year old machines in sector *i* in period *t*, are then given by

$$u_{it}^{j} = \frac{Y_{it}^{j}}{\beta_i M_{it}^{j}}, \qquad j = 0,1,\ i = 1,2 \qquad \text{.....(3)}$$

M^j_{it} is the number of *j*-year old machines used in production in sector *i* in period *t* and β_i is the output capacity of a machine in sector *i*. It is assumed that this is the same for new (*j* =0) and used (*j* = 1) machines, so that variation in efficiency over the life of machines amounts to more labour being required by older machines to produce the same output as new machines. u^j_{it} refers to the utilization rate in period t of *j*-year-old machines.

Differences in utilization rates on machines of different ages are treated in as simple a manner as possible by assuming a linear relation such that

$$u_{it}^{1} = \frac{u_{it}^{0}}{(1+\phi)} \qquad \text{.....(4)}$$

It is also assumed that there is a desired utilization rate in each sector, specifically a desired rate in relation to newly installed plant, denoted as u^{n0}_{it}. In view of (4) above, this effectively also implies a desired rate of utilization on older plant.

Output during period *t* is taken to be governed by the demand which materialises during period *t*. In other words, subject to the constraint provided by capacity, producers respond in the same period to the demand which is expressed. In this manner the utilization rate in each sector fluctuates in line with demand. For simplicity, inventories of finished goods are ignored.

Over the longer-run capacity in each sector is assumed to adjust to persistent variation in demand. Specifically, the decision about the size of capacity in each sector at the end of period *t* involves an estimate of demand through period *t+1*, and on that basis an estimate of the capacity required assuming a utilization rate of u^{n0}_{it+1} on newly installed capacity; and hence an estimate of the extent to which capacity at the end of period *t* is deficient or excessive. Hence, with demand D^e_{it+1} expected in sector *i* in period *t+1*, the firm would choose a capacity comprising one-year old machines in *t+1*, which were new in t, and new machines to be installed for use in *t+1*, such that

$$D^e_{it+1} = \left(u^n_i . M^0_{it+1} + \frac{u^n_i . M^1_{it+1}}{(1+\phi)} \right) . \beta_i \qquad \text{…..(5)}$$

Since $M^1_{it+1} = M^0_{it}$ and assuming that investment demand I_{it} during t leads to the installation of an equivalent amount of new capacity for use in *t+1*, M^0_{it-1} then expression (5) can be written as

$$D^e_{it+1} = \left(u^n_i . I_{it} + \frac{u^n_i . M^0_{it}}{(1+\phi)} \right) . \beta_i \qquad \text{…..(6)}$$

Demand expected in t+1 is assumed to be based an extrapolation of demand observed in period *t-1*, so that

$$D^e_{it+1} = D_{it-1} . \left(1 + g^d_{it-1}\right)^2 \qquad \text{…..(7)}$$

where D_{it-1} and g^d_{it-1} are demand for commodity *i* during *t-1* and the rate of growth of demand for commodity *i* in *t-1* respectively. It follows from (6) and (7) that investment demand in sector *i* at time *t* can be expressed as

$$I_{it} = \frac{D_{it-1} . \left(1 + g^d_{it-1}\right)^2 . (1+\phi) - u^n_i . M^0_{it} . \beta_i}{u^n_i . \beta_i . (1+\phi)} \qquad \text{…..(8)}$$

Demand

It is assumed that the components of demand which depend on income are expressed with a lag of one period. In other words, consumption demand of workers in period *t* is based on wage income earned in period *t-1*. Likewise, consumption by capitalists in period *t* is based on profit flows generated in period *t-1*.[1]

Demand for new machines - commodity 2 - during period *t* is

[1] This assumption effectively allows one to dispense with changes in inventories as a means by which supply is adapted to demand. On the other hand this treatment cannot avoid the problem that meeting demand could require stocks because utilization cannot be greater than 100%.

$$D_{2t} = I_{1t} + I_{2t} + I_t^{Aut} = \sum_{i=1}^{2} \left(\frac{D_{it-1}.\left(1+g_{it-1}^{d}\right)^2.(1+\phi) - u_i^n.M_{it}^0.\beta_i}{u_i^n.\beta_i.(1+\phi)} \right) + I_t^{Aut} \quad \text{.....(9).}$$

$I^{Aut}{}_t$ represents autonomous investment demand for commodity 2 expressed during period t. Precisely what makes up $I^{Aut}{}_t$ is taken up below (section IV), suffice to note here that this demand will be non-capacity creating demand for new machines. This assumption allows one to identify newly installed capacity with what might be called "induced" investment demand, viz., I_{it}.[2]

Expression (9) can be rearranged given $M_{it+1}^1 = M_{it}^0$, the assumption $M_{it}^0 = I_{it-1}$ and that $D_{2t-1} = I_{1t-1} + I_{2t-1} + I_{t-1}^{Aut}$, so that

$$D_{2t} = I_t^{Aut} + \frac{I_{t-1}^{Aut}}{(1+\phi)} + D_{2t-1}.\left(\frac{D_{12t-1}.\left(1+g_{1t-1}^d\right)^2}{u_1^n.\beta_1} + \frac{\left(1+g_{2t-1}^d\right)^2}{u_2^n.\beta_2} + \frac{1}{(1+\phi)} \right) \quad \text{.....(10)}$$

where D_{12t-1}=D_{1t-1}/D_{2t-1}. In turn, by dividing through by D_{2t-1}, equation (10) can be transformed into an expression for the growth rate of demand for commodity 2:

$$g_{2t}^d = \frac{D_{12t-1}.u_2^n.\beta_2.(1+\phi)\left(1+g_{1t-1}^d\right)^2 + u_1^n.\beta_1.\left\{1+\phi+u_2^n.\beta_2.\left[ID_{2t-1}^{Aut}.a + ID_{2t-1}^{Aut}(a\phi+\phi+2) - \phi - 2\right]\right\}}{u_1^n.u_2^n.\beta_1.\beta_2.(1+\phi)}$$

.....(11)

where $ID^{Aut}{}_{2t}$ is the ratio of autonomous demand for commodity 2 to total demand for commodity 2 at *t* and *a* is the exogenously given growth rate of autonomous demand I^{Aut}.[3]

Demand for commodity 1 – consumption demand – during period t can be written as

$$D_{1t} = (1-s_w)Y_{t-1}^w + (1-s_c)P_{t-1}^c + D_{1t}^{Aut} \quad \text{.....(12)}$$

with s_w, and s_c the saving propensities of workers and capitalists respectively, $Y^w{}_{t-1}$ the income of workers and $P^c{}_{t-1}$ the profit flow to capitalists. $D^{Aut}{}_{1t}$ represents autonomous demand for commodity 1.

---

2 Effectively in the present paper autonomous demand for machines which is non-capacity creating entails the early truncation of machines (*cf.* pp.).

3 In effect, for this model, where income-expenditure multipliers exist between each commodity and each component of autonomous demand, $ID^{Aut}{}_2$ represents the "Sraffian supermultiplier" relating output of commodity 2 to autonomous demand for commodity 2.

As is well known with $s_w > 0$ some part of total profit will accrue to workers. In this case, income of workers in period *t* can be written as

$$Y_t^w = \frac{s_c.w}{s_c - s_w}\left(L_{1t} + L_{1t}\right) \qquad \text{.....(13)}$$

Here *w* is the real wage in terms of commodity 1 while the parenthetical term represents the total labour requirement for the economy in period *t*. For simplicity it is assumed for the rest of the paper that $s_w = 0$, so that total workers' income is equal to the total wages.

A first complexity which arises with the treatment of fixed capital as a joint product together with the assumption of variable efficiency of machines relates to the representation of the total labour requirement of equation (13). In effect the labour required in each industry in order to produce a given output will depend in part on the age-composition of the stock of physical capital. In other words, and bearing in mind equations (1) and (4) the total labour requirement for each sector can be written as

$$L_{it} = \beta_i\left(M_{it}^0.l_{it}^0.u_{it}^0 + M_{it}^1.l_{it}^1.u_{it}^1\right) = \left(M_{it}^0.l_{it}^0.u_{it}^0 + \frac{M_{it}^1.l_{it}^0(1+\alpha)u_{it}^0}{(1+\phi)}\right),\ i = 1,2 \qquad \text{.....(14)}$$

A second complication introduced by the joint production treatment of fixed capital relates to capitalists' consumption, via the flow of net profit. The profit available to capitalists for consumption expenditure will be influenced by the size of depreciation allowances and these are dependent on relative prices and the rate of profit. More precisely, depreciation for each of the two sectors can be written as

$$De_{it} = M_{it}^0\left(p_{21t} - p_{it}^{m11}\right) + M_{it}^1.p_{it}^{m1} \qquad i = 1,2 \qquad \text{.....(15)}$$

where p^{m1}_{it} refers to the price of one-year old machines used in sector *i* in period *t* relative to the price of commodity 1. Taking commodity 1 as the numeraire, profit available for consumption expenditure by capitalists is given by

$$P_t = \left(1 - l_{1t}^0\right)Y_{1t}^0 + \left(1 - l_{1t}^1\right)Y_{1t}^1 + \left(p_{21t} - l_{2t}^0\right)Y_{2t}^0 + \left(p_{21t} - l_{2t}^1\right)Y_{2t}^1 - \left(De_{1t} + De_{2t}\right) \qquad \text{.....(16)}$$

Taking c_p as the propensity of capitalists to consume, equations (13)-(16), together with equations (1)-(4) allow one to transform equation (12), representing the demand for commodity 1, as

$$D_t^1 = \frac{w}{(1+\phi)}.\sum_{i=1}^{2} l_i^0.u_{it-1}^0.\beta_i.\left\{M_{it-1}^0.(1+\phi)+M_{it-1}^1.(1+\alpha)\right\}+c_p.\left[A^0+A^1\right]+D_{1t}^{Aut}$$

with …..(17) [4]

$$A^0 = \sum_{i=1}^{2}.M_{it-1}^0\left\{p_{i1}.u_{it-1}^0.\beta_i - l_i^0.u_{it-1}^0.w.\beta_i - p_{i1} + p_i^{m1}\right\}$$

$$A^1 = \sum_{i=1}^{2} M_{it-1}^1\left\{\frac{p_{i1}.u_{it-1}^0.\beta_i}{(1+\phi)} - \frac{l_i^0.u_{it-1}^0.w.\beta_i.(1+\alpha)}{(1+\phi)} - p_i^{m1}\right\}$$

Prices

Before proceeding to clarify demand further, it is necessary to explicitly deal with the price system. This will allow for some simplification in the expression for consumption demand. As noted in the Introduction, it is assumed that relative prices and the real wage are given at their long-period equilibrium levels, consistent with a uniform rate of profit. With fixed capital treated as a joint product and considering the capital stocks and outputs of period *t*, evaluated at long-period equilibrium prices, one can write down the following price system:

$$\begin{aligned}
&M_{1t}^0.p_{21t}.(1+\pi_t)+w_t.l_{1t}^0.Y_{1t}^0 = Y_{1t}^0 + M_{1t}^0.p_{1t}^{m1}\\
&M_{1t}^1.p_{1t}^{m1}.(1+\pi_t)+w_t.l_{1t}^1.Y_{1t}^1 = Y_{1t}^1\\
&M_{2t}^0.p_{21t}.(1+\pi_t)+w_t.l_{2t}^0.Y_{2t}^0 = Y_{2t}^0 + M_{2t}^0.p_{2t}^{m1}\\
&M_{2t}^1.p_{2t}^{m1}.(1+\pi_t)+w_t.l_{2t}^1.Y_{2t}^1 = Y_{2t}^1.p_{21t}
\end{aligned} \quad …..(18)\ [5]$$

Taking account of equations (1) - (4), and assuming that utilization rates are at their desired (normal) level in both sectors, price equations (18) can then be rewritten as

$$\begin{aligned}
&p_{21}.(1+\pi) = u_1^n.\beta_1.\left(1-w.l_{1t}^0\right)-p_1^{m1}\\
&p_1^{m1}.(1+\pi) = u_1^n.\beta_1.\left(1-w.l_{1t}^0.(1+\alpha)\right)/(1+\phi)\\
&p_{21}.(1+\pi) = u_2^n.\beta_2.\left(p_{21}-w.l_2^0\right)-p_2^{m1}\\
&p_2^{m1}.(1+\pi) = u_2^n.\beta_2.\left(p_{21}-w.l_2^0.(1+\alpha)\right)/(1+\phi)
\end{aligned} \quad …..(19)$$

---

[4] The term p_{i1} in the expressions for A^0 and A^1 is of course equal to 1 for commodity 1 since the latter is numeraire. Additionally, since prices are assumed to be a long-period equilibrium levels (see next sub-section), the time-subscripts have been omitted.

[5] The value of the used machine is equal to the discounted profit per unit of output on the machine, where the discount rate is the rate of profit. It is also assumed here that equilibrium is maintained in the market for used machines to the extent that they exist.

where u^n refers to the desired / normal utilization rate on newly installed capacity, which, with ϕ also given, implies a "desired" utilization rate on older machines. The price system (19) provides 4 equations to solve for 3 relative prices and either the rate of profit or the real wage rate. In the present case, the rate of profit is taken as exogenous – and specifically is assumed to be governed by long-term rates of interest. Equations (19) therefore solve for the real wage rate and relative prices for a given technique of production.[6]

As noted above, these prices equations provide a means of simplifying the expression for the demand for commodity 1(equation (17)). In particular, the second term on the right-hand side of equation (17) can be simplified in light of equations (19), so that

$$D_t^1 = \frac{w}{(1+\phi)}.\sum_{i=1}^{2} l_i^0 .u_{it-1}^0 .\beta_i .\left\{M_{it-1}^0.(1+\phi)+M_{it-1}^1.(1+\alpha)\right\}+c_p.\pi.\left\{p_{21}.\left(\sum_{i=1}^{2} M_{it-1}^0\right)+\sum_{i=1}^{2} p_i^{m1} M_{it-1}^1\right\} \quad \text{.....(20)}$$

Defining the growth rate of induced investment in sector *i* between t and t-1 as

$$g_{it}^m = \frac{M_{it}^0}{M_{it-1}^0} - 1 \quad \text{.....(21)}$$

and given equations (2), (3) and (4) then the ratio of demand to new capacity installed in sector 1 for use in period *t-1*, can be written as

$$\frac{M_{1t-1}^0}{D_{1t-1}} = \frac{\left(g_{1t-1}^m + 1\right)(1+\phi)}{u_{1t-1}^0 .\beta_1 .\left(\phi.g_{1t-1}^m + g_{1t-1}^m + \phi + 2\right)} \quad \text{.....(22)}$$

Likewise one can write the following expressions

6 The obvious deficiency with such an approach is that it ignores the dependence of the desired rate of capacity utilization on relative prices and thus on the rate of profit. The more satisfying approach – not pursued here in the interest of simplicity – is one where normal utilization rates (meaning those which are implicit in the rate of profit used as a guide for investment decisions), are determined simultaneously with long-period equilibrium prices (*cf*., White, 1996 and more recently Franke (2000)).

$$\frac{M^{0}_{2t-1}}{D_{1t-1}}=\frac{1-\left(\frac{1}{\left(\phi.g^{m}_{11-1}+g^{m}_{1t-1}+\phi+2\right)}\right)}{M^{0}_{12t-1}u^{0}_{it-1}.\beta_{i}.}$$

$$\frac{M^{1}_{1t-1}}{D_{1t-1}}=\frac{(1+\phi)}{u^{0}_{it-1}.\beta_{i}.\left(\phi.g^{m}_{1t-1}+g^{m}_{1t-1}+\phi+2\right)} \qquad \text{.....(23)}$$

$$\frac{M^{1}_{2t-1}}{D_{1t-1}}=\frac{1-\left(\frac{1}{\left(\phi.g^{m}_{11-1}+g^{m}_{1t-1}+\phi+2\right)}\right)}{\left(1+g^{m}_{2t-1}\right)M^{0}_{12t-1}u^{0}_{it-1}.\beta_{i}.}$$

Making use of (22) and (23) one can transform equation (20) into an expression for the rate of growth of demand for commodity 1 between *t* and *t-1*:

$$g^{d}_{1t}=\frac{\left(1+g^{m}_{1t-1}\right)X^{d}_{1}+\left(1+g^{m}_{2t-1}\right)M^{0}_{12t-1}.c_{p}.X^{d}_{2}}{\left(1+g^{m}_{2t-1}\right)M^{0}_{12t-1}u^{0}_{it-1}.\beta_{i}.} \qquad \text{.....(24)}$$

where

$$X^{d}_{1}=\left[l^{0}_{2}.u^{0}_{2t-1}.w.\beta_{2}.(Z2+\alpha)+c_{p}.\left(g^{m}_{1t-1}\left(1+p_{21}\right)+p^{m1}_{2}\right)\pi.(1+\phi)\right]$$

$$X^{d}_{2}=\left(g^{m}_{1t-1}\left(1+p_{21}\right)+p^{m1}_{1}\right)\pi.(1+\phi)$$

$$+u^{0}_{1t-1}.\beta_{1}.(2.l^{0}_{1}.w+l^{0}_{1}.\phi.w+l^{0}_{1}.\alpha.w)+CD^{Aut}_{t-1}\left(1+g^{a}\right)Z1-\phi+g^{m}_{1t-1}.\left(l^{0}_{1}.w\right)(1+\phi)-2$$

and

$$Z1=\left(\phi.g^{m}_{1t-1}+g^{m}_{1t-1}+\phi+2\right)$$

$$Z2=\left(\phi.g^{m}_{2t-1}+g^{m}_{2t-1}+\phi+2\right)$$

while CD^{Aut}_{1t-1} refers to ratio of autonomous demand for commodity 1 to total demand for commodity 1 at *t-1*, while g^{a} is the growth rate of this autonomous demand, assumed constant.

The growth rate of induced investment for each sector – i.e. the growth rate of new capacity – can be derived on the basis of equations (8) and (21). Thus

$$g^{m}_{it}=\frac{\beta_{i}.\left(u^{n0}_{i}+u^{n1}_{i}\right)\left[D_{it-2}.\left(1+g^{d}_{it-2}\right)^{2}-M^{0}_{it-1}.u^{n1}_{i}.\beta_{i}\right]-D_{it-1}.\left(1+g^{d}_{it-1}\right)^{2}.u^{n0}_{i}.\beta_{i}}{u^{n0}_{i}.\beta_{i}.\left[M^{0}_{it-1}.u^{n1}_{i}.\beta_{i}-D_{it-2}.\left(1+g^{d}_{it-2}\right)^{2}\right]},$$

i = 1,2

so that, in view of expression (22),

$$g_{it}^{m} = \frac{\beta_i.\left(u_i^{n0}+u_i^{n1}\right)\left(1+\phi\right)\left(1+g_{it-1}^{d}\right)\left(1+g_{it-1}^{m}\right)+u_{it-1}^{0}\left(1+g_{1t-1}^{d}\right)^2.u_i^{n0}.\beta_i-\left(1+g_{t-2}^{d}\right)^2.\beta_i.\left(u_i^{n0}+u_i^{n1}\right)Z_{it-1}}{u_i^{n0}.\beta_i.\left[\left(1+g_{it-2}^{d}\right)^2 u_{it-1}^{0}.Z_{it-1}-\left(1+\phi\right)\left(1+g_{it-1}^{d}\right)\left(1+g_{it-1}^{m}\right)u_i^{n1}\right]}$$

i = 1,2 …..(25)

where

$$Z_{it-1} = \left(\phi.g_{i1-1}^{m}+g_{it-1}^{m}+\phi+2\right).$$

As noted above (p.4) u^{n0}_i represents the normal or desired utilization rate on newly installed plant. This rate implies a normal or desired rate on older plant, denoted in equation (25) as u^{n1}_i.

Completing the model requires modeling the behavior of utilization rates, the ratio of investments of the two sectors (M^0_{12}) and the ratios of autonomous demand to total demand ($ID^{Aut}$and CD^{Aut}).

Considering utilization rates first, equations (2), (3) and (4) imply that for sector i

$$Y_{it} - M_{it-1}^{0}.u_{it}^{0}.\beta_i = \frac{M_{it-1}^{0}.u_{it}^{0}.\beta_i}{1+\phi} \qquad i = 1,2$$

and therefore that

$$u_{it}^{0} = \frac{Y_{it}.\left(1+\phi\right)}{\beta_i.\left(M_{it}^{0}.\left(1+\phi\right)+M_{it-1}^{0}\right)} = \frac{D_{it}.\left(1+\phi\right)}{\beta_i.\left(I_{it-1}^{0}.\left(1+\phi\right)+M_{it-1}^{0}\right)} \qquad i = 1,2 \quad …..(26).$$

In view equation (8) for induced investment,

$$u_{it}^{0} = \frac{D_{it}.u_i^{n0}}{D_{it-2}.\left(1+g_{it-2}^{d}\right)^2} = \frac{\left(1+g_{it}^{d}\right)\left(1+g_{it-1}^{d}\right)u_i^{n0}}{\left(1+g_{it-2}^{d}\right)^2} \qquad i = 1,2 \quad …..(27)$$

Substituting expressions (24) and (11) for period *t* rate of growth of demand for commodities 1 and 2 respectively in the corresponding version of equation (27) allows one to express utilization on newly installed plant in each sector as a function of variables in *t-1* and *t-2*.

Regarding the ratio of induced investment of sector 1 relative to sector 2 at time *t*, M^0_{12t}, this can be expressed as

$$M_{12t}^{0} = \frac{\left(1+g_{1t}^{m}\right)M_{12t-1}^{0}}{1+g_{2t}^{m}} \qquad …..(28)$$

Substituting the corresponding version of expression (25) for g_{1t} and g_{2t} respectively will yield an expression for M^0_{12t} as a function of growth rates of both demands in *t-1* and *t-2*, growth rates of capacity in *t-1* and utilization rates in *t-1*.

Finally, regarding the ratio of autonomous demand to total demand for each sector, it is assumed for simplicity that both autonomous demands grow at a uniform rate, so that the ratio of the two autonomous demands is constant. The relation between the proportions of autonomous in total demand for the two sectors can be written as

$$CD_t^{Aut} = \frac{ID_t^{Aut}.\mu}{D_{12t}} \qquad \text{…..(29)}$$

where μ is the ratio of the two autonomous demands and D_{12t} is the ratio of total demands for the two sectors at time t. Equations (22) and (23) allow for a relation between D_{12t} and the ratio of the new capacities in the two sectors:

$$D_{12t} = \frac{\left(g_{2t}^m + 1\right)M_{12t}^0.u_{1t}^0.\beta_1.\left(\phi.g_{1t}^m + g_{1t}^m + \phi + 2\right)}{\left(g_{1t}^m + 1\right)u_{2t}^0.\beta_2.\left(\phi.g_{2t}^m + g_{2t}^m + \phi + 2\right)} \qquad \text{…..(30)}$$

In effect, equations (29) and (30) allow us to eliminate CD^{Aut}_{t-1} from the expressions for g^d_{1t} and u^0_{1t} by substituting (in equations (24) and (27)) with a term in ID^{Aut}_{t-1}, μ, M^0_{12t-1}, and utilization and capacity growth rates for $t-1$. There remains to express the time path of ID^{Aut}:

$$ID_t^{Aut} = \frac{ID_{t-1}^{Aut}.(1+g^a)}{(1+g_{2t}^d)} \qquad \text{…...(31)}$$

Substituting from equation (30) to eliminate D_{12t-1} in expression (11) and in turn substituting for g^d_{2t} in equation (31) provides for a relation between ID^{Aut}_t and growth and utilization rates in $t-1$.

III Equilibrium: endogenous or exogenous growth?

The model outlined above essentially provides a system of equations describing the time path of eight variables: two growth rates of demand (g^d_{1t}, g^d_{2t}), two growth rates of new capacity (g^m_{1t}, g^m_{2t}), two utilization rates on newly installed machines (u^0_{1t}, u^0_{2t}), the ratio of induced investments in the two sectors (M^0_{12t}) and the ratio of autonomous investment demand to total investment demand (ID^{Aut}_t). More precisely, defining the following two additional variables

$$g_{it}^L = g_{it-1}^d \quad i = 1,2 \qquad \text{…..(32)}$$

one can substitute g^L_{it-1} for g^d_{it-2} in the expressions above. This then allows the model to be written as

$$x_t = f\left(x_{t-1}\right) \qquad \text{…..(33)}$$

where x is the vector

$$x = \left(g_1^d, g_2^d, g_1^m, g_2^m, M_{12}^0, u_1^0, u_2^0, ID^{Aut}, g_1^L, g_2^L\right)$$

An equilibrium (fixed point) of the recursive system (32) is a situation characterised by $g_{it}^d = g_{it-1}^d = g_{it}^m = g_{it-1}^m = g*$ for $\forall i$ and $u_{it} = u_i^n$, for $\forall i$.

However, the existence of autonomous demand introduces a considerable complication in thinking about equilibria for system (33). In fact one of two distinct equilibria seem possible and these two possibilities correspond to two cases arising in recent literature on a long-run Keynesian approach to growth and autonomous demand. The first possibility is that the rate of growth of autonomous demand is sufficiently low relative to "induced" growth of the economy that the ratio of autonomous demand to total demand declines over time ultimately reaching zero. This appears to be the approach of Park (2000). In this case, the equilibrium growth rate is *endogenous*. Equations (11) and (24) (after taking account of equation (30)) effectively provide two expressions in $g*$ and the relative size of the two sectors given by D_{12t}.

The alternative approach, which seems to underlie recent arguments by Trezzini (1995, 1998) and which is adopted here, does not place the same restriction on the rate of growth of autonomous demand. Here the argument is that the equilibrium rate of growth would be determined by the exogenous rate of growth of autonomous demand.[7] We then have what could be appropriately termed "*exogenous growth*". However, the system (33) in this case determines endogenously the ratio of autonomous demand to total demand for each sector, along with the relative size of the two sectors.[8]

It should be added however that although the second possibility seems more appropriate for the analysis of systems with autonomous demand, insofar as it does not seek to arbitrarily fix the value of a key exogenous variable, it does implicitly assume something about the possibility of equilibrium growth rates in excess of the rate of growth of autonomous demand. In particular it would seem to imply either that no such equilibria exist or that the economy finds itself in a position growing at a rate lower than that required in such an equilibrium but higher than the growth rate of autonomous demand and where the former is an unstable equilibrium.

Therefore, although in this paper the discussion of stability is limited to an equilibrium where growth is equal to the rate of growth of autonomous demand, a more general treatment of stability

---

7 It should be noted however that Trezzini's main point concerns the difficulty in the economic system adjusting to changes in the growth rate of autonomous demand in such a way as to restore normal (desired) capacity utilization.

8 In other words, setting all growth rates equal to g^a, equations (11) and (24) would determine ID^{Aut} and D_{12} and equation (29) would determine CD^{Aut} for a given μ.

needs to consider the stability of multiple equilibria in the case where the rate of growth of autonomous demand is comparatively low.[9]

VI Autonomous demand and technical progress

Before proceeding to discuss stability, some comments are required regarding the nature of autonomous demand, in particular autonomous demand for the capital good, specifically non-capacity creating autonomous demand. In the context of the present model, such additional demand for the capital good, other than from net export or government sources, would presumably arise as a result of technical progress. Yet, as has been noted elsewhere (e.g. Caminati, 1986), such demand effects of technical progress cannot be taken for granted. Certainly if one is seeking to be consistent with a joint production treatment of fixed capital, such a stimulus to demand without capacity effects would seem to require the early truncation of fixed capital. In other words, non-capacity creating investment demand would involve substitution of newer machines for older machines without an enlargment of output capacity, but a speeding up of investment demand used machines are replaced "earlier".

In the context of the present model this process would conceivably take the form of a purchase of new machines to replace not only two-year old machines due for replacement at the end of period t, but also replacement of machines which are only one-year old at the end of t by machines embodying newer technology. The quantity of new machines purchased would be determined by the capacity of the newer machines, the cost-minimizing average utilization rate of newer machines, and the expected demand for output. If demand for output is anticipated to grow at the same rate as in the preceding period, investment would nonetheless show a faster growth rate compared with the earlier period, since "replacement investment" will be larger.

More significantly, this investment demand effect, to the extent that it does entail the early truncation of older machines, thereby requires such changes in technical coefficients as would generate negative prices for used machines which either embodied or were combined with labour in production using the old technology (*cf.*, *ibid.*, and also Levrini, 1988). In other words, there are constraints on technology, which would have to be satisfied in order that the above-mentioned stimulus to demand accompanies technical change.

For the present model, for example these constraints may be illustrated for the simplest case of technical change – where less labour is required in the production of a given output in both sectors and

---

9 That multiple equilibria exist is implied by the fact that with the present model the case where the ratio of autonomous demands to total demand approaches zero, yields a quartic equation, the roots of which are equilibrium growth rates.

for both new and older machines. From the set of price equations (19) it is possible to express the price of used machines in the two sectors as

$$p_1^{m1} = \frac{u_1^n.\beta_1.\left(1 - w.l_{1t}^0.(1+\alpha)\right)/(1+\phi)}{(1+\pi)}$$

$$p_2^{m1} = \frac{u_2^n.\beta_2.\left(p_{21} - w.l_2^0.(1+\alpha)\right)/(1+\phi)}{(1+\pi)} \qquad \text{……(19a)}$$

It is useful also to derive each of the relative prices and the real wage in terms of technical coefficients and the rate of profit. From the set of price equations (19) it is possible to express the three relatives prices and the real wage – assuming a profit rate calculated on the basis of normal capacity utilization in each sector set at 100% - in the following form:

$$p_{21} = \frac{l_2^0.\beta_1.\beta_2.Q}{l_2^0.\beta_2\,(1+\pi)(Q-1) - l_1^0\beta_1\left[(1+\pi)(Q-1) - \beta_2.Q\right]}$$

$$w = \frac{\beta_1.Q\left[(1+\pi)(Q-1) - \beta_2.Q\right]}{(Q+\alpha)\left\{l_1^0.\beta_1\left[(1+\pi)(Q-1) - \beta_2.Q\right] - l_2^0\beta_2\,(1+\pi)(Q-1)\right\}} \qquad \text{…..(19b)}$$

$$p_1^{m1} = \frac{\beta_1.\left\{l_2^0.(1+\pi)\beta_2.(Q+\alpha) + \left[(1+\pi)(Q+1) - \beta_2.Q\right]l_1^0.\alpha.\beta_1\right]}{(Q+\alpha)\left\{l_2^0.\beta_2\,(1+\pi)(Q-1) - l_1^0\beta_1\left[(1+\pi)(Q-1) - \beta_2.Q\right]\right]}$$

$$p_2^{m1} = \frac{l_2^0.\beta_1.(\alpha.(1+\pi-\beta 2) + 1 + \pi)\beta 2.Q}{(Q+\alpha)\left\{l_2^0.\beta_2\,(1+\pi)(Q-1) - l_1^0\beta_1\left[(1+\pi)(Q-1) - \beta_2.Q\right]\right]}$$

where $Q = (\pi\phi + \phi + \pi + 2)$.

The constraints on technology implied by the early truncation of fixed capital used with older technologies can be expressed as the conditions under which the prices of one-year old machines used with the old technology (i.e. the old labour coefficients) are negative when calculated using the prices of new machines with the new technology and the real wage rate generated by the new technology. Thus, combining the expressions for w and p_{21} in (19b) with the expressions (19a) the conditions in question can be written as

$$\frac{l_{1old}^0\,Q.\beta_1\left\{(1+\pi)(Q-1) - Q.\beta_2\right\}}{(Q+\alpha)\left\{l_{1new}^0.\beta_1\left[(1+\pi)(Q-1) - Q.\beta_2\right] - l_{2new}^0.(1+\pi).(Q-1).\beta_2\right\}} - \frac{1}{1+\alpha} > 0$$

$$\frac{Q.\beta_1\beta_2.\left\{l_{2new}^0\,(Q+\alpha).\beta_2 + l_{2old}^0.(1+\alpha)\left[(1+\pi)(Q-1) - Q.\beta_2\right]\right\}}{(1+\pi).(1+\alpha)(Q+\alpha)\left\{l_{2new}^0.\beta_2\,(1+\pi)(Q-1) - l_{1old}^0.\beta_1\left[(1+\pi)(Q-1) - \beta_2.Q\right]\right\}} < 0 \qquad \text{…..(34)}$$

where the subscripts old and new refer to the labour coefficients associated with the old and new technologies; and where the two inequalities (34) represent the conditions under which the price of one-year old machines for sectors 1 and 2 respectively used with the old technology are negative at the prices of the new technology.

Three points are worth noting in relation to these conditions. First, since the relation between the old and new labour coefficients is presumably exogenous, along with the rate of profit (by assumption) then the inequalities (34) represent two constraints on technology which need to be met in order for it to be profitable for producers to undertake additional investment aside from that associated with expanding capacity in line with expected demand growth. Second, an analysis of growth which includes an important role for autonomous investment demand in determining the economy's long-run growth path must at ultimately demonstrate consistency between the conditions required for the stability of that growth path and the constraints on technology implied by investment demand stimulated by technical change. Third, it is worth recalling the assumption underlying the present analysis that relative prices are at their long-period equilibrium levels. As has been pointed out elsewhere (Salanti, 1985), not all forms of technical change can be considered profitable "whatever the price system ruling"(p.115). A more general analysis would seek to consider technical choice in terms of producers evaluating technical choices at market prices, as distinct from long-period equilibrium prices. Indeed, this raises the wider issue of how to consider technical choice in the context of a process of gravitation of prices around long-period levels.[10]

V Disequilibrium and local stability

The following tentative discussion of stability is limited to some observations about the local stability of equilibria characterized by normal capacity utilization in both sectors and uniform steady growth at a rate equal to the assumed uniform rate of growth of autonomous demands. This involves constructing the Jacobian matrix for difference equation system (33) and evaluating its components at equilibrium. Stability in the vicinity of equilibrium requires that the eigenvalues of the Jacobian are less than unity in absolute value.

The Jacobian matrix, *J*, is presented on the following page. For the sake of simplicity the normal utilization rate on newly installed capacity, u^{n0} is assumed to be 100% for both sectors. The uniform growth rate of autonomous demands is denoted as *a*, the steady state ratio of newly installed capacity

[10] An additional point worth making is that effectively the discussion here is limited to the effects on demand of a "once-over" change in technology. The situation with regard to the demand effects of technical progress is considerably more complex when one seeks to consider persistent technical progress and whether this brings with it persistent demand effects (*cf*., Caminati, 1986).

in the two sectors, (M^0_{12} in the discussion so far) is denoted as k, and the steady state ratio of autonomous demand for commodity 2 to total demand for commodity 2 (ID^{Aut}) is denoted as A^D.

Since the Jacobian is 10 x10 and some elements are unambiguously negative[11], it becomes difficult to establish the whether *sufficient* conditions for all of its eigenvalues to be less than unity are met. Following the discussion of necessary and sufficient conditions in in Gandolfo (1980, pp. 136-39), two means of establishing sufficient conditions where the elements of J are arbitrary, are available. The first involves establishing that all leading principal minors of the matrix $[I - J^+]$ are positive, where I is the identity matrix and the elements of $J+$ are the absolute values of the corresponding elements of J (Gandolfo, 1980, pp.136-39). This in turn involves dealing with polynomials of the 5th degree and higher.

The alternative procedure is to consider the sum of the absolute values of elements in each column of J. A set of sufficient stability conditions is that the (n = 10) sums be less than unity in absolute value. Clearly this condition would not be satisfied for columns 1, 2 and 5 of J.

In the remainder of this section the discussion is limited to some thoughts on necessary conditions for the eigenvalues of J to be less than unity in absolute value. One of these is the condition that the determinant of J be less than unity in absolute value. Again this involves the solution to a rather difficult polynomial. One other necessary condition which is somewhat easier to deal with is that

$$\left|\sum_{i=1}^{n} j_{ii}\right| < n \qquad \text{.....(35)}$$

where the j_{ii} represent the elements along the main diagonal of J and n is the order of J. This condition is clearly met if all j_{ii} are less than unity. Given that a and ϕ are both positive and therefore that Z-1 > 1, the first, third and fourth elements on the main diagonal will be less than unity.

The second element on the main diagonal will be less than unity provided that $\beta 2 > 2G$. It is worth noting that the condition $\beta 2 > 2G$ in fact corresponds to one of the abovementioned sufficient conditions for the eigenvalues of J to be less than unity in absolute value. Specifically the condition that the second principal minor of the matrix $[I - J^+]$ be positive implies that $1-(2G/\beta 2) > 0$. This condition implies a constraint on technologies consistent with economically feasible outcomes of the model, since although G is exogenous, reflecting the growth rate of autonomous demand, the set of

---

11 That is, $Z > 1$, while G, $\beta 2 > 0$.

feasible (positive ratios of outputs and of autonomous demands to total demands) will imply some restriction on technologies.

Assuming that the seventh element on the main diagonal is also less than one places a further restriction on feasible technologies. Together with the condition that $\beta 2 > 2G$, the condition that $Gk/\beta_2<1$sets an upper limit on $\beta 2$.More precisely, *k*, represents the relative size of new capacities in each sector, and, given β1 and β2, will, in the steady state, reflect the relative size of the outputs in each sector. *k* therefore depends in a complex way on the technology of the system, both directly and indirectly through the price system.

The fifth element on the main diagonal is equal to one and the last two elements are each zero. For the sum of the elements on the main diagonal to be less than *n*, and assuming the second and seventh elements are less than 1, it is sufficient to for the remaining two elements, *J66* and *J88* to be less than unity.

Taking first the condition that J88<1, little in general can be said about it being satisfied. Under reasonable assumptions about the growth rate *a* and the parameter ϕ and given the condition above $\beta 2 > 2G$, the numerator of J88 will be negative with *G* and *Z* - 1 both slightly larger than 1 and a sufficiently small *k*. However, a small *k* relative to β2 may render *S* negative so that J88 < 1 is not guaranteed. Satisfying this condition in the absence of specific values for technical coefficients remains ambiguous.

Turning to the condition J66 < 1, this effectively entails a restriction on the size of autonomous demand for commodity 1 relative to total demand for commodity 1. Specifically, J66 < 1 can be written as the condition that

$$A^D.G.Z.\beta 2.\mu + R + (Z+\alpha)\omega 2 < G.k.Z.\beta 1 \qquad \ldots..(36)$$

Bearing in mind that A^D refers to the ratio of autonomous demand for commodity 2 to total demand for commodity 2; that β2 refers to the output capacity of a machine in sector 2; that μ refers to the ratio of autonomous demands for the two commodities; and that output (equal to demand) in the steady state is assumed to be at full capacity, then equation (36) can be rewritten as

$$\frac{D_2^{Aut}}{D_2}\cdot\frac{Y_2}{M_2^0+M_2^1}\cdot\frac{D_1^{Aut}}{D_2^{Aut}}.G.Z + R + (Z+\alpha)\omega 2 < G.k.Z.\beta 1$$

Bearing in mind also equation (30) which gives the relation between k and the ratio of the two demands, D_{12} (and setting growth rates equal), the RHS of inequality (35) can be rewritten so that

$$\frac{D_2^{Aut}}{D_2}.\frac{Y_2}{M_2^0+M_2^1}.\frac{D_1^{Aut}}{D_2^{Aut}}.G.Z+R+(Z+\alpha).\omega 2<G.Z.\frac{D_1}{D_2}.\frac{\beta 2}{\beta 1}.\beta 1$$

$$\left(=G.Z.\frac{D_1}{D_2}.-\frac{Y_2}{M_2^0+M_2^1}.\frac{M_1^0+M_1^1}{Y_1}.\frac{Y_1}{M_1^0+M_1^1}\right)$$

and

$$R+(Z+\alpha).\omega 2<\frac{G.Z.}{M_2^0+M_2^1}\left(D_1-D_1^{Aut}\right) \qquad \text{.....(37)}$$

Interestingly, inequality (37) implies that a necessary condition for stability is linked to the relative size of the induced component of demand for commodity 1. Effectively this condition amounts to a lower limit on the size of the “Sraffian supermultiplier” for sector 1. Interesting also is the fact that this lower limit is determined by some of the same factors which would govern the size of this multiplier, viz., relative prices and the distribution of income (via R and ω2).

VI Concluding remarks

The preceding discussion is an attempt to clarify some of the issues arising out of recent debate about the contours of a revitalized Keynesian approach to long-run growth. A simplified fixed capital model has been constructed as a means of considering the relation between long-run growth and the existence of autonomous demand. The model allows for two alternative “closures”: one corresponding to exogenous growth – where the warranted growth path adapts itself to the growth rate of autonomous demand; one corresponding to endogenous growth – where the warranted growth path, driven by the multiplier and investment geared to expected growth in demand – dominates autonomous components of demand. In the present case, the choice between these two closures really requires a more exhaustive analysis of the multiple equilibria associated with the second type of closure.

The discussion above also highlights the need to consider consistency between the constraints on technology which are implied by conditions necessary for local stability on the one hand, and on the other, the constraints on technology required for autonomous demands associated with technical progress.

References

Caminati M.(1986), “Technical innovations, life of equipment and effective demand”, *Political Economy: Studies in the Surplus Approach*, Vol. 2, No.2.

Franke R. (2000), “Optimal utilization of capital and a financial sector in a classical gravitation process”, *Metroeconomica*, Vol. 51, No. 1.

Gandolfo G. (1980), *Economic Dynamics: Methods and Models*. Netherlands: North-Holland.

Levrini E. (1988), “Joint production: Review of some studies on Sraffa’s system”, *Political Economy: Studies in the Surplus Approach*, Vol. 4, No.2.

Park M.S. (2000), “Autonomous demand and the warranted rate of growth”, *Contributions to Political Economy*, Vol. 19.

Salanti A.(1985), “Prices of production, market prices and the analysis of the choice of techniques”, *Metroeconomica*, Vol. 37, February.

Serrano F. (1985), “Long period effective demand and the Sraffian Supermultiplier”, *Contributions to Political Economy*, Vol. 14.

Trezzini, A. (1995), “Capacity utilization in the long-run and the autonomous components of aggregate demand”, *Contributions to Political Economy*, Vol. 14.

------------- (1998), “Capacity utilization in the long-run: some further considerations”, *Contributions to Political Economy*, Vol. 17.

White G. (1996), “Capacity utilization, investment and normal prices: some issues in the Sraffa-Keynes synthesis”, *Metroeconomica*, Vol. 47, No. 3, October.