On the Mechanics of Economic Development and Non–Development

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Abstract

Young adults observe wage rates, interest rates, and child mortality and decide about savings and the quantity and quality of their children. Expenditure on child quality causes human capital accumulation as an external effect. If mortality is high parents prefer to have many children and spend only essential rearing effort. Without human capital accumulation the economy may stabilize in an equilibrium of economic stagnation and high population growth. If mortality is low parents prefer to have only few children and spend comparatively large fractions of income on their quality. With human capital accumulation the economy is capable of long-run growth. The paper also shows the possibility of an endogenously explained demographic transition and discusses a development program on education.

Keywords: Demographic Transition, Stages of Development, Economic Growth

JEL: J10, J13, O11, O12

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1. Introduction

In the wake of New Growth Theory models have been developed which use the parental decision to invest in human capital of their children as an explanation for long-run economic growth as well as an explanation for economic stagnation. The centerpiece of the approach originates from Becker's (1960) theory on fertility and child quality: Young adults have to decide how many children they would like to have and how much effort they would like to spend on each child. Parents decide to rear only a few children and spend much effort on each of them when economic circumstances are favorable and to have more children with less effort spent on each of them otherwise. Under the assumption that parental effort, which may be measured in time or money or both, expand human capital, the decision in favor of a few well educated children can produce economic growth whereas the decision to have many children can produce stagnation. The states of stagnation and perpetual growth are connected by a path of demo-economic transition along which fertility decreases and economic development takes off.

Prominent contributions to this approach are by Becker, Murphy and Tamura (1990), Ehrlich and Lui (1991) and Tamura (1996).[1] The main difference of this paper is that economic stagnation as well as long-run growth are generated as external effects. Young adults do not foresee whether their individual choice of number of children and effort spent on them will produce stagnation or growth on the macroeconomic level. They observe the interest rate, wages, and the probability in their area or country that a child survives up to adulthood. Assuming that their children will face the same conditions as adults they decide about savings, number of children, and expenditure on each child.

A recent model that has much in common with the current paper is by Galor and Weil (2000). There it is also assumed that young adults do not take into account the impact of their decision about quantity and quality of children on the rate of technological progress. Galor and Weil, however, focus on the historic development path of today's well-developed countries. Consequently, their economy always converges towards steady growth. Technically, this feature is caused by the assumption that the rate of technological progress is increasing in population size. The current paper is more concerned to explain different economic performances in different parts of the world today. I assume that technological progress does not

[1]See Nerlove and Raut (1997), Robinson and Srinivasan (1997), or Tamura (2000) for a survey of the literature.

depend on population size and only indirectly and negatively on population growth. Consequently, besides long-run growth there exists a second equilibrium of long-run stagnation. A further important distinction is that the current paper emphasizes the role of mortality and the interplay of old-age support and altruism in the family planing decision. This feature in turn is shared with the approach of Ehrlich and Lui (1991).

2. Decision Problem of the Young Adult

Life is separated into three periods, childhood, young adulthood, and old–age. A young adult receives a working income w. At the beginning of young adulthood he plans consumption during this period, c_1, consumption in old–age, c_2, the number of children, n, and the expenditure per child Q. I do not consider gender differences. Every single young adult can have children. Rearing a child requires a fixed fraction of income, e, $0 < e < 1$. Furthermore, the parent may voluntarily increase expenditure per child by the amount qw.

As Srinivasan (1988) and Ehrlich and Lui (1991) I consider the implication of old-age support when survival is uncertain. I assume that spending the income fraction e is all a parent can (and will) do to rear up a healthy child. Nevertheless the child may die for reasons that are not under control of the parent. Expected probability for a child to survive to young adulthood is denoted by π. For simplicity, survival to the stage of old age is certain once the stage of young adulthood is reached. The young adult has to pay d units of his income, $0 < d < 1$ to old-age support of his parent and expects own old-age support of dw from each of his surviving children.[2]

A positive demand of children can be ensured by assuming that either no capital market is accessible or that the net return on children exceeds the net return on investment on the capital market. In either case it is trivial to show that it is optimal to finance old-age consumption by rearing children. The empirical literature, however, suggests that although the old-age security motive matters in child demand the net return on children is negligible or even negative.[3] The interesting question therefore is why people decide to extend their family size when child mortality decreases even though the net return on children is negative.

[2] A more complex model with uncertain survival to old-age generates qualitatively the same results. Modelling of old age support follows Ehrlich and Lui (1991) by assuming implicit contracts of extended families. This implies that old age support does not involve uncertainty: The parent can expect old age support πdwn for sure. Such an implicit contract is credible and time consistent if cheating young adults receive nothing from their children when old, see Tamura (2000).

[3] See e.g. Willis (1980) and Nugent (1985).

To provide an answer I assume that the parent can alternatively finance old-age consumption by savings with interest rate r and that the net monetary return from of a child, $\pi d - e(1+r)$, is negative.

An assumption that turns out to be crucial is that parents experience higher utility if more children survive up to adulthood. Utility from child quantity is therefore weighted by survival probability. This approach may indirectly also include disutility from dying children. Note, however, that I do not assume that only surviving children provide utility for the parent. In that case one would have to consider a more complex expression for child quantity in an expected utility function.[4]

At all stages of development young adults solve the following maximization problem:

$$\max_{s,n,q} U = b_1 \ln c_1 + b_2 \ln c_2 + \pi b_3 \ln n + b_4 \ln Q \quad , \tag{1}$$

$$c_1 = (1 - d - en - s - qn)w \quad , \tag{2}$$

$$c_2 = ((1+r)s + \pi n d)w \quad . \tag{3}$$

$$(a)\ \ q \geq 0 \quad , \qquad (b)\ \ n \geq 1/\pi \quad . \tag{4}$$

Control variables are the savings rate s, the number of children, n, and the fraction of income voluntarily spent on child quality, q. The notion of the young adult as being the average or representative young adult in an economy explains the omission of an integer constraint on n. It may be the case that the average adult decides to rear less than one surviving child. From a macroeconomic viewpoint this results in convergence of population size towards zero and the problem at hand disappears. Hence, I follow Ehrlich and Lui (1991) and assume that parents decide to have at least one surviving child.

Because individuals are facing two inequality constraints they generate three different solutions to the problem, labelled Lifestyle A, B, and C. Consider first Lifestyle A, which is the solution of the maximization problem when (4a) is binding with equality so that parents do not voluntarily spend income for child quality. Lifestyle A is summarized by the set of equations in (5) and (6).

[4] Sah (1991) considers utility from surviving children assuming a binomial distribution for child survival. Cigno (1998) assumes a general density function for child survival and discusses the case where parents can influence survival by spending more money on child care. Similar to the current paper he arrives at the result that mortality and fertility are negatively correlated when mortality is high and positively correlated at low levels of mortality.

LIFESTYLE A

$$
\begin{aligned}
n &= \frac{b_3\pi(1-d)(1+r)}{(b_1+b_2+b_3\pi)\left[e(1+r)-d\pi\right]} \ , \\
q &= 0 \ , \\
s &= \frac{(1-d)\left\{\left[b_2(e(1+r)-d\pi)\right]-b_3 d\pi^2\right\}}{(b_1+b_2+b_3\pi)\left[e(1+r)-d\pi\right]} \ , \\
c_1 &= \frac{b_1(1-d)w}{b_1+b_2+b_3\pi} \ .
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\frac{\partial n}{\partial \pi} &= \frac{b_3(1-d)(1+r)\left[(b_1+b_2)e(1+r)+b_3 d\pi^2\right]}{(b_1+b_2+b_3\pi)^2\left[e(1+r)-d\pi\right]^2} > 0 \ , \\
\frac{\partial n}{\partial r} &= \frac{b_3 d\pi^2(1-d)}{(b_1+b_2+b_3\pi)\left[e(1+r)-d\pi\right]^2} > 0 \ , \\
\frac{\partial s}{\partial r} &= \frac{b_3(1-d)de\pi^2}{(b_1+b_2+b_3\pi)\left[e(1+r)-d\pi\right]^2} > 0 \ , \\
\frac{\partial s}{\partial \pi} &= -\frac{b_1 d\pi(2e(1+r)-d\pi)+b_2 e^2(1+r+r^2)+b_3 de\pi^2(1+r)}{(b_1+b_2+b_3\pi)^2\left[e(1+r)-d\pi\right]^2} < 0 \ ,
\end{aligned}
\tag{6}
$$

The crucial outcome of Lifestyle A is reflected by the derivatives with respect to π. If survival probability increases parents decide to have *more* children and to save *less*. Because the net monetary return on children is less negative if more children survive and having children raises utility whereas having savings does not, parents partly substitute savings for children and thereby extend their family.

Consider now the case where neither the constraint on q nor the constraint on n is binding with equality. Parents then optimally select Lifestyle B as depicted in (7) and (8). For the existence of Lifestyle B the utility elasticity of child quality must be sufficiently large so that $b_4 > \pi b_3$. When survival probability is sufficiently low q according to (7) is negative and parents prefer Lifestyle A over Lifestyle B. The positive derivative $\partial q/\partial\pi$ shows that the possibility that individuals decide for Lifestyle B rather than A increases with increasing survival probability. Moreover, if π improves Lifestyle B parents want to have *less* children and spend *more* on their quality.

Lifestyle B

$$
(7) \qquad
\begin{aligned}
n &= \frac{(1-d)(b_4 - b_3\pi)(1+r)}{d\pi(b_1+b_2+b_3\pi)} \ , \\
q &= \frac{b_3\pi e(1+r) - b_4\left[e(1+r) - d\pi\right]}{(b_4 - b_3\pi)(1+r)} \ , \\
s &= \frac{(1-d)(b_2+b_3\pi-b_4)}{b_1+b_2+b_3\pi} \ , \\
c_1 &= \frac{b_1(1-d)w}{b_1+b_2+b_3\pi} \ .
\end{aligned}
$$

$$
(8) \qquad
\begin{aligned}
\frac{\partial n}{\partial \pi} &= -\frac{(1-d)(1+r)\left\{[b_4(b_1+b_2+2\pi b_3] - b_3^2\pi^2\right\}}{d\pi^2(b_1+b_2+b_3\pi)^2} < 0 \ , \\
\frac{\partial n}{\partial r} &= \frac{(b_4-b_3\pi)(1-d)}{d\pi(b_1+b_2+b_3\pi)} > 0 \ , \\
\frac{\partial q}{\partial \pi} &= \frac{b_4^2 d}{(b_4-b_3\pi)^2(1+r)} > 0 \ , \\
\frac{\partial q}{\partial r} &= -\frac{b_4 d\pi}{(b_4-b_3\pi)(1+r)^2} < 0 \ , \\
\frac{\partial s}{\partial \pi} &= \frac{(1-d)b_3(b_1+b_4)}{(b_1+b_2+b_3\pi)^2} > 0 \ ,
\end{aligned}
$$

Lifestyle C

$$
(9) \qquad
\begin{aligned}
n &= 1/\pi \ , \\
q &= \frac{\{(b_4\left[d\pi - e(1+r)\right] + b_4(1+r-d) - (b_1+b_2)e(1+r)\}}{(b_1+b_2+b_4)(1+r)} \ , \\
s &= \frac{b_2(1-d)(1+r) - (b_1+b_4)d\pi}{b_1+b_2+b_4} \ .
\end{aligned}
$$

$$
(10) \qquad \frac{\partial q}{\partial r} = -\frac{b_4 d\pi}{(b_1+b_2+b_4)(1+r)^2} < 0 \ ,
$$

When the (4b) binds with equality parents select Lifestyle C given by (9) and (10).[5]

Voluntary expenditure on child quality entails a positive external effect. It increases the child's productivity. In particular, I assume that each young adult supplies one unit of raw labor, that children inherit $(1-\delta)$ of their parents human capital, and that any additional

[5] In all cases I ignore the uninteresting non- negativity constraint on savings by assuming parameters b_i that ensure positive savings at all stages.

parental expenditure increases the child's human capital, so that the dynastic path of human capital per capita is given by

$$h_{t+1} = \max\{1, (1-\delta)h_t + f(q_t w_t, h_t)\} \quad , 0 \le \delta \le 1 \quad , h_0 = 1 \quad . \tag{11}$$

In a macroeconomic context one can interpret f as a schooling function, where the skills produced depend on parental expenditure and on the skill–level of teachers. The schooling function has a simple Cobb–Douglas form, $f(qw, h) = B(qw)^{\phi} h^{1-\phi}$. With z denoting wages per unit of human capital, $z \equiv w/h$, the dynastic growth rate of human capital per worker is obtained as

$$g_h = \begin{cases} B(zq)^{\phi} - \delta & \text{for } h > 1 \\ \max\left\{0, B(zq)^{\phi} - \delta\right\} & \text{for } h = 1 \end{cases} \quad , \text{ where } \quad 0 < \phi < 1 \quad . \tag{12}$$

In contrast to Lucas (1988), human capital production is allowed to be subject to decreasing returns with respect to its own stock. Linearity in the stock can be avoided because child expenditure is not bounded from above as it is the case for schooling time. If parents of successive generations are willing to spend a constant fraction of their earnings per unit of human capital on child quality, h will grow at a constant positive rate. In other words, to maintain a constant g_h total expenditure per child must increase with the wage rate.

Quantitatively, a dynasty grows at rate

$$g_L = \pi n(\pi) - 1, \tag{13}$$

which is increasing in π for Lifestyle A dynasties and decreasing for Lifestyle B dynasties.

Following the OLG literature young adults are assumed to work with a capital stock that has been accumulated by their parents. Hence, the dynastic capital labor ratio is $k_{t+1} = s_t w_t L_t / L_{t+1} = s_t w_t / (n_t \pi_t)$. Let wages and interest rates be defined as $w := (1-\alpha)Y/L$, $r := \alpha Y/K$ (These expressions are derived in the next section). Inserting factor shares and rents, and s and n from the different lifestyles, growth rate of the capital labor ratio is obtained as shown in equations (14) – (16). The important feature for economic development and stagnation is that g_k increases with improving survival probability in Lifestyle B- and C-economies but decreases in a Lifestyle A-economy.

Lifestyle A

(14)

$$g_k = \frac{(1-\alpha)r\left[b_2(e(1+r)-d\pi)-b_3d\pi^2\right]}{\alpha b_3\pi^2(1+r)} - 1 \ ,$$

$$\frac{\partial g_k}{\partial \pi} = \frac{(1-\alpha)rb_2\left[2e(1+r)-d\pi\right]}{\alpha b_3\pi^3(1+r)} < 0 \ , \ \frac{\partial g_k}{\partial r} = \frac{(1-\alpha)\left[b_2(e(1+r)^2-d\pi)-b_3d\pi^2\right]}{\alpha b_3\pi^2(1+r)^2} > 0 \ .$$

Lifestyle B

(15)

$$g_k = \frac{(1-\alpha)dr\left[b_2+b_3\pi-b_4\right]}{\alpha(b_4-b_3\pi)(1+r)} - 1 \ ,$$

$$\frac{\partial g_k}{\partial \pi} = \frac{(1-\alpha)b_2b_3dr}{\alpha(b_4-b_3\pi)^2(1+r)} > 0 \ , \ \frac{\partial g_k}{\partial r} = \frac{(1-\alpha)d\left[b_2+b_3\pi-b_4\right]}{\alpha(b_4-b_3\pi)(1+r)^2} > 0 \ .$$

Lifestyle C

(16)

$$g_k = \frac{(1-\alpha)r\left[b_2(1+r)(1-d)-(b_1+b_4)d\pi\right]}{\alpha\pi(b_1+b_2+b_4)(1+r)} - 1 \ ,$$

$$\frac{\partial g_k}{\partial r} = \frac{(1-\alpha)\left[b_2(1+r)^2(1-d)-(b_1+b_4)d\pi\right]}{\alpha\pi(b_1+b_2+b_4)(1+r)^2} > 0 \ .$$

3. Development Dynamics

Growth rates and interest rate obtained in the previous section are long term rates calculated over the span of young adulthood. Applying the geometric mean allows a more convenient interpretation of the macroeconomic outcome as growth rates (interest rates) per year:

(17)

$$\gamma_i = (1+g_i)^{1/\psi} - 1 \ , \ \psi > 0 \ ,$$

where ψ is the length of young adulthood.

The economy is populated by a large number of families with overlapping decision points in time. Since we cannot determine a natural unit for a discrete time interval for the macro–economy the qualitative outcome should be independent of the selection of such a unit. Letting $\Delta t \to 0$ the macro-economy will therefore be analyzed in continuous time.[6]

[6]For the continuity argument see e.g. Gandolfo, 1997, pp. 547-9, or Barro and Sala-i-Martin, 1995, pp. 130-3.

Having made the essential child rearing effort, e, survival of the child is exogenous from the parental point of view. From a macroeconomic viewpoint, however, it is endogenously explained by the performance of the economic system as a whole. Instead of introducing several new variables measuring health care, doctors per person, number of famines etc. I will assume that all the explanatory variables can be approximated by income per capita $y \equiv Y/L$. Figure 1 displays GDP per capita for the five year subperiod 1980-1984 and the corresponding life expectancy at birth for 112 countries.

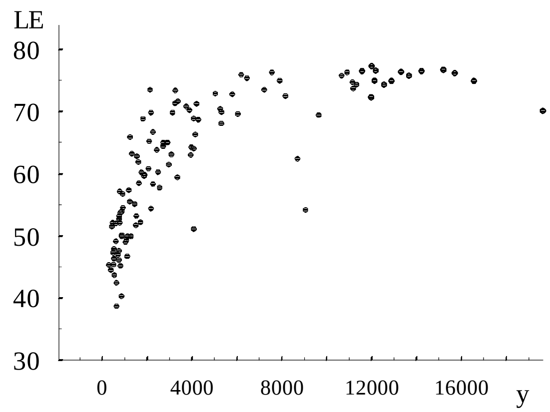

Figure 1: Life Expectancy at Birth Against Income for 112 Countries
Source: Barro and Lee (1993)

The assumption of a unique correspondence between life expectancy at birth and the probability to reach the stage of young adulthood suggests a survival probability function of the form

$$\pi = \pi(y), \quad \partial\pi/\partial y > 0, \quad \sup(\pi) \leq 1 \quad . \tag{18}$$

Employing physical capital, K, and workers, L, endowed with human capital, h, a large number of competitive firms produces output under constant returns to scale using a Cobb-Douglas production $A(Lh)^{1-\alpha}K^{\alpha}$. Factors are paid according to their marginal product:

$$w = (1-\alpha)y \quad , \qquad r = \alpha y/k \quad . \tag{19}$$

Productivity, which is taken as given by the individual firm, is determined by $A(t) = [e^{xt}A_0]/[L(t)^\beta]$, $x \geq 0$. The numerator of the expression allows for a positive rate of technological progress. From viewpoint of a less developed economy this progress is exogenous (produced in already developed economies) and can be thought of as quasi land-augmenting. For discussion of the fully-fledged economy x will be set to zero. The denominator assumes congestion. This assumption can be interpreted as originating from economy-wide limited (arable) land. It constitutes the Malthusian part of the model by generating decreasing returns to scale with respect to the physical factors of production.[7] In conclusion, the macroeconomic production function is given by

$$Y(t) = \mathrm{e}^{xt}A_0[L(t)h(t)]^{1-\alpha}K(t)^\alpha L(t)^{-\beta} \ , \ \ x \geq 0 \quad , 0 < \alpha, \beta < 1 \ . \tag{20}$$

It describes a variety of production possibilities. When per capita income and survival probability are low, and parents do not voluntarily spend income on child quality, human capital is constant, and there are economy-wide decreasing returns to scale with respect to the (possibly) growing factors, L and K. The economy is in the Malthusian phase of development. On the other end of possible production techniques, we have an economy populated by a constant work force and increasing human capital per worker. In this case production exhibits constant returns to scale with respect to the growing factors, h and K. Such an economy in which land plays a negligible role can be thought of as an industrial or modern economy. Between these border cases production exhibits *temporarily* increasing returns when all three factors, K, L, and h, are growing. In this phase the economy experiences a demographic transition and an economic take-off.

At all stages of development income per capita $y = Y/L$ grows at rate

$$\gamma_y = x + \alpha\gamma_k + (1-\alpha)\gamma_h - \beta\gamma_L \ . \tag{21}$$

and the interest rate evolves according to

$$\gamma_r = \gamma_y - \gamma_k \ . \tag{22}$$

[7]Congestion is a straightforward method to introduce Malthusianism into the model. Alternatively, one could explicitly consider land as a limiting factor of production. This would produce a more complicated model with rents on land as third source of income. Since land is fixed the more complicated model generates the same qualitative results as the congestion approach.

Dynamics of the economy are described by the system (21), (22), and (12). The growth rates γ_k, γ_L, and γ_h are generated differently for different lifestyles of young adults and determine whether the differential equation system describes economic stagnation, take-off, or steady growth.

3.1. **The Less Developed Economy and the Population Trap.** Consider an economy with initially low income per capita. If the survival probability is sufficiently low, the economy is populated by Lifestyle A individuals which do not invest voluntarily in child quality. It is then possible that Malthus' (1798) mechanism works so that increases in income due to exogenous technological progress are eventually absorbed by increasing fertility and population growth. The following proposition presents the conditions under which a stable Malthusian state of stagnation exists.

PROPOSITION 1. *Let $\tilde{y}$ define the range of income levels $[0,\ \tilde{y}]$ where the survival probability $\pi(y)$ is small enough so that individuals select Lifestyle A. If an (y, r) exists so that $x/\beta < \gamma_L(y, r)$ and if fertility reacts comparatively stronger on changes in survival probability than savings,*

$$\frac{(\partial s/\partial \pi)\pi}{(\partial n/\partial \pi)\pi + n} < \frac{(\partial s/\partial r)r + s}{(\partial n/\partial r)r} \quad , \tag{23}$$

then the economy stagnates in an equilibrium of zero economic growth and high population growth with rate $\gamma_L{}^\star = x/\beta$.

Proof: See Appendix. It is plausible to assume preferences according to which an increase in interest rates causes a more pronounced increase in savings than in fertility and an increase in survival probability causes a more pronounced increase in fertility, so that (23) holds. Nevertheless the Malthusian equilibrium may not exist. In accordance with Malthus, decreasing returns to scale with respect to the variable factors of production, K and L, is a necessary condition for stagnation. In contrast to Malthus, however, decreasing returns are not sufficient for stagnation. If the congestion externality is sufficiently small (which can be interpreted as a relative minor role of cultivable land in production), or if x is sufficiently large, (which can be interpreted as a high rate of land–augmenting technological progress), then the population growth rate $\gamma_L^\star$ may be not attainable. If for example $x = 0.05$ and $\beta = 0.1$ a stagnation equilibrium would require that individuals generate an annual population growth rate of 50 percent.

There exists, however, a wide range of reasonable parameter specifications where a stable state of stagnation exists. To demonstrate convergence dynamics towards such a state I consider an economy parameterized as in Table 1 and specify the survival function as

$$\pi(y) = 0.9 - \frac{0.25}{1 + \exp(0.0012y)} \quad . \tag{24}$$

The specification ensures a shape of the survival function that corresponds to the observable correlation between income per capita and life expectancy at birth in Figure 1. For the least developed countries the survival probability is around 0.78. At low income levels income improvements have large effects on survival probability but with further rising income the income dependency of survival decreases, so that at an income level of \$ 5000 survival probability has almost balanced at a constant level of 0.9.

The scaling factor ψ is approximated by the length of the fecundity period with $\psi = 20$. As an experiment of thought we can imagine that young adulthood begins at the age of 16 and assume that the annual probability to survive is independent from age. From the data of Figure 1 we then compute a survival probability of approximately $1-16/80 = 0.8$ for countries at the lower end of the income per capita scale and of approximately $1 - 16/160 = 0.9$ for countries with income above 5000.

The preference parameters from Table 1 ensure that individuals prefer consumption when young over consumption in old age and savings are positive with a rate smaller than one. With $e = 0.08$ and $d = 0.1$ net monetary return from children is negative for all income levels and all reasonable interest rates. Since b_4 exceeds $b_3\pi$ for all π, individuals give birth to a positive number of children at all stages of income. But at low survival probabilities q would be negative according to (7), and parents opt for the corner solution and spend no voluntary effort on child quality.

Table 1. Model Parameterization

b_1	b_2	b_3	b_4	α	β	x	d	e	B	ϕ	δ	ψ
0.5	0.47	0.242	0.25	0.4	0.3	0.01	0.05	0.08	0.0165	0.5	0.1	20

Preference parameters and congestion externality are determined so that the same set of preferences generates high population growth rates at income levels of today's less developed economies as well as an almost stable population at very high income levels. With $\beta = 0.3$ and $x = 0.01$ a stagnation equilibrium occurs if individual preferences support a population growth rate of $0.01/0.3 = 3.33$ percent. Parameter values of the education technology support an endogenously generated long-run growth rate around 2 percent. The value of α of 0.4 implies a long-run annual interest rate of 8.3 percent in the equilibrium of stagnation and of 6.7 percent in the equilibrium of long-run growth. The difference in interest rates reflects the higher relative scarcity of capital in less developed economies.

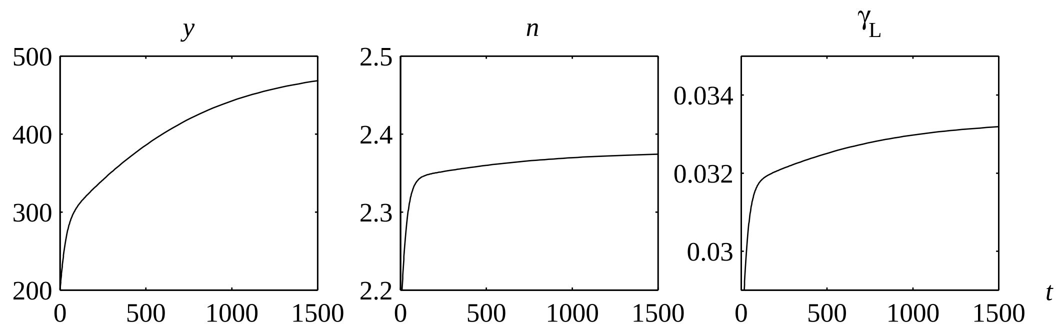

Figure 2: Stagnation of the Less Developed Economy

Figure 2 shows the development process starting at an initial income level of $y_0 = 200$. With technological progress at work income per capita rises so that the probability to rear up a child to young adulthood increases. When more children survive the monetary return on children becomes less negative. Since parents receive utility from having children but not from having savings they react (according to (6)) with an increase in fertility and a decrease in savings. Since more children are born and more children reach young adulthood the labor force grows. With further rising income decreasing returns to scale at the economy-wide level increasingly compensate the positive effect of technological progress on output per capita. At the Malthusian equilibrium the effect of population growth and decreasing returns exactly balance the effect of technological progress and the economy stagnates with constant income per worker and high population growth. Since technological progress is exogenous this state of stagnation and population growth cannot describe the state in which today's modern economies have been settled before the onset of the industrial revolution. But it may characterize the state of today's least developed countries.

3.2. **Demographic Transition and Economic Take-off.** Figure 3 show the development path when technological progress grows at a slightly higher rate of $x = 0.012$. This implies that an equilibrium of stagnation requires a population growth rate of $0.012/0.3 = 4$ percent. Preferences according to Table 1, however, support only a maximum population growth rate of 3.5 percent. Consequently, the Malthusian equilibrium does not exist. The path of demo-economic transition never reaches a point where population grows at a rate of 4 percent.

The first phase of development for this economy is qualitatively identical to the case of stagnation. At an income level of \$ 800 where income is higher than ever before and its growth rate lower than ever before but still positive the qualitative behavior of the system changes. Life circumstances are now favorable enough that parents optimally decide to switch to Lifestyle B and to spend voluntary additional units of income on child quality.

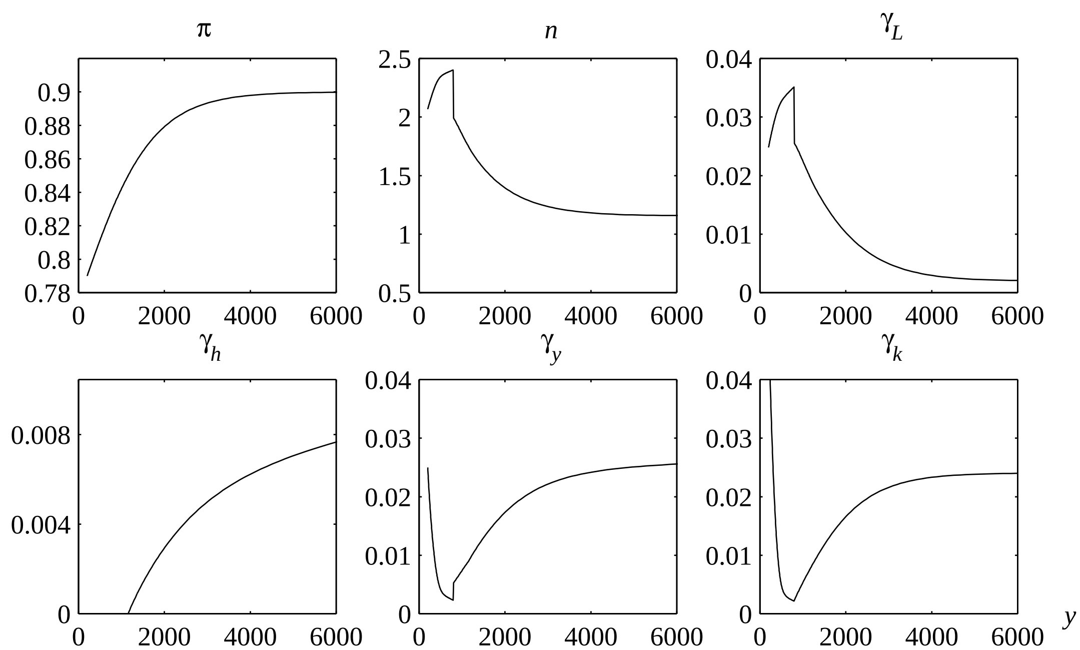

Figure 3: Demographic Transition and Economic Take-off
Parameters from Table 1 except $x = 0.012$.

Rising child quality expenditure evokes a prolific cycle of demo-economic development. Since human capital grows, output per worker begins to grow endogenously. With further rising income, survival probability increases further, and parents continue to substitute child quality for child quantity. The importance of the old-age security motive for having children decreases and parents increase their savings in physical capital. With decreasing number of children and rising savings the capital labor ratio in (15) rises. All three effects, a rising growth rate of human capital per capita, γ_h, a decreasing population growth rate, γ_L, and an increasing capital labor ratio, γ_k, work in the same direction of further increases in income per worker. With decreasing γ_L and increasing γ_h raw labor becomes relatively less important in production and human capital plays an increasingly dominant role. The demo-economic transition comes to its end when there is no more (perceptible) effect of income improvements on child survival, and population and income grow with constant rates.

3.3. **The Fully Fledged Economy and Endogenous Growth.** From a very long-run perspective a positive population growth rate is not feasible because of the finite size of earth.[8] The case of constant population therefore deserves special attention and I proceed assuming preferences so that parents in high-income countries decide on Lifestyle C. Under the assumption that x is developed in today's most advanced economies it is reasonable to regard x as exogenous from the viewpoint of the less developed economy. For a fully developed economy, however, the assumption of exogenous technological progress becomes unreasonable. Consequently, I set x to zero.

After inserting (12) into (21) and (22), using $\gamma_L = 0$, $\pi = \bar{\pi}$, and $x = 0$, and the definition of wages per unit of human capital, the system (21) and (22) can be rewritten as

$$\dot{z}(r,z) = \alpha \left[(\gamma_k(r) - \gamma_h(z,r) \right] z \ , \tag{25}$$

$$\dot{r}(r,z) = -(1-\alpha) \left[(\gamma_k(r) - \gamma_h(z,r) \right] r \ . \tag{26}$$

The following proposition shows that the fully fledged economy produces endogenous growth.

Proposition 2. *An economy populated by Lifestyle C–individuals converges towards a path where the interest rate is constant and parents spend a constant share of income on child quality. On this path the economy grows at a steady rate* $\gamma_y = \gamma_k = \gamma_w = \gamma_h$.

The proof is given in the Appendix. Because of decreasing returns of human capital accumulation with respect to its own stock, perpetual growth requires perpetually increasing amounts that parents are willing to spend in children's quality. At a steady-state, interest rate and survival probability are constant, and parents spend a constant fraction q of their income on child quality. Income in turn is rising at a constant rate because of the external

[8] I focus on economic growth on earth and ignore the possibility of future migration to outer space.

effect of child expenditure on human capital accumulation and growth. Hence, parents indeed spend increasing amounts on their children and generate thereby perpetual growth.

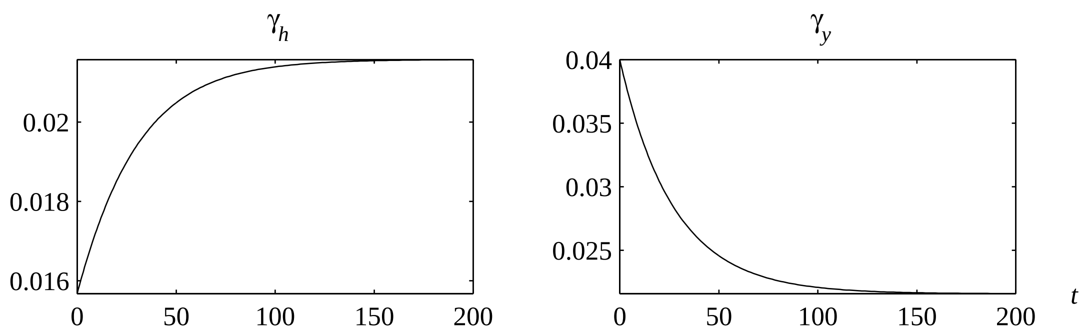

Figure 4: Convergence of the Fully Fledged Economy
Parameters from Table 1 except $x = 0$, $b_3 = 0.245$

To demonstrate adjustment dynamics, I consider an economy specified as in Table 1 and set x to zero and b_3 to 0.245 so that the corner solution for n becomes binding at high income levels. The economy starts at income level \$ 10,000. Resulting time paths are displayed in Figure 4. While the growth rate of human capital adjusts from below, the growth rate of output adjusts from above, implying that increasing efforts in the accumulation of skills are necessary to maintain the same growth rate of output. If we interpret q and h in a broad sense that includes education to become researchers, the picture is consistent with recent observations on research and growth, as e.g. presented by Jones (1995).

The half–time of the adjustment process is about 25 years which is consistent with empirical observations. While the adjustment speed is slower than the corresponding value for the economy in transition, it is, however, much faster than the slow pace obtained for the economy before the onset of the demographic transition.

Finally, I provide a brief sensitivity analysis of long–run growth with respect to parental preferences. Since γ_h depends positively on q and it is straightforward to show that q depends positively on b_4 but negatively on b_1 and b_2, long-run growth rate depends positively on the relative importance of child quality in parental utility. Table 2 displays the impact of alternative utility parameters on the long-run growth rate. Although the decrease in long-run growth is less pronounced if b_2 increases because the reduction in human capital is partly compensated by an increase in physical capital, the general finding is: The more selfish the country's population the lower its long-run growth rate.

TABLE 2. SELFISHNESS AND LONG–RUN GROWTH

	Basic	$b_4 = 0.2$	$b_4 = 0.15$	$b_4 = 0.1$	$b_1 = 0.6$	$b_2 = 0.6$
γ_y	2.16	1.92	1.55	0.78	2.00	2.10

Growth rates in percent. $x = 0$, $b_3 = 0.245$. All other parameters from Table 1.

3.4. **Successful Development Policy: An Example.** This section resumes the analysis of the less developed economy of Section 3.1 which ended up in a Malthusian equilibrium of stagnation. Section 3.2 has shown that the same economy would escape from the population trap if technological progress improves, so that $\gamma_L{}^\star$ becomes unattainable. An escape would also be possible under enforced population control policy which makes $\gamma_L{}^\star$ unattainable or by manipulation of preferences towards a higher preference for child quality at earlier stages of development. In this section, however, I would like to continue to treat technologies and preferences as exogenous and investigate a meaningful development policy.

Given that the vicious cycle of the population trap collapses as soon as people start to invest in child quality I introduce a child quality (education program) into a Lifestyle A-economy. The crucial parameter of such an education program is not the magnitude of annual development aid but its duration. The education program must continue until parents start to invest *voluntarily* in child quality. Any policy of shorter duration does not break the vicious cycle but only stabilizes the economy at a higher level of income per capita and a higher population growth rate.

The following example introduces development aid into the economy from Section 3.1, which has almost stabilized at the population trap at an income of \$ 500. As long as parents do not voluntary invest in child quality, each young adult receives per child q_e units of his wage income as development aid. This amount must be spent on education. The generated growth rate of human capital is $\gamma_h = (q_e(1-\alpha)z)^\phi$ during this period.

Figure 5 shows development paths for an example where $q_e = 0.10$ implying that each child receives extra investment of about \$ $0.1(1-\alpha)500 = 30$. Although equipped with increasing human capital endowment young adults still select Lifestyle A during the phase of development aid. Eventually, however, at the point where population grows at a rate higher than ever before, income is sufficiently high and mortality is sufficiently low so that parents

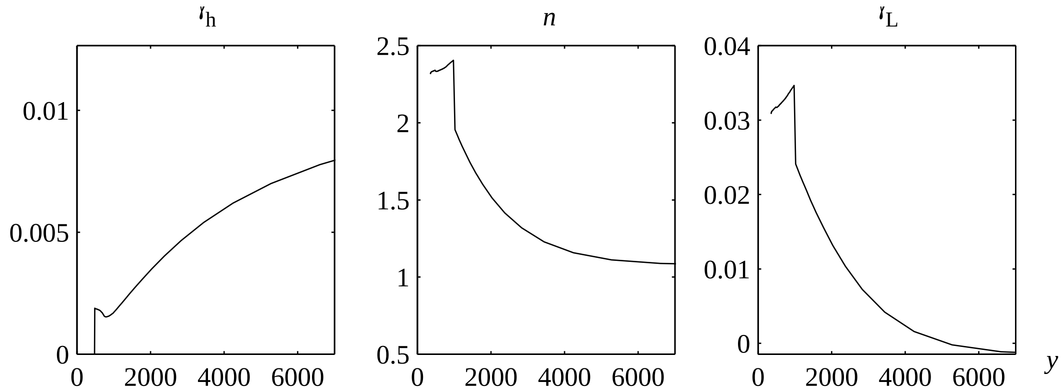

Figure 5: A Successful Development Policy
Parameters from Table 1.

begin to prefer Lifestyle B. The development aid program is terminated and the economy undergoes a demo-economic transition as described in Section 3.2.

4. Conclusion

The paper has offered an alternative view on the problem of economic development. In contrast to related work parents do not rationally choose to converge towards long-run stagnation or growth. Parents decide rationally on their microeconomic planning level but they do not take into account the external effects that their behavior may have on the macroeconomy. They do not consider that their family planning may produce stagnation due to congestion on limited arable land or long-run growth via human capital accumulation.

The same parents that produce perpetual growth in a fully developed economy produce stagnation when situated in a less developed economy. In a less developed economy parents react on improvements in income and survival probability by increasing fertility since surviving children become relatively less costly. In a developed economy where mortality is almost constant parents react on income improvements by holding the number of children constant and spending more income on them.

When income per capita is low, and survival is uncertain parents decide that they cannot afford expenditure on child quality. Without spending on child quality the economy may get stuck in a population trap. In contrast to pure Malthusian reasoning stagnation does not necessarily occur. If parents do not generate the demographic transition endogenously, a development aid program can manage the escape from stagnation. The crucial point of a successful development program is not the amount of money spent but its duration. If the program ends before parents begin to invest voluntarily in child quality it is meaningless in the long-run.

The demographic transition is accompanied by an economic transition. With decreasing importance of raw labor in production and increasing importance of human capital the main focus of production shifts gradually from agriculture to manufacturing where limiting land becomes insignificant. At the end of this process population growth stabilizes and the economy catches up towards a fully developed economy.

5. Appendix: Proof of Propositions

Proof of Proposition 1

In an economy populated by Lifestyle A–individuals equations (21) and (22) constitute a two–dimensional differential equation system in r and y. If individual preferences according to (5) support an y with $n(\pi(y))$ such that $\gamma_L(y) = x/\beta$ then the system has an equilibrium with $\gamma_h = \gamma_y = \gamma_k = \gamma_r = 0$. Since $n(\pi(y))$ is monotonous for $y \leq \tilde{y}$ the equilibrium is unique. Let J denote the Jacobian of (21) and (22) evaluated at the steady–state. The trace $\text{tr}J = \alpha\partial\gamma_k/\partial y - (1-\alpha)\partial\gamma_k/\partial r - \beta(\partial\gamma_L/\partial y + \partial\gamma_L/\partial r)$ is negative since $\partial\gamma_k/\partial y < 0$, $\partial\gamma_k/\partial r > 0$ from (14) and (17), and $\partial\gamma_L/\partial y > 0$, $\partial\gamma_L/\partial r > 0$ from (12), (14), and (17).

The Jacobian determinant is computed as

$$\det J = \beta\left[\partial\gamma_L/\partial y\partial\gamma_k/\partial r - \partial\gamma_L/\partial r\partial\gamma_k/\partial y\right] \quad .$$

After insertion of (17) we find the determinant to be positive if

$$\frac{\partial g_k/\partial r}{\partial g_k/\partial\pi} > \frac{(\partial g_n/\partial r)\pi}{n + \pi(\partial n/\partial\pi)} \quad .$$

Note, that stability is independent from the specification of the survival function. For Lifestyle A–individuals g_k can be written as $g_k = (1-\alpha)s(y,r)r/\left[(\alpha\pi(y)n(y,r)\right] - 1$ with

$$\frac{\partial g_k}{\partial k} = \frac{1-\alpha}{\alpha}\frac{ns - rs(\partial n/\partial r) + rn(\partial s/\partial r)}{n^2\pi}$$

$$\frac{\partial g_k}{\partial k} = \frac{1-\alpha}{\alpha}r\frac{n\pi(\partial s/\partial y) - \pi s(\partial n/\partial y) - ns(\partial\pi/\partial y)}{\alpha n^2\pi^2}$$

Employing these derivatives the stability condition can be written as in (23). □

Proof of Proposition 2

The function

$$V(z,r) = \left[\gamma_k(z,r) - \gamma_h(z,r)\right]^2 \geq 0 \;\text{ with }\; = 0 \;\text{ for }\; \gamma_k = \gamma_h \Leftrightarrow \gamma_z = \gamma_r$$

is a Ljapunov–function for the system (25) and (26) since

$$\begin{aligned}\dot{V}(r,z) &= \frac{\partial V(z,r)}{\partial z}\dot{z}(z,r) + \frac{\partial V(z,r)}{\partial r}\dot{r}(z,r) \\ &= 2(\gamma_k - \gamma_h)^2\left[\alpha\gamma_z\left(\frac{\partial\gamma_k}{\partial z} - \frac{\partial\gamma_h}{\partial z}\right) - (1-\alpha)r\left(\frac{\partial\gamma_k}{\partial r} - \frac{\partial\gamma_h}{\partial r}\right)\right] \leq 0\end{aligned}$$

with $= 0$ for $\gamma_k = \gamma_h \Leftrightarrow \gamma_z = \gamma_r$, and $\partial \gamma_k / \partial z = 0$, $\partial \gamma_h / \partial z > 0$, and $\partial \gamma_k / \partial r = 0$, $\partial \gamma_h / \partial r < 0$ since $\partial q / \partial r < 0$ in (10). $\square$

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