Interaction between trend and cycle in "Keynesian" and "Neo-Marxian" dynamical models of the economy*

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1 Introduction

The problem of the interrelation of economic growth and economic cycles has recently been at the core of research on macroeconomics. In the last two decades or so, indeed, the integration of growth and business cycle theory has been the first item on the real business cycle (RBC) theorists' agenda.[1]

In this framework, the attempt to achieve such an integration turns into the attempt to explain the observed fluctuations of economic activity as a by-product of the same economic forces that lead to growth in the neoclassical model of capital accumulation, the latter being the *benchmark model* which "... must be at the core of *any* understanding economists will provide of business cycles" (Plosser, 1989, p. ; my emphasis).

Two considerations seem appropriate in this regard.

First, there is no doubt that RBC theorists believe that the integration of growth and business cycle theory is achieved in their models. Yet, the resulting dynamics is barely discussed from this angle and one has the impression that the supposed integration between growth and business cycle theory is taken for granted rather than proved.

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[1]See, for example, Kydland and Prescott (1982, p. 1345) and King, Plosser and Rebelo (1988, p. 196) .

Second, it is a fact that in the RBC literature no reference is made to earlier contributions on the topic. These two considerations are somehow linked, in the sense that it could well be that, in the light of earlier theories of cyclical growth, one can conclude that a proper integration of growth and business cycle theory is not in fact achieved in RBC models.

To pave the way for an investigation of this matter, as part of a more comprehensive project (see also Fiaschi and Sordi, 2001), the present paper reviews some of the earlier formalizations of cyclical growth models concentrating, in particular, on *Keynesian* ("multiplier-accelerator") and *neo-Marxian* ("growth cycles") dynamical models of the economy.

Two authors, Marx (1887/1954) and Schumpeter (e.g., 1939), we believe, more than any others, have enhanced our ability to tackle the problem of why, when capitalist economies develop, they do not do so steadily but rather do so by means of fluctuations of economic activity.

For both Marx and Schumpeter the growth and the economic fluctuations observable in the dynamics of capitalist economies are *strictly interrelated phenomena*.

In Marx's theory,[2] the business cycle is the basic way in which capitalist economies develop due to interaction between the accumulation process (and the resulting growth of productive capacity) and the conflict over income distribution between capitalists and workers.

A crucial role in this (endogenous) mechanism is played by the size of the *reserve army of labour*. In periods of high rates of accumulation, the reserve army of labour decreases, so bringing a reduction in the number of unemployed and, as a consequence, an increase in labour's bargaining power. This, in turn, causes a change in income distribution in favour of workers, implying a decline of profits and a consequent decline of accumulation. But this increases the unemployment rate again, pushes the wage share down and restores the profitability of real investment. As a consequence, the rate of capital accumulation goes up and the sequential mechanism just described can start again.

Schumpeter, on the other hand, produced an integration of growth and business cycle theory in which economic fluctuations are nothing other than the "form which progress takes in capitalist society".

A basic role in this explanation is played by the concept of "pioneering entrepreneur" who innovates, for example by introducing a new method of production, a new product, a new market or a new source of supply. This opens up new profitable avenues such that more entrepreneurs are induced

[2] See, in particular, Chapter 25, "The general law of capitalist accumulation" in Marx (1887/1954).

to innovate, giving rise to an investment boom and driving growth for the economy as a whole.[3] Once the innovations are fully exploited, however, the economy relapses into a depression until the accumulation of new ideas creates a favourable climate for a new burst of innovating investment and so on.

Thus, when evaluating a model with regard to its ability to represent cyclical growth of the economy as a whole, it is useful, first, to check whether or not it allows for some role for the Marxian ("distributional") and/or the Schumpeterian ("innovative") mechanisms we have just described. This is the perspective from which we intend to investigate the various models in what follows.

To pursue this task, the paper is organized as follow.

In the next section, a simple Keynesian ("multiplier-accelerator") prototype model is presented and analysed with respect to its cyclical growth properties. In Section 3 we outline a simple neo-Marxian dynamical model, which consists in a generalization of Goodwin's growth cycle model. An integration of the two is sketched in Section 4. Section 5 contains some conclusions and indications for further research.

2 Keynesian "multiplier-accelerator" models

2.1 A linear formulation of the model

It is a fact that — in spite of important contributions by "pioneers" of economic dynamics modelling such as Goodwin (e.g., 1953, 1955, 1967), Harrod (e.g., 1939, 1951), Kaldor (1954) and Kalecki (1968) — in most of the earlier contributions to economic dynamics which followed the publication of Keynes' *General Theory* (1936), the prevalent attitude was that of a separate handling of business cycles and growth.[4] They were usually models which considered purely endogenous relationships explaining the aggregate behaviour of consumers (through the *multiplier*) and that of entrepreneurs (through some version of the *principle of the accelerator* or some other theory of aggregate investments). The dynamics of the resulting standard "multiplier-accelerator" interaction model was either cyclical or monotonic and did not succeed then in representing the observed cyclical growth of real economies.

The only way out of this puzzle was to assume that the parameters of the model were such that the solution was cyclical (with fluctuations of con-

[3] See Kaldor (1954).

[4] See, for example, Samuleson (1939) and Hicks (1950).

stant amplitude) and then to add to the model an autonomous component of aggregate investment, which grew exogenously in time. As a result, the solution of the model described cyclical fluctuations of constant amplitude around a growth trend, but, *by construction*, there cannot be any interaction between the growth and the cycle components of the dynamics.

This can be easily illustrated by using a (prototype) model of the "multiplier-accelerator" interaction, which, in order to guarantee continuity with what follows, we formulate in continuous-time.[5]

Apart from a Keynesian consumption function

$$C = cY, \; 0 < c < 1 \tag{1}$$

where

$$\begin{aligned} C &= \text{aggregate consumption} \\ Y &= \text{national income} \end{aligned}$$

the basic dynamical ingredients of the model are two *error-adjustment mechanisms*, according to which aggregate supply and investment in fixed capital adjust to their desired levels, determined by total demand and the principle of accelerator respectively.

In the case in which such adjustments take place with a simple exponential lag, we can write

$$\dot{Y} = \frac{1}{\varepsilon}\left[(C+I) - Y\right], \; \varepsilon > 0 \tag{2}$$

$$\dot{I}^i = \frac{1}{\theta}\left(v\dot{Y} - I^i\right) = \frac{1}{\theta}\left\{v\dot{Y} - \left[I - I^a(t)\right]\right\} =, \; v > 0, \theta > 0 \tag{3}$$

where

$$\begin{aligned} &\varepsilon,\, \theta && = \text{lengths of the adjustment lags} \\ &1/\varepsilon,\, 1/\theta && = \text{speeds of adjustment} \\ &I && = \text{total investment} \\ &I^i && = \text{induced investment} \\ &I^a(t) && = \text{autonomous investment} \\ &v && = \text{the capital-output ratio} \end{aligned}$$

and where a dot over a variable (e.g., $\dot{x}$) indicates the derivative with respect to time (dx/dt).[6]

[5] That is, rather than Samuelson's (1939) original formulation, we consider the version of the model studied by Phillips (1954). See also Flaschel (1993, pp. 100-101) and Gandolfo (1997, pp. 71-74).

[6] By "autonomous" investment we mean in general terms that component of aggregate investment which is independent of existing capacity and of the existence or otherwise of any excess capacity.

Intervals of values of v	Type of solution
$v \leq (\sqrt{\varepsilon} - \sqrt{s})^2$	monotonic, convergent (**A**)
$(\sqrt{\varepsilon} - \sqrt{s})^2 < v < \varepsilon + s$	oscillating, damped (**B**)
$v = \varepsilon + s$	oscillating, constant amplitude (**C**)
$\varepsilon + s < v < (\sqrt{\varepsilon} + \sqrt{s})^2$	oscillating, divergent (**D**)
$v \geq (\sqrt{\varepsilon} + \sqrt{s})^2$	monotonic, divergent (**E**)

Table 1: Intervals of parameter values and type of solution

With simple algebraic calculations we then obtain

$$\ddot{Y} + \frac{(\varepsilon + s - v)}{\varepsilon}\dot{Y} + \frac{s}{\varepsilon}Y = \frac{1}{\varepsilon}(D+1)\, I^a(t) \tag{4}$$

where $s = 1 - c$, $D = d/dt$ and where we have chosen the time unit so as to have $\theta = 1$

In the case in which there is no autonomous investment ($I^a(t) = 0$, $\forall t$), simple considerations allow us to conclude that the relation between the values of the parameters of the model and the resulting dynamics is the one summarized in Table 1 and shown, for two different cases,[7] in Figure 1.

The implication of the analysis is that the dynamics of the model is *either* monotonic (if the pair (v, ε) is in regions **A** or **E**) *or* oscillating (if the pair (v, ε) is in regions **B**, **C**, or **D**).

On the other hand, it possible to assume (see, for example, [30, p.]) that *there is* an autonomous component of investment which grows in time, for example, such that

$$I^a(t) = a_0 + a_1 t$$

In this case equation (4) becomes a non-homogenous differential equation, such that

$$\ddot{Y} + \frac{(\varepsilon + s - v)}{\varepsilon}\dot{Y} + \frac{s}{\varepsilon}Y = \frac{1}{\varepsilon}(a_2 + a_1 t) \tag{5}$$

where $a_2 = a_0 + a_1$.

Thus, for example choosing a combination of values for the parameters on the straight line **C** of Figure 1, we obtain the representation of fluctuations of constant amplitude around a growing trend. By construction, however, there cannot be any interaction between the cycle component (Figure 2(i)) and the growth component (Figure 2(ii)) of the dynamics in that they are simply superimposed (Figure 2(iii)).

[7] See Phillips (1954), Allen (1967) and Flaschel (1993). In the figure on the left, we have fixed s at the value 0.25, whereas in the figure on the right we have fixed ε at 0.5.

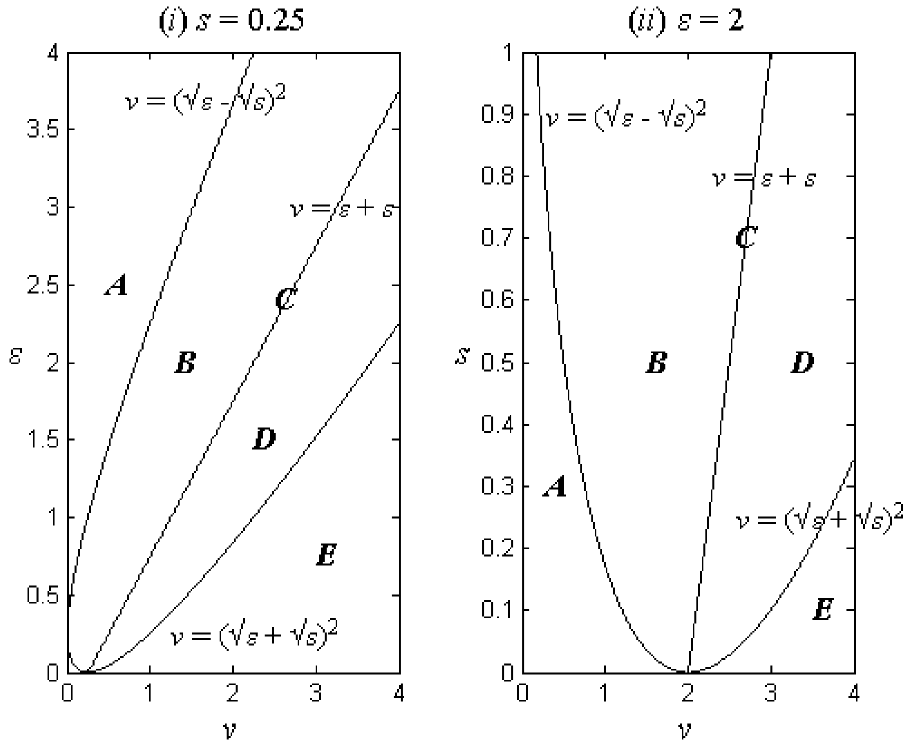

Figure 1: Regions of parameter values and type of solution.

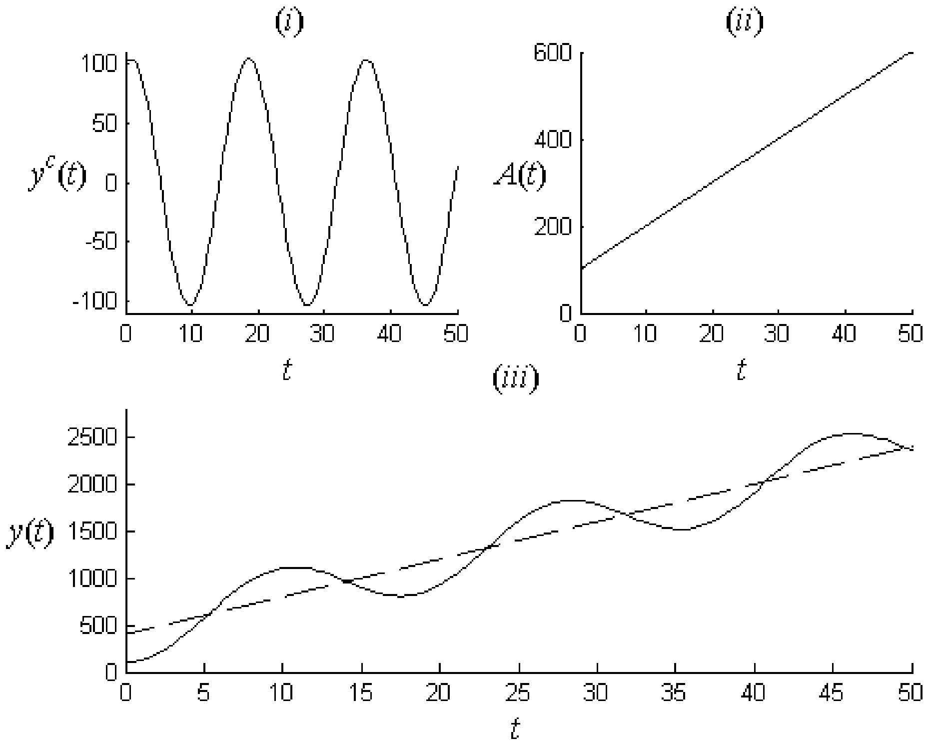

Figure 2: The graphs of the solutions of the linear multiplier-accelerator model with a linear trend and oscillations of constant amplitude (for $\varepsilon = 2$, $s = 0.25$, $v = 2.25$).

2.2 A nonlinear formulation of the model with "innovative" investment

The "multiplier-accelerator" model we have just analysed proves to be a pure (linear) cyclical model, such that the representation of cyclical growth can only be achieved by superimposing a trend on it.[8]

On the basis of the discussion in the introduction, however, this is hardly surprising given that it is a model (i) which is built around the product market (and neglects the labour market altogether) and (ii) in which any kind of "innovational" investment is ignored.

Given (i), it appears that the consideration of a conflict-over-distribution mechanism would require a complete reformulation of the model.

It remains to be seen whether it is possible to introduce in it a "Schumpeterian" element.

In order to do that, let us consider a nonlinear version of the model (Goodwin, 1951).This is easily done by replacing the desired level of investment $v\dot{Y}$ in the error-adjustment mechanism (3) with the nonlinear function $\phi\left(\dot{Y}\right)$, where ϕ is as shown in Figure 3(i).[9]

Doing that, equation (4) becomes

$$\ddot{Y} + \frac{1}{\varepsilon}\left\{(\varepsilon + s)\dot{Y} - \phi(\dot{Y})\right\} + \frac{s}{\varepsilon}Y = \frac{1}{\varepsilon}\left(D + 1\right) I^a(t) \tag{6}$$

so that, assuming — as in Goodwin (1951, p. 12) — that autonomous investment is constant and equal to I^{a*} for all t, we obtain

$$\ddot{y} + \frac{1}{\varepsilon}\left\{(\varepsilon + s)\dot{y} - \phi(\dot{y})\right\} + \frac{s}{\varepsilon}y = 0 \tag{7}$$

where $y = Y - Y^* = Y - I^{a*}/s$.

In the case in which $v > \varepsilon + s$ (i.e., in the case in which the equilibrium is locally unstable) the solution of equation (7) is a *limit cycle,* describing *persistent* fluctuations of national income around the (constant) equilibrium (see Figure 4).

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[8] See Kaldor (1954, pp. 61-65). This is stressed also by Aghion and Howitt in their recent book on endogenous growth (1998, p. 234).

[9] More precisely, for use in the numerical simulations below, we choose the following functional form for the investment function:

$$\phi\left(\dot{y}\right) = \left(\frac{I^{\max} + I^{\min}}{I^{\max}\exp\left\{-\frac{I^{\max}+I^{\min}}{(I^{\max})(I^{\min})}v\dot{y}\right\} + I^{\min}} - 1\right) I^{\min}$$

As can be easily proved, this functional form satisfies all that is required by the nonlinear

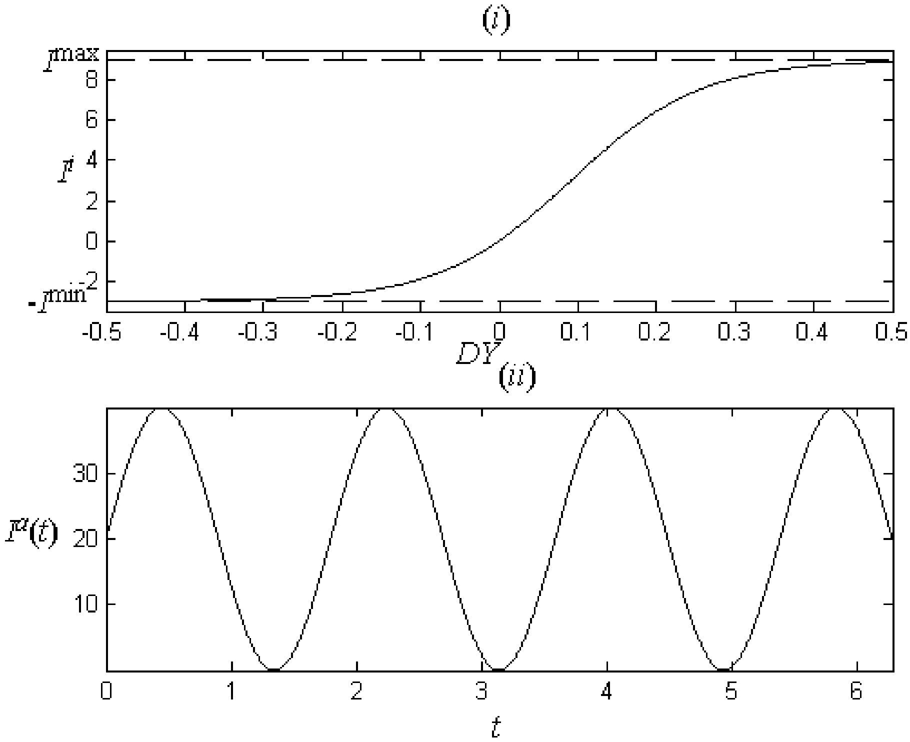

Figure 3: The (i) induced and (ii) autonomous component of investment.

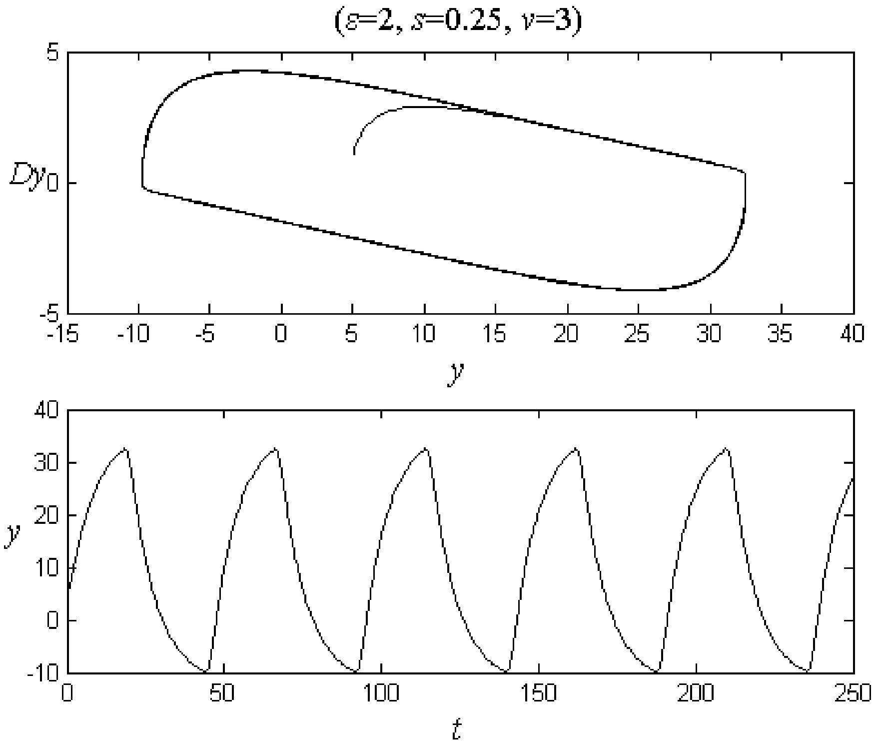

Figure 4: The limit cycle of the nonlinear accelerator model (in deviations from equilibrium).

To understand the relevance of this result, however, it is useful to go back to the meaning of *autonomous* — as opposite to *induced* — investment.

In the model we are considering, such a component includes all that investment in fixed capital that is not explained by the acceleration principle, mainly, therefore, *innovational* investment in the Schumpeterian sense. It does not appear satisfactory at all, then, to assume that is constant in time. Rather, a better, although rough, way of formalizing Schumpeter's idea of clustering of innovations is to assume that the autonomous component of investment is a *periodic* function of time of the kind (Figure 3(*ii*))

$$I^a(t) = b\left[1 + \sin(ct)\right] \tag{8}$$

where the two parameters b and c determine the amplitude and the frequency of innovational investment respectively.

An example of the simulation of the model with the "forcing" effect of innovational investment — as formalized in (8) — is given in Figure 5.

3 The "neo-marxian" approach to growth cycles

Starting from the late Sixties, a different approach ("neo-Marxian") to growth cycles was developed, based on Goodwin's 1967 model.[10] The reason why we call it "neo-Marxian" is to stress the fact that in it a basic role is played by the reserve-army-of-labour mechanism we described in the introduction.

Taking account of the vast literature that has appeared since the publication of the original contribution (from now on, OVM = Original Version of the Model), we intend in what follows to propose and analyse two generalisations of the OVM.

Before doing that, however, let us briefly recall that the OVM gives rise to the following dynamical system of the Lotka-Volterra type

$$\dot{v} = (g - g_n)\, v = \left(\frac{1}{\sigma} - g_n - \frac{1}{\sigma}u\right) v \tag{9}$$

$$\dot{u} = \left[f(v) - \alpha\right] u = \left[-(\gamma + \alpha) + \rho v\right] u \tag{10}$$

accelerator. In particular, one has $\phi(0) = 0$, $\phi'(0) = v$, $\phi(-\infty) = -I^{\min} < 0$, and $\phi(+\infty) = I^{\max}$.

[10] In Aghion's and Howitt's opinion (1998, p. 234), this is perhaps the first model in which the occurrence of economic fluctuations was modelled as a deterministic consequence of the accumulation (i.e., growth) process; more specifically, of the variations in income distribution this process induces over time.

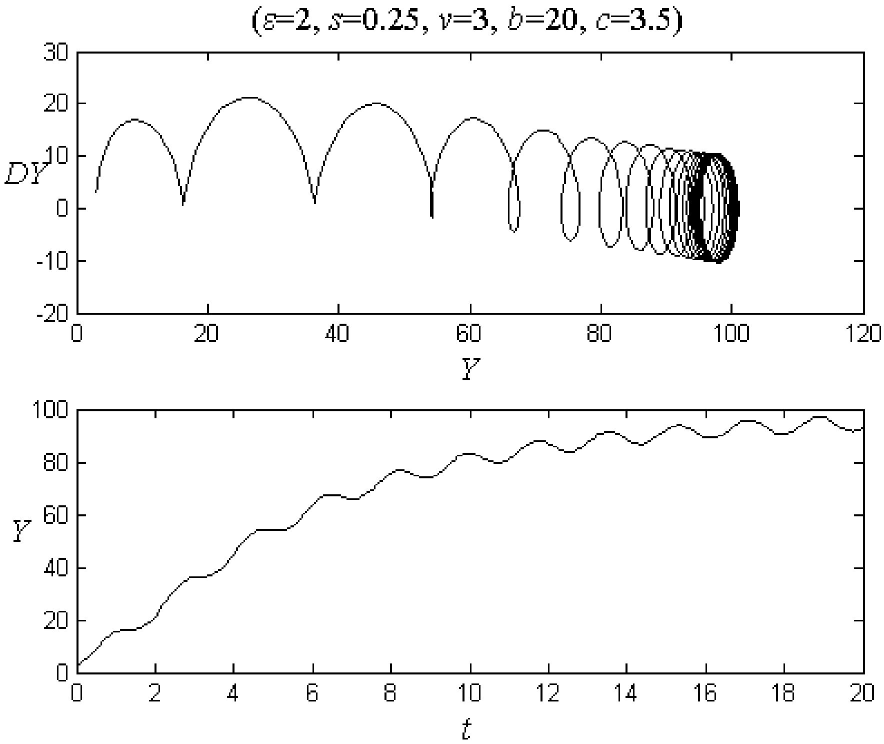

Figure 5: Dynamics of the nonlinear accelerator model with innovational ("forcing") investment.

where

l	$=$ employment
q	$=$ output
$q/l = a$	$=$ labour productivity
n	$=$ labour force
$g_n = \alpha + \beta$	$=$ natural rate of growth
w	$=$ real wage
$u = wl/q = w/a$	$=$ *share of wages*
$v = l/n$	$=$ *employment rate*
k	$=$ capital stock
$\sigma = k/q$	$=$ capital-output ratio
$g = \hat{q} = \hat{k} = S/k$	$=$ rate of growth

such that

$$a = a_0 \exp(\alpha t)$$
$$n = n_0 \exp(\beta t)$$
$$\sigma, \text{ constant}$$

Equations (9) and (10) easily follow from these basic assumptions together with the assumption of a Phillips curve for the *real* wages dynamics

$$\frac{\dot{w}}{w} = f(v) \approx -\gamma + \rho v, \;\; \gamma, \rho > 0 \tag{11}$$

and a classical assumptions about savings behaviour (all profits saved and invested, all wages consumed)

$$S = (1 - u)\, q \tag{12}$$

As is well known (see Figures 6(i) and 6(ii)), the solutions in v and u of equations (9)-(10) are cyclical (the positive equilibrium point being a *centre*) and, given that

$$\frac{\dot{q}}{q} = g = \frac{1 - u}{\sigma}$$

this implies that the solution in g is also cyclical (Figure 6(ii)).

Thus, the output is subject to cyclical growth as shown in Figure 6(iii).

It is obvious that, in this case, the capacity to generate (growth) cycles is "*intrinsic*" to the model and is not due to the choice of any particular functional form for the functions of the model.

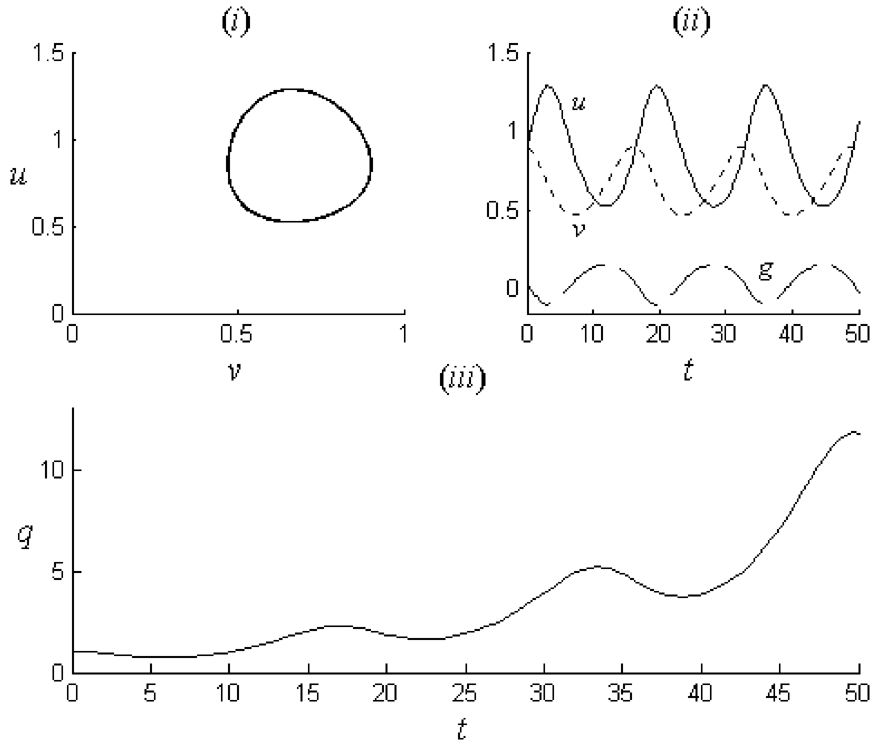

Figure 6: Growth cycles in the OVM.

Two problems, however, appear to reduce the relevance of this result.

First, the fact that the positive equilibrium point of the model is a *centre* implies that the fluctuations of v and u around their equilibrium values are of an amplitude which fully depends on initial conditions. It is not too difficult, however, to modify the OVM in such a way that the resulting dynamics is a *limit cycle* of the relevant variables. One of the possible modifications of the model in this direction is briefly presented and discussed in section 3.1.

Second, and more importantly, the OVM neglects altogether the product market. There cannot be, therefore, any role — in the generation of the cycles — for effective-demand problems and/or for mechanisms of adjustment to product market disequilibria. This is in sharp contrast with the Keynesian model we have presented in section 2. An "hybrid" version of the OVM, which attempts to introduce into it some Keynesian flavour, *while preserving the capacity of the model to generate growth cycles*, is sketched in section 4.

3.1 A modified version of the model with differential savings

It is worth analysing whether the OVM can be modified in such a way that income distribution plays a role in the dynamics also *via* savings behaviour (in the Cambridge tradition).

As shown in Sordi (2001), if, in relaxing the classical assumption about savings behaviour,

(i) we take account of Pasinetti's criticism (e.g., Pasinetti, 1962) of Kaldor's approach to differential savings (Kaldor, 1956), and

(ii) we consider a more general version of the Phillips curve, according to which the rate of growth of real wages depends not only on *level* of the rate of employment, but also on its *rate of change*

then the model can produce persistent cyclical (*limit cycle*) behavior and even chaotic dynamics.

First of all, to consider this first "modified version of the model" (MVM1), we need the following additional notation

$\varepsilon = k_c/k$	= proportion of capital held by capitalists
$1-\varepsilon = k_w/k$	= proportion of capital held by workers
P_w	= workers' profits
P_c	= capitalists' profits
$P = P_c + P_w$	= total profits
$r = P/k$	= rate of profit
s_w, S_w	= workers' propensity to save and savings
s_c, S_c	= capitalists' propensity to save and savings
$S = S_w + S_c$	= total savings
$\Delta s = s_c - s_w > 0$	
$\Delta s_\sigma = \Delta s/\sigma$	
$s_{w\sigma} = s_w/\sigma$	
$s_{c\sigma} = s_c/\sigma$	

Then, (i) and (ii) can be taken into account by writing

$$\begin{aligned} S_c &= s_c P_c = s_c r k_c \\ S_w &= s_w (wl + P_w) = s_w (wl + r k_w) \\ q &= wl + P_w + P_c \\ r &= \frac{P}{k} = \frac{q - wl}{k} = \frac{1-u}{\sigma} \\ g &= \frac{s_w + \Delta s(1-u)\varepsilon}{\sigma} \end{aligned}$$

and

$$\hat{w} = f(v, \hat{v}) = h(v) + \delta\hat{v},\ \ h'(v) > 0, h''(v) > 0, \delta > 0 \tag{13}$$

It is then easy to show (Sordi, 2001, p.) that the MVM1 reduces to the following 3D-dynamical system in v, u, and ε**:**

$$\begin{aligned} \dot{v} &= [s_{w\sigma} - g_n + \Delta s_\sigma \varepsilon (1-u)]\, v & (14) \\ \dot{u} &= [h(v) + \delta\hat{v} - \alpha]\, u & (15) \\ \dot{\varepsilon} &= [\Delta s_\sigma - s_{c\sigma} u - \Delta s_\sigma \varepsilon (1-u)]\, \varepsilon & (16) \end{aligned}$$

with singular point $P_1 \equiv (v_1, u_1, \varepsilon_1) \equiv (0,0,0)$, $P_2 \equiv (v_2, u_2, \varepsilon_2) \equiv (0,0,1)$ and $P^* \equiv (v^*, u^*, \varepsilon^*) = (v^*, (s_{c\sigma} - g_n)/s_{c\sigma}, s_{c\sigma}(g_n - s_{w\sigma})/\Delta s_\sigma g_n)$.

It is worth noticing that the positive equilibrium P^* guarantees *steady state results* that have a Pasinettian-Kaldorian "flavour", in particular

- it guarantees a steady-state growth of the system at a warranted rate equal to the natural rate and is such that the *Cambridge equation* is satisfied

$$
\begin{aligned}
g^* &= g_n \\
r^* &= \frac{1-u^*}{\sigma} = \frac{g_n}{s_c}
\end{aligned}
$$

- in order to be economically meaningful, it requires that the *Pasinettian case* holds

$$0 \leq s_w < \sigma g_n < s_c \leq 1 \tag{17}$$

- it is such that the steady-state growth path is characterized by a positive (constant) rate of unemployment equal to $(1-v^*)$ rather than by full employment

However, and more importantly given our purposes, when condition (17) is satisfied, the system may not converge to P^*, but *rather persistently fluctuates around it.*[11] A limit cycle solution of the model with the linear approximation $h(v) \approx -\gamma + \rho v$ is shown in Figures 7, 8, 9 and 10.

4 An hybrid version of the growth cycle model

Apart from the generalized Phillips curve and the assumption about savings behaviour, the MVM1 maintains all the other simplifying assumptions of the OVM. In particular, as in the OVM, it assumes a permanent product market equilibrium and does not consider an independent investment function.

Finally, in the attempt to link up the two different types of dynamic model we have considered in this paper ("Keynesian" and "neo-Marxian"), we propose, starting again from the OVM, a second modified version of the model (MVM2), in which investment in fixed capital is explained by an "accelerator-type" mechanism and in which the product market does not clear. Rather it is governed by an error-adjustment mechanism as in (2).[12]

[11] See Sordi (2001), where this is shown by applying to the dynamical sistem (14)-(16) the Hopf bifurcation theorem.

[12] Other contributions in which an independent investment function has been introduced into the "growth-cycle" framework are, for example, Glombowski and Krüger (1988), Rampa and Rampa (1988), Wolfstetter (1982). See also Flaschel (1988, 1993).

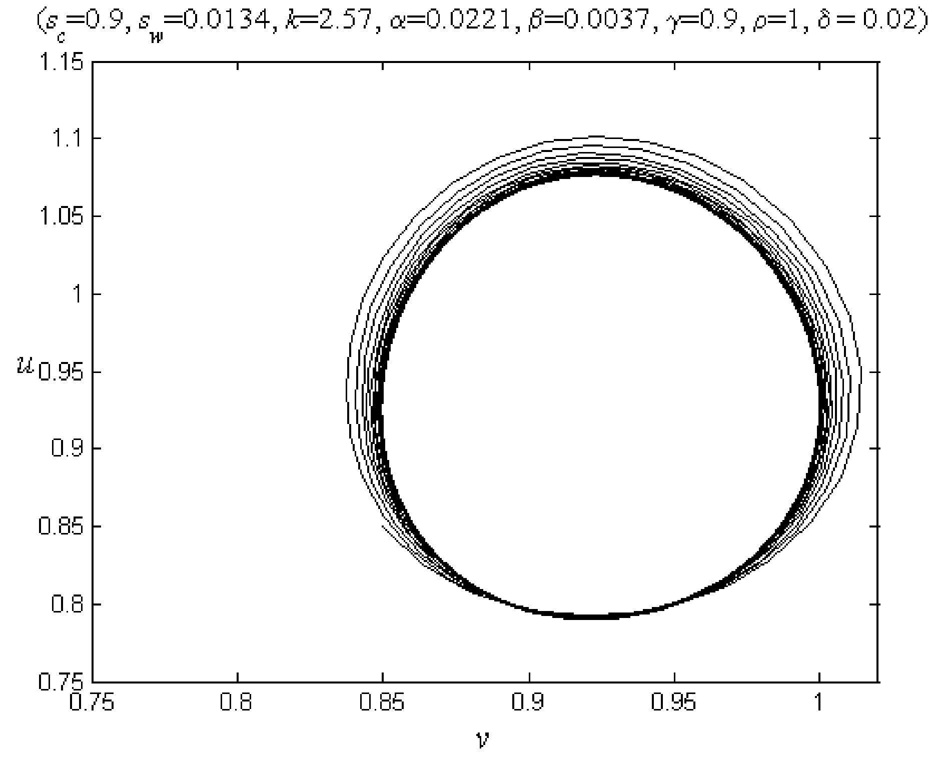

Figure 7: Growth cycles in the MVM1 (2D).

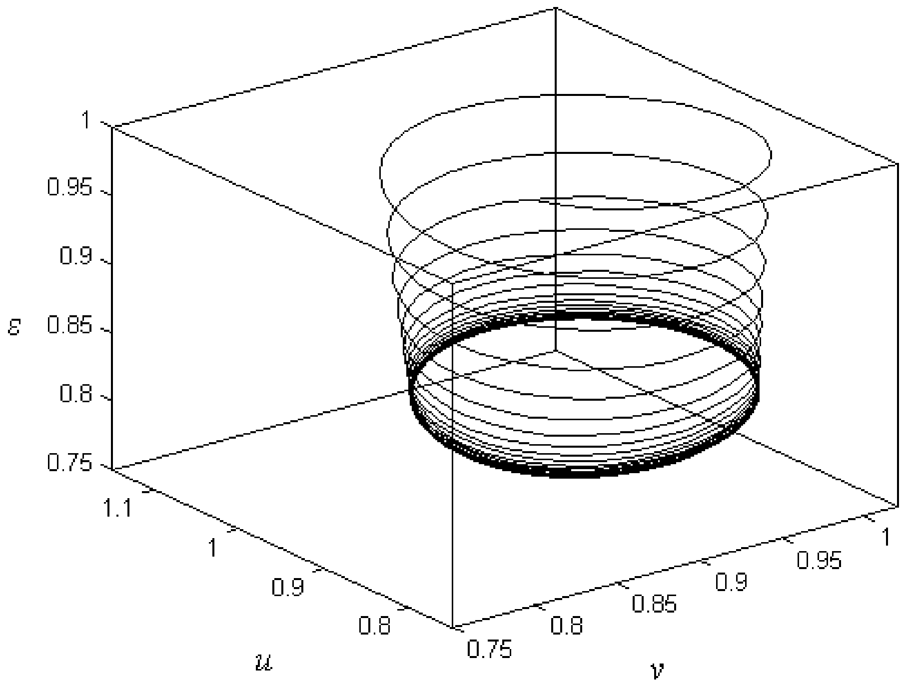

Figure 8: Growth cycles in the MVM1 (3D), with initial conditions such that $\varepsilon = 1$.

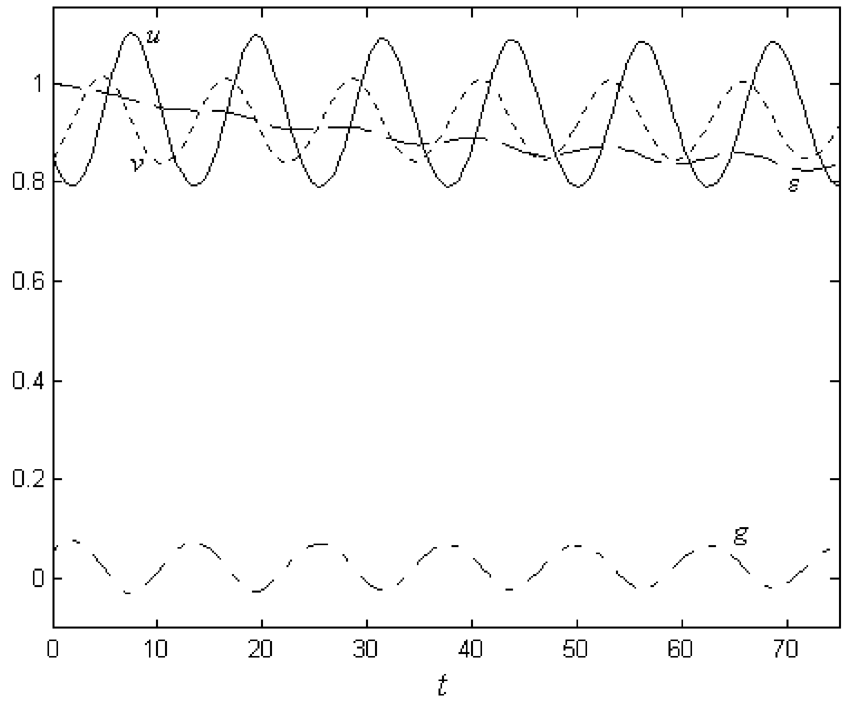

Figure 9: Growth cycles in the MVM1.

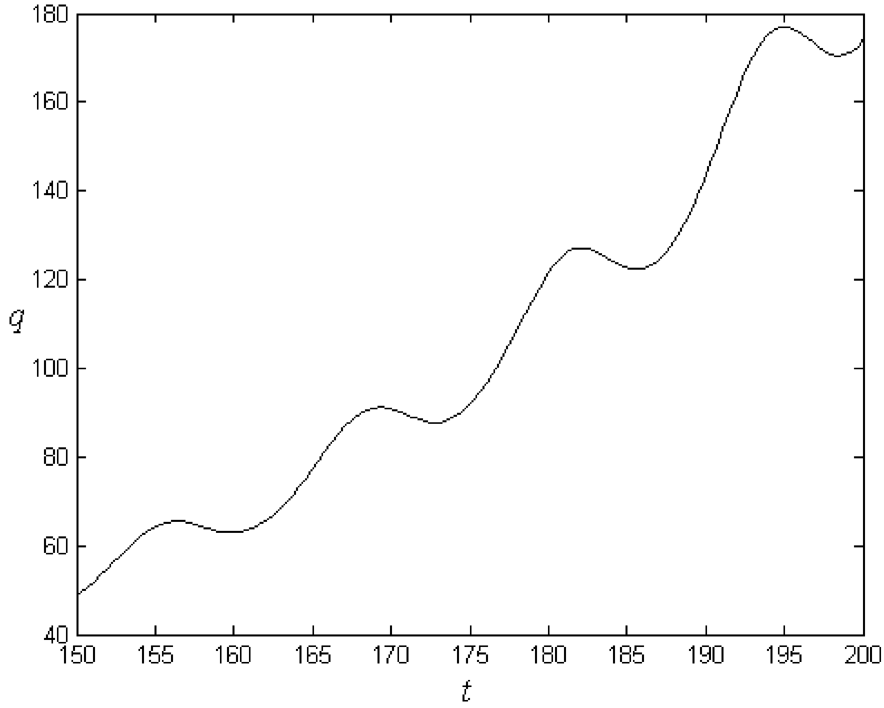

Figure 10: Growth cycles in the MVM1.

5 Conclusions

[to be written]

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