Theories of Economic Growth – Old and New

Heinz D. Kurz & Neri Salvadori

I. INTRODUCTION

Ever since the inception of systematic economic analysis at the time of the classical economists from William Petty to David Ricardo the problem of economic growth – its sources, forms and effects – was high on the agenda of economists. In the real world the problem and the fact of economic growth is, of course, of much longer standing. Even in the more or less stationary economies of antiquity the possibility, if not the fact, of economic expansion lingers at the back of certain considerations. Brick plates from Mesopotamia provide information about social productivity by means of a simple input-output calculation in terms of barley. The main question concerned the *surplus product* of barley the ancient society was able to generate, that is, the excess of total output in a year with a normal harvest over the amount of input of barley as seed or as means of subsistence of labourers plus any other inputs needed in the society measured in terms of barley. In other words, what the plates contained was a primitive system of social accounting of the following type:

$$\begin{aligned} \text{Surplus Product} &= \text{Gross Output} - (\text{Seed} + \text{Necessary Consumption} + \text{Other Inputs}) \\ &= \text{Gross Output} - \text{Necessary Input} \end{aligned}$$

where all magnitudes are measured in units of barley. This represents what recently, with regard to some ideas encountered in classical political economy, became known as the core of the 'corn model'. Relating the Surplus Product to the Necessary Input as a whole (or to some of its components) Thomas Robert Malthus, at the beginning of the nineteenth century, was to speak of a 'material rate of produce'. Renger (1991) writes about such rates that towards the end of the third millennium B.C. the average returns of barley with which the administration in Mesopotamia calculated were twenty, in some especially fertile regions even up to thirty times as large as the seed input. Later the ratio tended to fall due to a variety of factors, including the exhaustion of the land. Compared with flourishing Mesopotamia, classic Greece is said to have exhibited a ratio of output to seed between 4,5 and 7, while according to the Roman author Columella in Italy the figure was as low as 4.

From the Surplus Rate, that is, the ratio of Surplus Product to Necessary Input,

$$\text{Surplus Rate} = \frac{\text{Surplus}}{\text{Necessary Input}}$$

it is obviously only a small step intellectually, but a huge step historically to the concept of the *rate of growth*. This step was taken, at the latest, by economists in the seventeenth century, most notably William Petty. Corn model considerations played also an important role in the writings of David Ricardo and Robert Torrens who came close to discovering, with regard to the institutional setting of competitive capitalism, the fundamental duality of the (inverse) relationship between the rate of growth of the economy and consumption per capita, on the one hand, and the (inverse) relationship between the rate of profit and the real wage rate, on the other. Indeed, with free competition and assuming that the entire social surplus will be saved and invested, that is, accumulated, in conditions characterized by constant returns to scale, the surplus rate gives both the general rate of profit in the economy and its rate of growth. The corn model therefore provided useful services as a starting point of a probing into the laws of capital accumulation, economic growth and income distribution.

Other authors sought to render clear the origins of people's motivation to accumulate capital. Adam Smith claimed:

> the principle which prompts to save, is the *desire of bettering our condition*, a desire which, though generally calm and dispassionate, comes with us from the womb, and never leaves us till we go to grave. (WN II.iii.28; emphasis added)

To this he added that man's 'love of distinction', and the easiness with which that love can be satisfied in terms of fortunes and riches that can openly be displayed, provides a strong incentive to accumulate.

These few lines on the beginnings of an analysis of economic growth and some very complex problems related to it must suffice. Let us now turn to a brief discussion of the characteristic features of a selection of contributions to the problem under consideration. Section II summarizes some crucial features of Adam Smith's views on capital accumulation and economic growth. The emphasis is on two contradictory effects of capital accumulation contemplated by Smith: a tendency of the rate of profit to fall due to the intensification of competition among capital owners; and a tendency of the rate of profit to rise due to the increase in productivity associated with the division of labour. Section III turns to David Ricardo's approach to the theory of distribution and capital accumulation. We argue that in Ricardo the growth rate is endogenous and may fall to zero when, during capital accumulation and population growth, the rate of profit tends to fall due to diminishing returns in agriculture and the exhaustion of some natural resources. Section IV deals with linear models of economic growth: the authors

discussed include Robert Torrens, Karl Marx, Georg von Charasoff and John von Neumann. Section V provides a taxonomy of 'classical' cases in which the rate of profit, and thus the rate of growth, need not fall to zero. We consider three cases: (i) the absence of scarce non-accumulable factors of production; (ii) the existence of a 'backstop technology'; and (iii) increasing returns to capital that are external to firms. Section VI discusses 'neoclassical' ideas or models of exogenous growth, especially those of Alfred Marshall, Gustav Cassel, Knut Wicksell, Robert Solow, Trevor Swan and James Meade. Section VII classifies the recent literature on the so-called 'new' growth models (NGMs) into three groups according to the route by means of which they try to avoid diminishing returns to capital. Section VIII draws some conclusions and argues that the 'new' growth theory (NGT) shares some crucial elements of the classical approach to the problem of growth and distribution. Hence, it can be said that there is a 'revolution' in the proper sense of the word, that is, present-day growth theory is partly returning to the roots of the classical approach.

II. ADAM SMITH ON GROWTH

A characteristic feature of the classical approach is the view that production involves labour, produced means of production and natural resources. In contrast to some contributions to modern growth theory none of these factors – labour, capital and land – were considered negligible other than in thought experiments designed 'to illustrate a principle' (Ricardo). To understand real growth processes one had to come to grips with the interrelated laws governing the growth of population, the pace of accumulation and the rate and bias of technical innovation in an environment characterized by the scarcity of natural resources. At stake was an understanding of the working of a highly complex system.

1. *Capital Accumulation and the Division of Labour*

Adam Smith viewed the growth process as strictly *endogenous* (see also Lowe,[1954] 1987, p. 108, and Eltis, 1984, p. 69), placing special emphasis on the impact of capital accumulation on labour productivity. He began his inquiry into the *Wealth of Nations*, first published in 1776, by stating that income per capita

> must in every nation be regulated by two different circumstances; first, by the skill, dexterity, and judgment with which its labour is generally applied; and, secondly, by the proportion between the number of those who are employed in useful labour, and that of those who are not so employed (WN I.3).

According to Smith there is no upper limit to labour productivity. This is why Smith maintained that an investigation of the growth of income per capita is first and foremost an inquiry into 'The causes of this improvement, in the productive powers of labour, and the order, according to which its produce is naturally distributed among the different ranks and conditions of men in the society' (WN I.5).

Smith's attention focused accordingly on the factors determining the growth of labour productivity, that is, the factors affecting 'the state of the skill, dexterity, and judgment with which labour is applied in any nation' (WN I.6). At this point the accumulation of capital enters into the picture, because of Smith's conviction that the key to the growth of labour productivity is the division of labour which in turn depends on the extent of the market and thus upon capital accumulation. 'The greatest improvement in the productive powers of labour', we are told, 'seem to have been the effects of the division of labour' (WN I.i.1), both *within* given firms and industries and, even more significantly, *between* them. In his analysis in the first three chapters of book I of *The Wealth of Nations* Smith established the idea that there are *increasing returns* which are largely *external* to firms, that is, broadly compatible with the classical hypothesis of a uniform rate of profit. In the first chapter he made clear how powerful a device the division of labour is in increasing labour productivity, and analysed in some detail its major features: (i) the improvement of the dexterity of workers; (ii) the saving of time which is otherwise lost in passing from one sort of work to another; and, most importantly, (iii) the invention of specific machinery (cf. WN I.i.6-8). In the second chapter he argued that there is a certain propensity in human nature 'to truck, barter and exchange one thing for another', which appears to be rooted in 'the faculties of reason and speech', that gives occasion to the division of labour (WN I.ii.1-2). In the third chapter the argument is completed by stressing that the division of labour is limited by the extent of the market (cf. WN I.iii.1): a larger market generates a larger division of labour among people and, therefore, among firms, and a larger division of labour generates a larger productivity of labour for all firms.

Despite the presence of increasing returns, Smith retained the concept of a *general* rate of profit. His argument appears to be implicitly based on the hypothesis that each single firm operates at constant returns, while total production is subject to increasing returns. Even though some examples provided by Smith relate more to the division of labour within firms than to the division of labour among firms, Smith appears to be correct in sustaining that some of the activities which were originally a part of the division of labour within the firm may eventually become a different 'trade' or 'business', so that the division of labour *within* the firm is but a step towards the division of labour *amongst* firms. In the example of pin making at the beginning of chapter I, Smith pointed out that 'in the way in which this business is now carried on, not only

the whole work is a peculiar trade, but it is divided into a number of branches, of which the greater part are likewise peculiar trades' (WN I.i.3).

Smith's analysis foreshadows the concepts of *induced* and *embodied* technical progress, *learning by doing*, and *learning by using*. The invention of new machines and the improvement of known ones is said to be originally due to the workers in the production process and 'those who had occasion to use the machines' (WN I.i.9). At a more advanced stage of society making machines 'became the business of a peculiar trade', engaging 'philosophers or men of speculation, whose trade it is, not to do any thing, but to observe every thing; and who, upon that account, are often capable of combining together the powers of the most distant and dissimilar objects'. Research and development of new industrial designs becomes 'the principal or sole trade and occupation of a particular class of citizens' (ibid.). New technical knowledge is systematically created and economically used, with the sciences becoming more and more involved in that process. The accumulation of capital propels this process forward, opens up new markets and enlarges existing ones, increases effectual demand and is thus the main force behind economic and social development:

> The increase of demand ... never fails to lower [prices] in the long run. It encourages production, and thereby increases the competition of the producers, who, in order to undersell one another, have recourse to new divisions of labour and new improvements of art, which might never otherwise have been thought of (WN V.i.e.26).

Here we have a *dynamic* notion of competition, conceived of as rivalry, which anticipates in important respects the views on competition of authors such as Karl Marx and Joseph Alois Schumpeter. Smith also anticipates the following two ideas that are prominent within the 'new' growth theory literature:

(1) 'new improvements of art' are generated within the economic system by specialized activities;
(2) new technical knowledge is or eventually will become a *public good*, that is, nonrival and nonexcludable.

However, whilst, as we shall see, the advocates of the 'new' growth theory are bold enough to postulate a production function of new technical knowledge – for example, the concept of 'research technology' in Romer (1986) – that is, a definite quantitative relationship between output (additional knowledge) and some inputs, and to provide a formalisation of the positive externality, Smith, did not put his ideas into algebra.

2. *Are There Clear and Obvious Limits to Growth in Smith?*

Did Smith expect the endogenous growth factors to loose momentum as capital accumulates? He considered three potential limits to growth: an insufficient supply of workers, the scantiness of nature, and an erosion of the motives of accumulation. Smith saw that the scarcity and potential depletion of renewable and the depletion of exhaustible resources may constrain human productive activity and the growth of the economy, and pointed out that 'useful fossils and minerals of the earth, &c. naturally grow dearer as the society advances in wealth and improvement' (WN I.xi.i.3; see also I.xi.d). Yet, it cannot be claimed that he paid a lot of attention to the scarcity of natural resources and its impact on economic growth. At the time when he wrote, the limits to growth deriving from nature were apparently still considered rather distant and thus negligible. This was to change soon, with authors like West, Malthus and Ricardo placing emphasis on the scarcity of land as the main barrier to economic growth. But in Smith there are not yet clear signs of any growth pessimism.[1]

Smith also saw no danger that the process of accumlation might come to an end because of an insufficient supply of labour and the ensuing diminishing returns to capital. He rather advocated a view which was to become prominent amongst the classical economists: the supply of labour is generated within the socio-economic system, that is, *endogenously*. Interestingly, Smith was of the opinion that the size of the workforce is regulated by the demand for labour. He drew an analogy between the multiplication of animals and that of the inferior ranks of people. He wrote: 'Every species of animals naturally multiplies in proportion to the means of their subsistence, and no species can ever multiply beyond it' (WN I.viii.39). A similar principle is said to govern the multiplication of men: the 'liberal reward of labour', by enabling workers to provide better for their children, adjusts the workforce

> as nearly as possible in the proportion which the demand for labour requires. ... It is in this manner that the demand for men, like that for any other commodity, necessarily, regulates the production of men; quickens it when it goes too slowly, and stops it when it advances too fast. It is this demand which regulates and

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[1] According to Eltis (1984, p. 70), Smith 'clearly believed that growth would eventually cease when a country's potential for development was fully realised.' However, in Smith it is not sufficiently clear how a country's potential is defined. Ultimately, a falling trend in the rate of profit is taken to indicate that the potential is getting exhausted. Yet, as we shall see, Smith's explanation of that trend is difficult to sustain.

> determines the state of propagation in all the different industries of the world (WN I.viii.40).

Smith envisaged the growth of the labour force as endogenous, the determinant being the rate of capital accumulation. Real wages are higher, the more rapidly capital accumulates. As to the impact of high and rising real wages on the rate of profit, it appears that we cannot say anything definite, given Smith's opinion that 'the same cause ... which raises the wages of labour, the increase of stock, tends to increase its productive powers, and to make a smaller quantity of labour produce a greater quantity of work' (WN I.viii.57). However, surprisingly, Smith came up with a definitive answer in chapter IX of book I. He introduced the chapter in the following terms: 'The rise and fall in the profits of stock depend upon the same causes with the rise and fall in the wages of labour, the increasing or declining state of the wealth of the society; but those causes affect the one and the other very differently' (WN I.ix.1). He added:

> The increase of stock, which raises wages, tends to lower profit. When the stock of many rich merchants are turned into the same trade, their mutual competition naturally tends to lower its profit; and when there is a like increase of stock in all the different trades carried on in the same society, the same competition must produce the same effect in them all (WN I.ix.2).

This explanation of a falling tendency of the rate of profit in terms of 'competition' does not stand up to close examination.[2] First, since Smith commonly presupposed free competition, a fall in profitability cannot be traced back to an intensification of competition. Second, Smith erroneously tried to carry an argument that is valid in a partial framework over to a general framework. A shift of capital from one trade to another, other things equal, will tend to reduce the rate of profit obtained in the latter (and increase it in the former); this mechanism was referred to by Smith in his explanation of the 'gravitation' of actual or 'market' prices to their 'natural' levels (see Kurz and Salvadori, 1995a, ch. 1). An increase in the economy's capital stock as a whole need not have an adverse effect on the general rate of profit. It all depends on how the real wage rate and the technical conditions of production are affected in the course of the accumulation of capital. This problem was tackled by David Ricardo.

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[2] For an interesting different view placing special emphasis on Malthus's interpretation of Smith according to which Smith had ruled out constant and diminishing returns, see Negishi (1993).

Adam Smith explained economic growth thoroughly as an *endogenous* phenomenon. The growth rate depends on the decisions and activities of agents. Special emphasis is placed on the endogenous creation of new knowledge that can be used economically. New technical knowledge is treated as a good, which is or in the long run tends to become a public good. There are no clear and obvious limits to growth. The additional work force required in the process of accumulation is generated by that process itself: labour power is a commodity the quantity of which is regulated by the effectual demand for it. Diminishing returns due to scarce natural resources are set aside or taken to be compensated by the increase in productivity due to the division of labour.

III. DAVID RICARDO ON DIMINISHING RETURNS

Ricardo set aside what may be called *statically and dynamically increasing returns*. The beneficial effects of capital accumulation on productivity mediated through the extension of the division of labour play hardly any role in his analysis. In modern parlance, the problems of externalities which figured prominently in Smith's analysis are given only sparse attention. Much of Ricardo's argument was developed in terms of the implicit assumption that the set of (constant returns to scale) methods of production from which cost-minimizing producers can choose, is given and constant. In such a framework the question then is how scarce natural resources, such as land, affect profitability as capital accumulates. The resulting vision is reflected in what Ricardo called the 'natural course' of events.

1. *Diminishing Returns in Agriculture*

As capital accumulates and population grows, and assuming the real wage rate of workers given and constant, the rate of profit is bound to fall; due to extensive and intensive diminishing returns on land, 'with every increased portion of capital employed on it, there will be a decreased rate of production' (Ricardo, *Works* I, p. 98). Since profits are a residual income based on the surplus product left after the used up means of production and the wage goods in the support of workers have been deducted from the social product (net of rents), the 'decreased rate of production' involves a decrease in profitability. On the assumption that there are only negligible savings out of wages and rents, a falling rate of profit involves a falling rate of capital accumulation. Hence, Ricardo's 'natural course' of events will necessarily end up in a stationary state.

2. *Technical Progress: A Counteracting Factor*

This path should not be identified with the *actual* path the economy is taking because technical progress will repeatedly offset the impact of the 'niggardliness of nature' on the rate of profit:

> The natural tendency of profits then is to fall; for, in the progress of society and wealth, the additional quantity of food required is obtained by the sacrifice of more and more labour. This tendency, this gravitation as it were of profits, is happily checked at repeated intervals by the improvements in machinery, connected with the production of necessaries, as well as by the discoveries in the science of agriculture which enable us to relinquish a portion of labour before required, and therefore to lower the price of the prime necessary of the labourer (Ricardo, *Works* I, p. 120).

By contrast, Smith was of the opinion that the accumulation of capital will systematically lead to improvements in the productive powers. Ricardo did not see an intimate connection; he rather treated those improvements as the outcome of singular events – special scientific discoveries and the like – not necessarily tied up with capital accumulation. Put more strongly, whereas Smith considered technological progress essentially an endogenous phenomenon, Ricardo treated it as largely exogenous. There is, however, also an important similarity: neither of them was of the opinion that technical progress will always be such that any tendency of the rate of profit to fall will be effectively counteracted. The classical authors' view is perfectly compatible with phases of falling and phases of rising profitability in any particular economic system. Ricardo was one of the first to stress that technological progress can take several *forms* associated with different implications for the performance of the system, its growth, employment and the sharing out of the product between wages, rents and profits.[3] The idea of 'neutrality' of technical progress as it is necessarily entertained in steady-state growth theory was alien to Ricardo's thinking.

3. *The Endogeneity of Growth*

Like Smith, Ricardo thought that saving and investment, that is, accumulation, would largely come from profits, whereas wages and rents played a negligible role. Hence, as regards the dynamism of the economy attention should focus on profitability. Assuming that the marginal propensity to accumulate out of profits, *s*, is given and constant, a 'classical' accumulation function can be formulated

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[3] Ricardo's discussion of different forms of agricultural improvements in chapter 2 and of the 'gross produce reducing' form of technical progress in chapter 31 of the third edition of the *Principles.*

$$g = \begin{cases} s(r - r_{min}) & \text{if } r \geq r_{min} \\ 0 & \text{if } r \leq r_{min} \end{cases}$$

where is $r_{min} \geq 0$ is the minimum level of profitability, which, if reached, will arrest accumulation (cf. Ricardo, *Works* I, p. 120).

Ricardo saw the rate of accumulation as endogenous. The demand for labour is governed by the pace at which capital accumulates, the long-term supply of labour by the 'Malthusian Law of Population'. Real wages may rise, that is, the 'market price of labour' may rise above the 'natural' wage rate. This is the case in a situation in which capital accumulates rapidly, leading to an excess demand for labour. As Ricardo put it, 'notwithstanding the tendency of wages to conform to their natural rate, their market rate may, in an improving society, for an indefinite period, be constantly above it' (ibid., pp. 94-5). If such a constellation prevails for some time a ratchet effect may make itself felt: it is possible, Ricardo observed, that 'custom renders absolute necessaries' what in the past had been comforts or luxuries. Hence, the natural wage is driven upward by persistently high levels of the actual wage rate. Accordingly, the concept of 'natural wage' in Ricardo is a flexible one and must not be mistaken for a physiological minimum of subsistence.

4. *A Graphical Illustration*

Setting aside the complex wage dynamics in Ricardo's theory, that is, assuming a given and constant real wage rate and setting the minimum rate of profit equal to zero, we may illustrate Ricardo's view of the long-run relationship between profitability and accumulation and thus growth in a schematic way. Figure 1, originally used by Kaldor (1955-56), shows the marginal productivity of labour-cum-capital curve *CEGH*. It is decreasing since land is scarce: when labour-cum-capital increases, either less fertile qualities of land must be cultivated or the same qualities of land must be cultivated with processes which require less land per unit of product, but are more costly in terms of labour-cum-capital. Let the real wage rate equal *OW*. Then, if the amount of labour-cum-capital applied is L_1, the area $OCEL_1$ gives the product, $OWDL_1$ gives total capital employed, and *BCE* total rent.

Profit is determined as a residual and corresponds to the rectangular *WBED*. As a consequence, the *rate* of profit can be determined as the ratio of the areas of two rectangulars which have the same basis and, therefore, it equals the ratio *WB/OW*. Let us now consider the case in which the amount of labour-cum-capital is larger, that is, L_2. Then $OCGL_2$ gives the product, $OWFL_2$ the capital, *ACG* the rent, and *WAGF* the profits. The rate of profit has fallen to *WA/OW*. Obviously, if a positive profit rate implies a positive growth rate, the economy will expand until labour-cum-capital has reached the level $\overline{L}$. At that point the profit rate is equal to zero and so is

the growth rate. The system has arrived at the so-called stationary state: growth has come to an end because profitability has.

Figure 1

For both Smith and Ricardo the required size of the work force is essentially generated by the accumulation process itself. In other words, labour power is treated as a kind of producible commodity. It differs from other commodities in that it is not produced in a capitalistic way in a special industry on a par with other industries, but is the result of the interplay between the growth of the working population and socioeconomic conditions. In the most simple conceptualization possible, labour power is seen to be in elastic supply at a given real wage basket. Increasing the number of baskets available in the support of workers involves a proportional increase of the work force. In this view the rate of growth of labour supply adjusts to any given rate of growth of labour demand without necessitating a variation in the real wage rate.

In a slightly more sophisticated conceptualization, higher rates of growth of labour supply presuppose higher levels of the real wage rate. But the basic logic remains the same: in normal conditions the pace at which capital accumulates regulates the pace at which labour, a *non-accumulable* factor of production, grows. Thus labour cannot put a limit to growth because it is generated within the growth process. The only limit to growth can come from other nonaccumulable factors of production: as Ricardo and others made clear, these factors are natural resources in general and land in particular. In other words, there is only endogenous growth in Ricardo. This growth is bound to lose momentum as the system hits its natural barriers, especially as soon as extensive and intensive diminishing returns make themselves felt and are not counteracted by a sufficient technical progress. There is no exogenous growth in Ricardo. Despite the fact that he did not give much attention to Smith's idea of increasing returns and positive externalities of selfish behaviour, Ricardo's theory fulfills the criterion of an endogenous explanation of growth. This shows also that it is not a necessary condition for a theory to be considered a theory of endogenous growth that it assumes some kind of increasing returns. This becomes clear in Section IV below. Interestingly, its main message was anticipated by Ricardo.

5. *Production and growth with land as a free good*

In one place Ricardo contemplated the implications for income distribution and the rate of expansion of the economic system in the hypothetical case in which land of the best quality is available in abundance. He wrote:

> Profits do not necessarily fall with the increase of the quantity of capital because the demand for capital is infinite and is governed by the same law as population itself. They are both checked by the rise in the price of food, and the consequent increase in the price of labour. If there were no such rise, what could prevent population and capital from increasing without limit? (Ricardo, *Works* VI, p. 301)

If land of the best quality were available in abundance it would be a free good and no rent would be paid for its use. In this case the curve of the graph showing the marginal productivity of labour-cum-capital would be a horizontal line and the rate of profit would be constant whatever the amount of labour-cum-capital employed. This case is illsutrated in Figure 2. As a consequence, other things equal, the growth rate would also be constant: the system could grow for ever at a rate that equals the given rate of profit times the propensity to accumulate. As the passage from Ricardo's *Works* just quoted shows, Ricardo was perfectly aware of this implication.

Figure 2

IV. LINEAR CLASSICAL MODELS OF PRODUCTION

Central elements of classical analysis are the concept of production as a circular flow and the related concept of *surplus* product left after the wage goods and what is necessary for the replacement of the used up means of production have been deducted from the annual output. This surplus can be consumed or accumulated. With constant returns to scale and setting aside the problem of scarce natural resources, the notion of an economy expanding at a constant rate of growth was close at hand. In this section we shall summarize some contributions to what may be called linear growth theory with a classical flavour.

1. *Robert Torrens*

Robert Torrens in his *Essay on the External Corn Trade* clarified that the concept of surplus provides the key to an explanation of the *rate* of profit. He put forward a 'corn model' in which the rate of profit can be determined as the ratio of two quantities of corn: the surplus product and the corn advanced as seed and as food in the support of workers (Torrens, 1820, p. 361). Torrens acknowledged his indebtedness to Ricardo's 'original and profound inquiry into the laws by which the rate of profit is determined' (ibid., p. xix). One year later he published his *Essay on the Production of Wealth*, in which he generalized the argument to the case of two sectors, each of which produces a commodity that is either needed as a means of production or

as a means of subsistence in both sectors. In the numerical example provided by him the surplus and the social capital consisted of the same commodities in the same proportions, so that the rate of profit can be determined without having recourse to the system of relative prices (Torrens, 1821, pp. 372-3).

Torrens made it clear that the physical schema of the production of commodities by means of commodities is not only important for the determination of the rate of profit and relative prices – it also provides the basis for assessing the *growth potential* of the economy. As Torrens stressed, 'this surplus, or profit of ten per cent they [that is, the cultivators and manufacturers] might employ either in setting additional workers to work, or in purchasing luxuries for immediate enjoyment' (ibid., p. 373). If in each sector the entire surplus were to be used for accumulation purposes in the same sector, then the rates of expansion of the two sectors would be equal to one another and equal to the rate of profit. Champernowne (1945, p. 10) in his commentary on von Neumann's growth model was later to call a constellation of equi-proportionate growth a 'quasi-stationary state.'

2. *Karl Marx*

Growth in the model by Torrens is both linear and endogenous; the rate of growth depends on the general rate of profit and the propensity to accumulate. The same can be said of Marx's theory of accumulation and expanded reproduction in chapter 21 of volume II of *Capital* (Marx, [1885] 1956). There Marx studied the conditions under which the system is capable of reproducing itself on an upward spiralling level. The expansion of the economy at an endogenously determined rate of growth is possible. This rate depends on the proportion of the surplus value ploughed back into the productive system to increase the scale of operation. Marx stressed that the accumulation of capital is 'an element *immanent* in the capitalist process of production' (ibid., p. 497; emphasis added). For, 'the aim and compelling motive of capitalist production' is 'the snatching of surplus-value and its capitalisation, i.e., accumulation' (ibid., p. 507).

Marx illustrated his argument in terms of numerical examples relating to an economy with two departments, one which produces the means of production, while the other produces the means of consumption. Commodities are exchanged according to their labour values and the accumulation of surplus value takes place within the same department in which the surplus value has been 'produced' and appropriated. Given the real wage rate, the rates of profit in the two sectors assessed on the basis of labour values are known magnitudes. Designating these rates of profit with π_1 and π_2, respectively, and the sectoral shares of surplus-value saved and invested with s_1 and s_2, a uniform rate of growth g involves

$$g = \pi_1 s_1 = \pi_2 s_2 \quad \text{and thus} \quad s_1/s_2 = \pi_1/\pi_2,$$

that is, a definite proportion between the two sectoral propensities to accumulate (cf. ibid., p. 516).

3. *Georg von Charasoff*

The Russian mathematician Georg von Charasoff elaborated on Marx's analysis and was possibly the first to provide a clear statement of the fundamental duality relationship between the system of prices and the rate of profit on the one hand, and the system of quantities and the rate of growth on the other, in Charasoff (1910). He developed his main argument within the framework of an interdependent model of (single) production exhibiting all the properties of the later input-output model, and which is fully specified in terms of use values (rather than labour values as in the case of Marx) and labour needed per unit of output. Let **C** be the *n*x*n* matrix of material inputs, let **d** be the *n*-vector giving the real wage rate, and let **l** be the *n*-vector of direct labour inputs in the different production processes. The *n*x*n* input matrix **A** used by Charasoff includes the means of subsistence in the support of workers and is therefore given by

$$\mathbf{A} = \mathbf{C} + \mathbf{l}\mathbf{d}^T$$

that is, it equals what later became known as the 'augmented input matrix'. For a given real wage rate he showed that the rate of profit and relative prices are simultaneously determined and that the former equals the maximum rate of growth of the system compatible with the given conditions of production (cf. ibid., p. 124). Although Charasoff refrained from using mathematics in his argument, it is clear from his verbal argument that the rate of profit is determined by

$$r = G = \frac{1 - \lambda}{\lambda}$$

where r is the rate of profit, G is the maximum rate of growth and λ is the dominant real eigenvalue of matrix **A**. He thus anticipated, albeit in a much less general framework, an important result of John von Neumann.

4. *John von Neumann*

The most sophisticated linear model of endogenous growth was elaborated by John von Neumann (1945) in a paper first published in German in 1937 and then translated into English in 1945. In it von Neumann assumed there are *n* goods produced by *m* constant returns–to–scale production processes. There is a problem of the choice of technique which consists of

establishing which processes will actually be used and which not, being 'unprofitable'. Von Neumann (1945, pp. 1-2) took the real wage rate, consisting of the 'necessities of life', to be given and paid at the beginning of the uniform period of production, that is, he considered wages as a part of the capital advanced and thus as a part of the *physical real costs of production*. In addition, he assumed 'that all income in excess of necessities of life will be reinvested'. The characteristic features of the model include:

(1) 'Goods are produced not only from "natural factors of production", but in the first place from each other. These processes of production may be circular'.
(2) Primary factors of production can be expanded 'in unlimited quantities'.
(3) The processes of production 'can describe the special case where good G_j can be produced only jointly with certain others, viz. its permanent joint products'.
(4) Both circulating and fixed capital can be dealt with: 'wear and tear of capital goods are to be described by introducing different stages of wear as different goods, using a separate P_i [process *i*] for each of these'.
(5) The Rule of Free Goods is applied to all primary factors of production, with the exception of labour, and to overproduced goods. (ibid., pp. 1-2)

To see the basic argument, let **A** and **B** be the *m*x*n* input and output matrices, respectively, where **A** includes, as in Charasoff's case, the means of subsistence in the support of workers. At the going real wage rate, labour is taken to be in perfectly elastic supply, that is, available in whichever amount is required by the growth of the system. Von Neumann demonstrated that there is a solution to his model, which determines (i) which processes will be operated; (ii) at what rate the economic system will grow; (iii) what prices will obtain; (iv) what the rate of interest (rate of profit) will be; and (v) that, given the special assumptions employed, the rate of interest equals the rate of growth.

In von Neumann's model the rate of growth is determined *endogenously*.[4] He set aside the problem of scarcity of all non-accumulable factors of production: while all primary factors other than labour (that is, all natural resources) were taken to be available at whichever amount was

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4 This is one of the reasons why the conventional interpretation of that model as belonging to the tradition established by the so-called 'Walras-Cassel model' cannot be sustained (see Kurz and Salvadori, 1993). Cassel (1932) took as exogenously given the rates of growth of all primary factors and assumed their continuous full employment (see Section VI below). Von Neumann never made this assumption.

needed at zero price, labour was assumed to be available at the required amount at a given real wage rate.

V. A TYPOLOGY OF CASES

We can now classify some broad cases in which the rate of profit, and therefore the rate of growth, does not fall to zero. There is perpetual growth provided that the premises underlying the different cases hold infinitely. It will be seen that while the cases discussed are all derived from a classical framework of the analysis as it was developed by Adam Smith and David Ricardo, the cases exhibit some striking similarities to the types of NGMs discussed in Section VII.

1. *Constant Returns to Capital*

As we have seen, the main ingredient to obtain a stationary state in the Ricardian model is the existence of land available in limited supply. If land were not needed as an input or if land of the best quality were available in abundance, then the graph giving the marginal productivity of labour-cum-capital would be an horizontal line and therefore the rate of profit would be constant whatever the amount of labour-cum-capital. This case is illustrated in Figure 2 above. As a consequence, the growth rate would also be constant.

Yet to assume that land is not useful in production or that it is available in given quality and unlimited quantity is unnecessarily restrictive. With the system growing infinitely, the point will come where land of the best quality will become scarce. This brings us to a case similar to one discussed in the economics of exhaustible resources, in which there is an ultimate 'backstop technology'. For example, some exhaustible resources are used to produce energy. In addition, there is solar energy which may be considered an undepletable resource. A technology based on the use of solar energy defines the backstop technology mentioned. Let us translate this assumption into the context of a Ricardian model with land.

2. *A Backstop Technology*

The case under consideration corresponds to a situation in which 'land', although useful in production, is not indispensable. In other words, there is a technology that allows the production of the commodity without any 'land' input; this is the backstop technology. With continuous substitution between labour-cum-capital and land, the marginal productivity of labour-cum-capital would be continuoulsy decreasing, but it would be bounded from below. This case is illustrated in Figure 3, with the dashed line giving the lower boundary. In this case the profit rate

and thus the growth rate are falling, but they could never fall below certain positive levels. The system would grow indefinitely at a rate of growth that asymptotically approaches the product of the given saving rate times the value of the (lower) boundary of the profit rate. In Figure 3 the latter is given by *WR/OW*.

Figure 3

3. *Increasing Returns to Capital*

The final case is that of increasing returns to labour-cum-capital (see Figure 4), as it was discussed, following Adam Smith, by Allyn Young (1928) and Nicholas Kaldor (1957 and 1966). Taking the wage rate as given and constant, the rate of profit and the rate of growth will rise as more labour-cum-capital is employed. (In Figure 4 it is assumed that there is an upper boundary to the rise in output per unit of labour-cum-capital given by *OR*.) To preserve the notion of a uniform rate of profit, it is necessary to assume that the increasing returns are *external* to the firm and exclusively connected with the expansion of the market as a whole and the social division of labour. This implies that while in the case of decreasing returns due to the scarcity of land (cf. Figures 1 and 3) the product was given by the area under the marginal productivity curve, now the product associated with any given amount of labour-cum-capital is larger than or equal to that amount multiplied by the corresponding level of output per unit of labour-cum-capital. In any case, the sum of profits and wages equals the product of the given amount of labour-cum-capital multiplied by the corresponding level of output per unit of labour-cum-capital.[5] Hence, in the case in which labour-cum-capital is L_2, the product is given by the

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[5] Let $x = f(L, L^*)$ be the product of the last unit of labour-cum-capital, when L represents the amount of labour-cum-capital employed and the division of labour is artificially kept fixed at the level appropriate when the amount of labour-cum-capital employed is L^*. Obviously, $f(L, L^*)$ as a function of L alone is either decreasing as in Figures 1 and 3 (if land is scarce) or constant as in Figure 2 (if land is not scarce). The product at L^* equals $\int_0^{L^*} f(L, L^*)\mathrm{d}L$, i.e., the area under the curve $f(L, L^*)$ in the range $[0, L^*]$. If $\frac{\partial f}{\partial L^*} > -\frac{\partial f}{\partial L}$ for $L^* = L$, then the curve $x = f(L, L)$, which is the curve depicted in Figure 4, is increasing, but the product is, as stated in the text, larger than or equal to the sum of profits and wages, which equals the product of the given amount of labour-cum-capital multiplied by the corresponding level of output per unit of labour-cum-capital.

corresponding rectangle. As a consequence, the product is larger than the area under the marginal productivity curve. The cases of decreasing and increasing returns are therefore not symmetrical: with increasing returns a rising real wage rate need not involve a falling general rate of profit.

Figure 4

VI. MODELS OF EXOGENOUS GROWTH

The marginalist or 'neoclassical' school of economic thought seeks to explain income distribution in a symmetrical way via the relative scarcities of the factors of production, labour, 'capital,' and land. Interestingly, the idea of *exogenous* growth which classical theory did *not* entertain is the starting point of important early works in the marginalist tradition.

1. *Alfred Marshall and Gustav Cassel*

The idea of an economic system growing exclusively because some exogenous factors make it grow has variously been put forward in the history of economic thought as a standard of comparison. For example, in chapter V of book V of his *Principles*, first published in 1890, Alfred Marshall ([1890] 1977, p. 305) introduced the 'famous fiction of the "Stationary state" ... to contrast the results which would be found there with those in the modern world'. By relaxing one after another of the rigid assumptions defining the stationary state, Marshall sought to get gradually closer to the 'actual conditions of life'. The first relaxation concerned the premise of a constant (working) population:

> The Stationary state has just been taken to be one in which population is stationary. But nearly all its distinctive features may be exhibited in a place where population and wealth are both growing, provided they are growing at about the same rate, and there is no scarcity of land: and provided also the methods of production and the conditions of trade change but little; and above all, where the character of man himself is a constant quantity. For in such a state by far the most important conditions of production and consumption, of exchange and distribution will remain of the same quality, and in the same general relations to one another, though they are all increasing in volume. (ibid., p. 306)

The resulting economic system grows at a constant rate which equals the exogenous rate of growth of population.[6] Income distribution and relative prices are the same as in the stationary economy. In modern parlance: the system expands along a steady-state growth path.

We encounter essentially the same idea in Gustav Cassel's ([1918] 1932) *Theory of Social Economy*. The model of exogenous growth delineated by Cassel can be considered the proximate starting point of the development of neoclassical growth theory. In chapter IV of book I of the treatise Cassel presented two models, one of a stationary economy, the other one of an economy growing along a steady-state path.

In his first model Cassel assumed that there are *z* (primary) factors of production. The quantities of these resources and thus the amounts of services provided by them are taken to be in given supply. The *n* goods produced in the economy are pure consumption goods, that is, there are no produced means of production or capital goods contemplated in the model: goods are produced exclusively by combining primary factor services at fixed technical coefficients of production. There are as many single-product processes of production as there are goods to be produced, that is, there is no choice of technique. General equilibrium is characterized by the following sets of equations:

(1) equality of supply and demand for each factor service;
(2) equality of the price of a good and its cost of production, that is, the sum total of factor service payments incurred in its production, and thus the absence of extra profits;
(3) equality of supply and demand for each good produced, where the demand for each good is conceived as a function of the prices of all goods.

The resulting sets of equations constitute what is known as the 'Walras-Cassel model' (Dorfman, Samuelson and Solow, 1958, p. 346). It satisfies the then going criterion of completeness: there are as many equations as there are unknowns to be ascertained.[7]

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6 It should be noted that Marshall (1977, book IV, ch. IV) saw reason to suppose that the growth of population depended, among other things, on socioeconomic factors and thus could not sensibly be treated, other than in a first step of the analysis, as exogenous.

7 As is well known, the approach to the theory of general equilibrium in terms of equations was attacked by Knut Wicksell, Hans Neisser, Heinrich von Stackelberg, Frederick Zeuthen, Karl Schlesinger and Abraham Wald and led to the development of the neoclassical theory of general equilibrium in terms of *in*equalities coupled with the

Cassel (1932, pp. 152-3) then turned to the model of a uniformly progressing economy. Although described only verbally, he introduced the model in the following way:

> We must now take into consideration the society which is progressing at a uniform rate. In it, the quantities of the factors of production which are available in each period ... are subject to a uniform increase. We shall represent by [*g*] the fixed rate of this increase, and of the uniform progress of the society generally.

In Cassel's view this generalization to the case of an economy growing at an exogenously given and constant rate does not cause substantial problems. The previously developed set of equations can easily be adapted appropriately, 'so that the whole pricing problem is solved'. Cassel thus arrived at basically the same result as Marshall.

2. *Knut Wicksell*

Prior to Cassel, Knut Wicksell had dealt with the problem of growth and income distribution in volume I of his *Lectures* (Wicksell, [1901] 1934). Wicksell assumed that production is carried out by means of labour, land and capital, that is, produced means of production, and that there was the possibility of substitution between these factors. He was very clear about the deficiency of the notion of capital in marginal productivity theory (see Kurz, 2000). With heterogeneous capital goods, 'social capital' had of necessity to be conceived of as a *value* magnitude. Trying to explain the rate of interest in terms of the marginal product of (value) capital implied 'arguing in a circle' (ibid., p. 149), since capital and the rate of interest enter as a cost in the production of capital itself. Hence the value of the capital goods inserted in the production function depends on the rate of interest and will generally change with it. Nevertheless Wicksell thought that the theory could be used in order to explain the long-run trend of profitability.

In the first two parts of volume I of the *Lectures* it is established that an increase in the 'amount of capital', given the amount of labour employed and the amount of land available, tends to diminish the marginal product of capital and thus the rate of interest. More precisely, different states of the economy characterized by different endowments of factors of production are compared. This, Wicksell (1934, p. 7) expounded, is the 'static point of view, i.e. we shall assume, in principle, a society which retains unchanged from year to year the same population, the same area of territory and the same amount of capital, and remains on the same level of technical achievement'. There is on the other hand 'a more dynamic point of view' which focuses

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introduction of the Rule of Free Goods (or free disposal assumption); see Kurz and Salvadori (1995a, ch. 13, section 7).

attention on 'the problem of saving or *accumulation of capital*.' Wicksell confronted this problem by first reformulating the findings of the static theory in the new 'dynamic' framework. He started from the premise that 'the progressive accumulation of capital must be regarded as economical so long as any rate of interest, however low, exists',[8] and added:

> Under such conditions, we should therefore expect a continual accumulation of capital – though at a diminishing rate – and, at the same time, a continual fall in the rate of interest (ibid., p. 209).

Here we have a clear expression that in neoclassical models without exogenous factors that make the system grow, the economy will asymptotically converge to a stationary state *strictu sensu*.

3. *Robert Solow, Trevor Swan and James Meade*

The neoclassical growth models of the 1950s and early 1960s differ from the growth version of the Walras-Cassel model in five important respects:

(1) they are macro-models with a single produced good only which could be used both as a consumption good and as a capital good;
(2) the number of primary factors of production is reduced to one, homogeneous labour (as in Solow, 1956 and 1963; Swan, 1956), or two, homogeneous labour and homogeneous land (as in Swan, 1956; Meade, 1961);
(3) the all-purpose good is produced by means of labour, capital, that is, the good itself, and possibly land;
(4) there is a choice of technique, where technical alternatives are given by a macroeconomic production function, which is homogenous of degree one with positive and decreasing marginal productivities with respect to each factor of production; and
(5) planned saving, which is taken to be equal to planned investment at all times, is proportional to net income, that is, a 'Keynesian' saving function is assumed.

Focusing attention on the models with a single primary factor (labour), in steady-state equilibrium

$$sf(k) = gk,$$

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[8] Wicksell implicitly assumed a zero rate of time preference. In an earlier part of his book he had rejected Böhm-Bawerk's arguments in favour of a positive rate of time preference as 'evidently untenable' (1934, p. 169).

where s is the (marginal and average) propensity to save, $f(k)$ is the per unit of labour or *per capita* production function, k is the capital-labour ratio (where labour is measured in terms of efficiency units), and g is the steady-state growth rate of capital (and labour, and income etc.). In steady-state equilibrium output expands exactly as the exogenous factors make it grow. Note that assuming $s > 0$ presupposes that the exogenous factors are growing at some positive rate. In these models the steady-state rate of growth is exogenous. Outside steady-state equilibrium the rate of growth can be shown to depend also on the behavioural parameter of the system, that is, the propensity to save (and invest), but that parameter plays no role in determining the long-term rate of growth.

While these models are aptly described as models of *exogenous* growth, they can also be described as models of *endogenous* profitability. Since in the one-good framework adopted by the authors under consideration the rate of profit r equals the marginal productivity of capital,

$$r = f'(k),$$

the two equations are able to determine a relationship between the rate of profit and the steady-state rate of growth. The following section shows that the NGMs essentially reverse what is endogenous and what is exogenous. In other words, without too much of an exaggeration they can be called models of *endogenous* growth and *exogenous* profitability.

VII. THE 'NEW' MODELS OF ENDOGENOUS GROWTH

One of the key properties of the NGMs emphasized by their advocates is the limitation of diminishing returns to capital. The first generation of NGMs defined the confines within which subsequent contributions to NGT were carried out. The attention focuses on the mechanism that prevents the returns to capital from falling (below a certain level).[9]

1. *Constant Returns to Capital*

The first class of models set aside all non-accumulable factors of production such as labour and land and assume that all inputs in production are accumulable, that is, 'capital' of some kind. The simplest version of this class is the so-called 'AK model', which assumes that there is a linear

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9 For a more detailed treatment of these models, see Kurz and Salvadori (1995b and 1998).

relationship between total output, Y, and a single factor capital, K, both consisting of the *same* commodity:

$$Y = AK, \tag{1}$$

where $1/A$ is the amount of that commodity required to produce one unit of itself. Because of the linear form of the aggregate production function, these models are also known as 'linear models'. This model is immediately recognized as the model dealt with in Subsection V.1 on the assumption that the technology to produce corn is the one illustrated in Figure 2. The rate of return on capital r is given by

$$r + \delta = \frac{Y}{K} = A, \tag{2}$$

where δ is the exogenously given rate of depreciation. There is a large variety of models of this type in the literature. In the two-sector version in Rebelo (1991) it is assumed that the capital good sector produces the capital good by means of itself and nothing else. It is also assumed that there is only one method of production to produce the capital good. Therefore, *the rate of profit is determined by technology alone*. Then the saving-investment mechanism jointly with the assumption of a uniform rate of growth, that is, a steady-state equilibrium, determines a relationship between the growth rate, g, and the rate of profit, r. Rebelo (1991, pp. 504 and 506) obtains either

$$g = \frac{A - \delta - \rho}{\sigma} = \frac{r - \rho}{\sigma} \quad , \tag{3}$$

or

$$g = (A - \delta)s = sr. \tag{4}$$

Equation (3) is obtained when savings are determined on the assumption that there is an immortal representative agent maximizing the following intertemporal utility function

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\sigma} [c(t)^{1-\sigma} - 1] \, dt,$$

subject to constraint (1), where ρ is the discount rate, or rate of time preference, and $1/\sigma$ is the elasticity of substitution between present and future consumption ($1 \neq \sigma > 0$), and where $Y = c(t) + \dot{K}$. Equation (4) is obtained when the average propensity to save s is given. Hence, in this model the rate of profit is determined by technology alone and the saving-investment mechanism determines the growth rate.

King and Rebelo (1990) essentially followed the same avenue. Instead of one kind of 'capital' they assumed that there are two kinds, real capital and human capital, both of which are accumulable. There are two lines of production, one for the social product and the real capital, which consist of quantities of the same commodity, and one for human capital. The production functions relating to the two kinds of capital are assumed to be homogeneous of degree one and strictly concave. There are no diminishing returns to (composite) capital for the reason that there is no nonaccumulable factor such as simple or unskilled labour that enters into the production of the accumulable factors, investment goods and human capital.[10] As in Rebelo's model the rate of profit is uniquely determined by the technology (and the maximization of profits which, because of the Non-substitution Theorem[11], implies that only one technique can be used in the long run); the growth rate of the system is then endogenously determined by the saving-investment equation. The larger the propensities to accumulate human and physical capital, the larger is the growth rate.

2. ***Returns to Capital Bounded from Below***

The second class of models preserve the dualism of accumulable and non-accumulable factors but restrict the impact of an accumulation of the former on their returns by a modification of the aggregate production function. Jones and Manuelli (1990), for example, allow for both labour and capital and even assume a convex technology, as the Solow model does. However, a convex technology requires only that the marginal product of capital is a decreasing function of its stock, not that it vanishes as the amount of capital per worker tends towards infinity. Jones and Manuelli assume that

$$h(k) \geq bk, \quad \text{each } k \geq 0,$$

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10 The assumption that the formation of human capital does not involve any unskilled labour as an input is not convincing: the whole point of education processes is that a person's capacity to perform unskilled labour is gradually transformed into his or her capacity to perform skilled labour. Adam Smith, for example, was perfectly aware of this. For an analytical treatment of the problem of human capital, taking Smith's discussion as a starting point, see Kurz and Salvadori (1995a, ch. 11).

11 We need a special case of the Non-substitution Theorem, because no primary factor (or a primary factor with a zero remuneration) is assumed; see Kurz and Salvadori (1995c).

where $h(k)$ is the per capita production function and b is a positive constant. The special case contemplated by them is

$$h(k) = f(k) + bk, \qquad (5)$$

where $f(k)$ is the conventional per capita production function. As capital accumulates and the capital-labour ratio rises, the marginal product of capital will fall, approaching asymptotically b, its lower boundary. With a given propensity to save, s, and assuming capital never wears out, the steady-state growth rate g is endogenously determined: $g = sb$. Assuming, on the contrary, intertemporal utility maximization, the rate of growth is positive provided the technical parameter b is larger than the rate of time preference ρ. In the case in which it is larger, the steady-state rate of growth is given by equation (3) with $r = b$.

It is not difficult to recognize that the difference between the model of Jones and Manuelli (1990) and that of Rebelo (1991) is the same as the one existing between the cases illustrated by Figures 3 and 2 above.

3. *Factors Counteracting Diminishing Returns to Capital*

Finally, there is a large class of models contemplating various factors counteracting any diminishing tendency of returns to capital. Here we shall be concerned only with the following two sub-classes: human capital formation and knowledge accumulation. In both kinds of models *positive external effects* play an important part; they offset any fall in the marginal product of capital.

A. Human Capital Formation

Models of the first sub-class attempt to formalize the role of human capital formation in the process of growth. Elaborating on some ideas of Uzawa (1965), Lucas (1988) assumed that agents have a choice between two ways of spending their (non-leisure) time: to contribute to current production or to accumulate human capital. It is essentially the allocation of time between the two alternatives contemplated that decides the growth rate of the system. For example, a decrease in the time spent producing goods involves a reduction in current output; at the same time it speeds up the formation of human capital and thereby increases output growth. With the accumulation of human capital there is said to be associated an externality: the more human capital society as a whole has accumulated, the more productive each single member will be. This is reflected in the following macroeconomic production function

$$Y = AK^{\beta}(uhN)^{1-\beta}h^{*\gamma}, \qquad (6)$$

where the labour input consists of the number of workers, N, times the fraction of time spent working, u, times h which gives the labour input in efficiency units. Finally, there is the term h^*. This is designed to represent the externality. The single agent takes h^* as a parameter in his or her optimizing by choice of c and u. However, for society as a whole the accumulation of human capital increases output both directly and indirectly, that is, through the externality. Here we are confronted with a variant of a *public good* problem, which may be expressed as follows. The individual optimizing agent faces constant returns to scale in production: the sum of the partial elasticities of production of the factors he or she can control, that is, his or her physical and human capital, is unity. Yet for society as a whole the partial elasticity of production of human capital is not $1 - \beta$, but $1 - \beta + \gamma$.

Lucas's conceptualization of the process by means of which human capital is built up is the following:

$$\dot{h} = \upsilon h(1 - u), \qquad (7)$$

where υ is a positive constant. (Note that equation (7) can be interpreted as a 'production function' of human capital.)

Interestingly, it can be shown that if the above mentioned externality is *not* present, that is, if γ in equation (6) equals zero, and therefore returns to scale are constant and, as a consequence, the Non-substitution Theorem holds, endogenous growth in Lucas's model is obtained in essentially the same way as in the models by Rebelo (1991) and King and Rebelo (1990): the rate of profit is determined by technology and profit maximization alone; and for the predetermined level of the rate of profit the saving-investment mechanism determines the rate of growth. Yet, as Lucas himself pointed out, the endogenous growth is positive *independently* of the fact that there is the above mentioned externality, that is, independently of the fact that γ is positive.[12] *Therefore, while complicating the picture increasing returns do not add substantially to it: growth is endogenous even if returns to scale are constant.* If returns to scale are not constant then the Non-substitution Theorem does not apply, implying that neither the competitive technique nor the associated rate of profit are determined by technical alternatives and profit maximization alone. Nevertheless, these two factors still determine, in steady states, a relationship between the rate of profit and the rate of growth. This relationship together with the relationship between the same rates obtained from the saving-investment mechanism determines both variables. Although

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[12] For a demonstration of this, see Kurz and Salvadori (1995b, pp. 13-19).

the analysis is more complex, essentially the same mechanism applies as in the models dealt with in Subsection VII.1.

B. Technical Change

Models of the second sub-class attempt to portray technological change as generated endogenously. The proximate starting point of this kind of models was Arrow's (1962) paper on 'learning by doing'. Romer (1986) focuses on the role of a single state variable called 'knowledge' or 'information' and assumes that the information contained in inventions and discoveries has the property of being available to anybody to make use of it at the same time. In other words, information is considered essentially a non-rival good. Yet, it need not be totally non-excludable, that is, it can be monopolized at least for some time. It is around the two different aspects of publicness – non-rivalry and non-excludability – that the argument revolves. Discoveries are made in research and development departments of firms. This requires that resources be withheld from producing current output. The basic idea of Romer's (1986, p. 1015) model is 'that there is a trade-off between consumption today and knowledge that can be used to produce more consumption tomorrow'. He formalizes this idea in terms of a 'research technology' that produces 'knowledge' from forgone consumption. Knowledge is assumed to be cardinally measurable and not to depreciate: it is like perennial capital.

Romer stipulates a research technology that is concave and homogeneous of degree one,

$$\dot{k}_i = G(I_i, k_i), \tag{8}$$

where I_i is an amount of forgone consumption in research by firm i and k_i is the firm's current stock of knowledge. (Note that the forgone consumption good is a capital good utilized in the production of 'knowledge'.) The production function of the consumption good relative to firm i is

$$Y_i = F(k_i, K, \mathbf{x}_i), \tag{9}$$

where K is the accumulated stock of knowledge in the economy as a whole and $\mathbf{x}_i$ are all inputs different from knowledge. The function is taken to be homogeneous of degree one in k_i and $\mathbf{x}_i$ and homogeneous of a degree greater than one in k_i and K. Romer (1986, p. 1019) assumes that 'factors other than knowledge are in fixed supply'. This implies that 'knowledge' is the only *capital good* utilized in the production of the consumption good. Spillovers from private research and development activities increase the public stock of knowledge K.

Assuming, contrary to Romer, that the above production function (9) is homogeneous of degree one in k_i and K involves a constant marginal product of capital: the diminishing returns to k_i are

exactly offset by the external improvements in technology associated with capital accumulation. In this case it can be shown that, similar to the NGMs previously dealt with, the rate of profit is determined by technology and profit maximization alone, provided, as is assumed by Romer, that the ratio K/k_i equals the (given) number of firms. The saving-investment relation then determines endogenously the growth rate. Once again endogenous growth does not depend on an assumption about increasing returns with regard to accumulable factors. Growth would be no more endogenous if increasing returns were to be assumed: such an assumption would only render the analysis a good deal more complicated. In particular, a steady-state equilibrium does not exist, and in order for an equilibrium to exist the marginal product of capital must be bounded from above. This is effected by Romer in terms of an *ad hoc* assumption regarding equation (8) (ibid., p. 1019). This assumption is not different from the one used in drawing Figure 4, where the marginal product of corn is shown to be increasing with the scale of production, but is bounded from above.

VIII. CONCLUSION

The NGMs revolve around a few simple and rather obvious ideas which have been anticipated by earlier economists, most notably Adam Smith and David Ricardo. Many of the interesting aspects of the NGMs are related to the classical perspective their authors (often unwittingly) take on the problem of growth, whereas some of their shortcomings derive from the lack of solutions to the problems of the neoclassical theory of growth which were put into sharp relief during the 1960s and 1970s. It has also been hinted that in some non-neoclassical approaches to the theory of accumulation and growth, the endogeneity of the growth rate has always been taken for granted. A brief look into the history of economic thought shows that from Adam Smith via David Ricardo, Robert Torrens, Thomas Robert Malthus, Karl Marx up to John von Neumann both the equilibrium and the actual rate of capital accumulation and thus both the equilibrium and the actual rate of growth of output as a whole were seen to depend on agents' behaviour, that is, endogenously determined. In this regard there is indeed nothing new under the sun.

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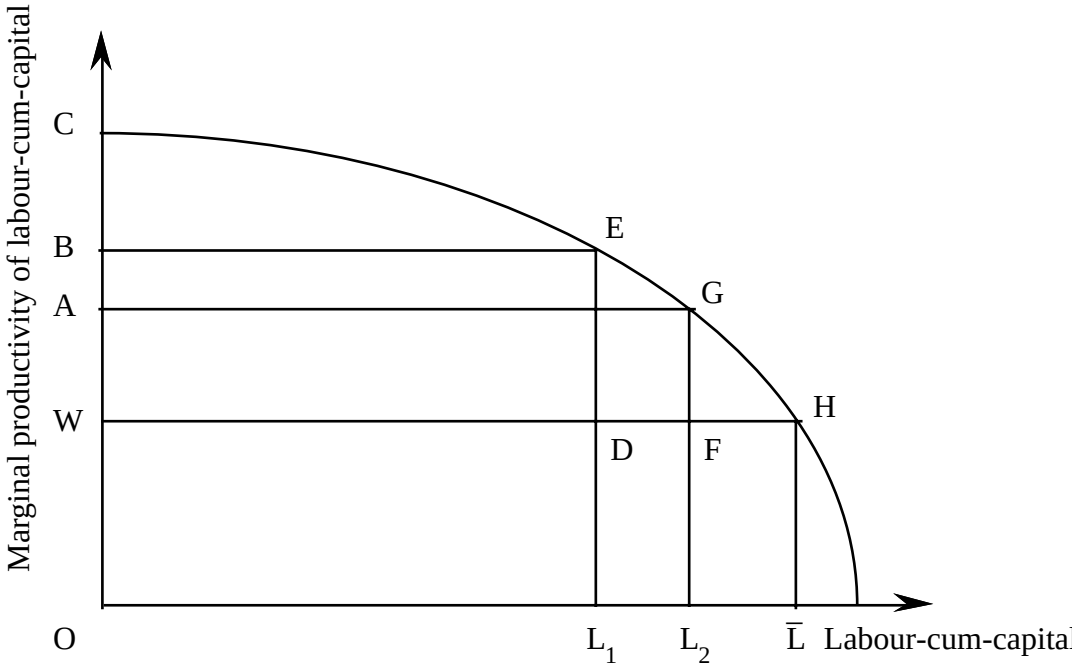

Figure 1: Land as an indispensable resource

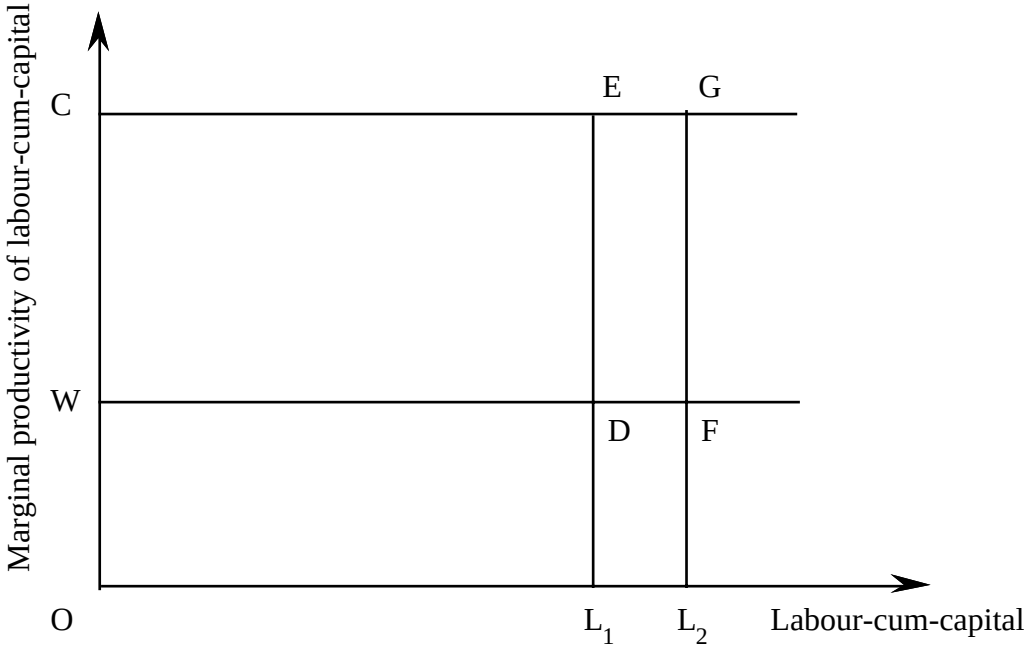

Figure 2: Land as a free good

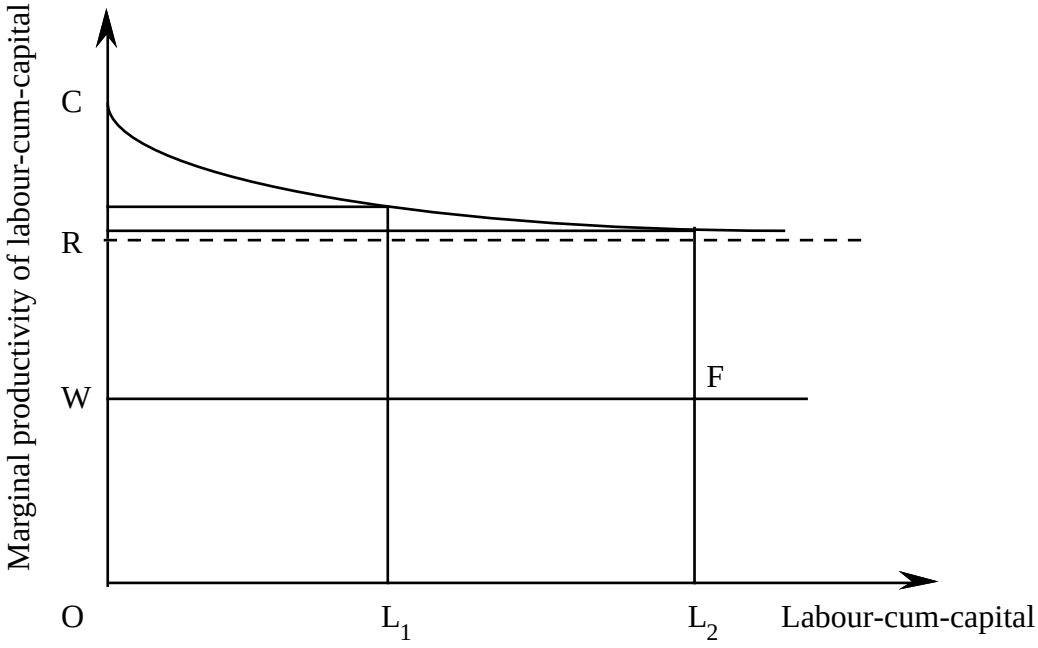

Figure 3: A backstop technology

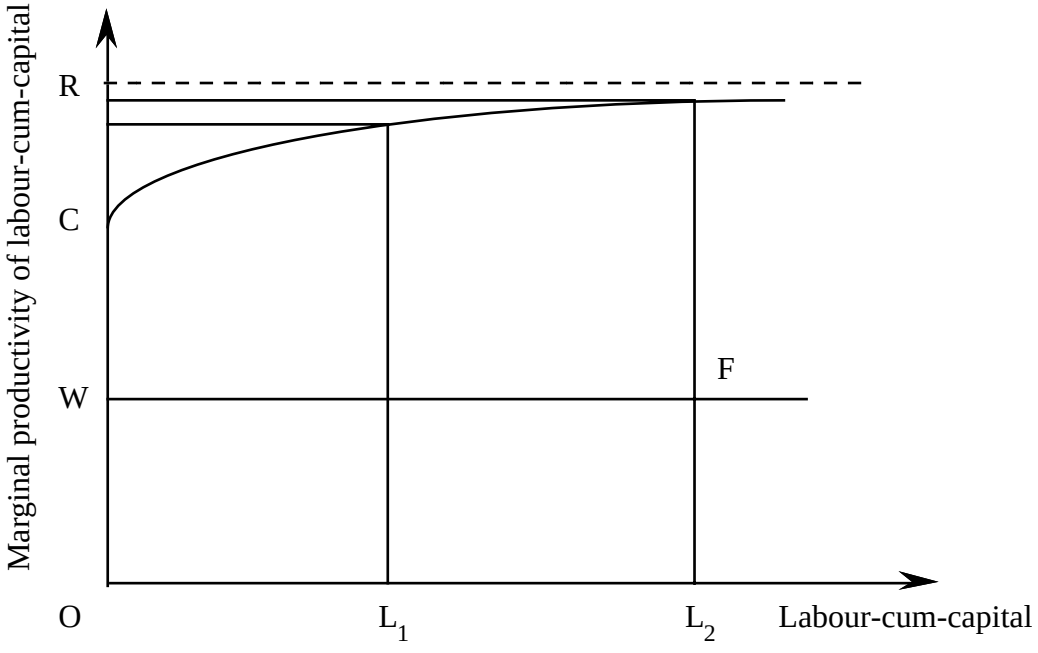

Figure 4: Increasing returns