Heterogeneous Mark-ups, Demand Composition, and the Inequality-Growth Relation

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Abstract

We explore the relationship between inequality and demand structure in an endogenous growth model where consumers expand consumption along a hierarchy of needs and desires. The consumption hierarchy is captured by non-homothetic preferences implying that the shape of the demand curves for various goods depends on the distribution of income. This setting enables us to study a mechanism that so far has been largely neglected in the literature: the role that inequality plays for the prices that innovators can charge and the corresponding quantities that innovators can sell. Thus, the influence of inequality on innovation incentive and growth can be analyzed.

We get the following results: (i) Changes in inequality affect the aggregate price structure and there may be market exclusion of the poor due to high prices. (ii) If there is exclusion, higher inequality tends to increase growth because the profit share increases. However, higher inequality due to a bigger group of poor people may reduce growth. (iii) If the innovators always sell to the whole population, inequality has an unambigously negative impact on growth. Prices are then determined by the willingness to pay of the poor. An even more egalitarian distribution allows the monopolist to set higher prices and earn higher profits as the poor are the 'critical' consumers that determine demand at the extensive margin.

1 Introduction

This paper studies the impact of hierarchic preferences on distribution and growth. When consumers have hierarchic preferences the structure of demand is affected by the distribution of income. Poor people concentrate most of their expenditures on basic needs, whereas richer people direct their expenditures to

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more luxurious goods. The empirical relevance of a hierarchic structure of demand is documented by 'Engel's law', one of the most robust empirical findings in economics. According to Engel's law the expenditure share for food decreases with income.

When demand is affected by the income distribution, inequality may be an important determinant of innovations and growth. The empirical importance of the inequality-growth relationship is a matter of discussion in the empirical literature. A number of earlier studies have found a robust negative correlation between growth rates and income inequality in cross-country regressions (Persson and Tabellini (1994), Alesina and Rodrik (1994), Clarke (1995), and in particular Perotti (1996)). While more recent work by Deininger and Squire (1998) cast doubt on the robustness of the relationship between growth and the distribution of income, empirical regularities in the inequality-growth relationship remain. In this paper we do not aim to directly address findings from this empirical literature. Our aim is to study the interesting meachanisms and show under which conditions we get a positive and when we get a negative impact of inequality on growth.

While recent research has extensively dealt with the question how income inequality affects the long-run growth performance of economies, little attention has been paid to the role of the income distribution for product demand and the resulting impact on innovations. Instead, much of the recent literature has either focused on the role of capital market imperfections, (see Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997), and others) or on political mechanisms (Bertola (1993), Persson and Tabellini (1994), Alesina and Rodrik (1994), and others). In contrast, the present paper focuses on the role of inequality for the dynamics of an innovator's demand and does neither rely on imperfect capital markets nor on politico-economic arguments.

In the standard Schumpeterian growth models consumers have homothetic preferences. By this assumption, the level of demand for the various goods - including the innovator's product - does not depend on the income distribution. Instead, we study a situation where preferences are non-homothetic and income distribution has an impact, both on the *composition of consumer demand* and on the *structure of prices* that innovators charge for their product. This yields a rich set-up that allows us to study the inequality growth-nexus via a channel that has not attracted much attention in the recent literature on innovation and growth.[1] That this channel has not attracted much attention is surprising given the empirical evidence. The vast majority of studies of consumer behavior reject the hypothesis of homothetic preferences (see Deaton and Muellbauer (1980)).

A hierarchy of wants implies that goods can be ranked according to their priority in consumption. In this paper, hierarchic preferences are introduced in a stylized way. In order to satisfy a certain want, consumers buy one unit of an indivisible good. This implies that poor consumers will only buy a small range of high priority goods, whereas richer people will consume a wider range

[1] A notable exception is Pasinetti (1980), he discusses extensively the influence of demand on the growth process.

including also goods of lower priority. Hence, the incentive to conduct R&D is affected by the distribution of income as inequality determines the level of demand and the optimal price of an innovator. Today, the good of an innovator may be purchased only by a small group of rich people and the willingness to pay may initially be low. But as incomes grow the size of the market grows as less wealthy people also become willing to buy. One novel aspect of this paper is to study how income distribution affects the time path of demand for the innovator's good; the other novel aspect of the paper is that the prices and mark-ups of innovators are determined by the distribution. This means we can study a situation where both depend on the income distribution and both affect the reward to an innovation. We have therefore a set-up where inequality affects growth via its impact on product demand.

The following three points are the main findings of our analysis.

First, inequality alters the degree of competition in the economy. With poor and rich consumers, it may be profitable for the monopolist only to sell to the rich, whose demand is inelastic (relative to the poor), and thus to charge higher prices. However, this strategy implies that in the aggregate we have a distortion in the price structure due to the fact that the poor are *excluded from consumption* due to too high prices.

Second, inequality has an a priori ambiguous impact on the incentive to innovate: On the one hand, with high inequality an innovator faces immediate demand with a high willingness to pay by the rich consumers; on the other hand, new markets are small for a long time since only the rich buy. However, we get the comparative-static result that the first effect dominates, if there is exclusion of the poor *and* if the increase in inequality is due to higher income of the rich group. Higher inequality increases the profit share of the economy what induces the agents to allocate more resources in R&D, this enhances growth. Instead, if higher inequality is due to an enlargement of the poor group although their relative wealth remains constant, higher inequality may reduce growth. The important message is that higher inequality *per se* is a too crude statement to decide how the demand structure is affected. The result suggests that higher inequality due to a smaller size of wealthy people is especially harmful for profits and thus for growth.

Third, if there is *no* exclusion of the poor, the inequality-growth relation changes its sign. The case of no exclusion can only arise, if some goods in the economy are supplied at marginal cost. If not all goods in the economy are supplied by monopolists, who receive the reward for their innovation, there exists a "non-innovative" sector in the economy, because the revenues of those goods do not create innovation incentives. The presence of those goods limits the scope for price setting by innovators, because the marginal willingness to pay for innovative products is bounded also for the very rich. Then, once a rather egalitarian distribution is considered, the innovator has no incentive to set prices that would exclude the poor. Thus, prices are determined by the willingness to pay of the poor. An even more egalitarian distribution allows the monopolist to set higher prices and earn higher profits as the poor are the 'critical' consumers that determine demand at the extensive margin.

The role of inequality and hierarchic preferences in the context of economic development has been studied in a few other papers. The present paper is related to that of Murphy, Shleifer, and Vishny (1989). Like in the present model, they show that the adoption of efficient methods of production requires large markets and excessive concentration of wealth may be an obstacle to economic development. However, Murphy, Shleifer, and Vishny (1989) focus on a static framework. As a consequence, changes in income distribution matter only if the demand of the marginal firm is affected. This is different from the present model where not only the level but also the time path of demand affects growth.[2] Models that study the impact of inequality and product demand on growth include Chou and Talmain (1996), Falkinger (1994), and Zweimüller (2000). These papers have in common that while income distribution affects there is no impact of the distribution on prices. Hence an important mechanism of the present model, namely that the poor may be excluded from the market as a result of the monopolists pricing decision, does not occur in these models.[3]

This paper is organized as follows. Section 2 presents the model and Section 3 studies the static equilibrium in detail. Section 4 deals with the supply side of the economy, Section 5 discusses the innovation process. In Section 6 we introduce our assumption on inequality and Section 7 we can study the general equilibrium of the model. Section 8 discusses the impact of inequality on growth. Section 9 concludes.

2 A model of hierarchic preferences and monopolistic competition

2.1 Hierarchic preferences and consumption choices

Consider an economy with many households earning different incomes (and owning different wealth levels). While there is heterogeneity with respect to income and wealth, all consumers have the same preferences. There exist many potentially produceable, differentiated products indexed by a continuous index $j \in [0, \infty)$. Consumers' preferences over these differentiated goods are 'hierarchic' in the sense that there is a clear priority in consumption: food has highest priority, clothing have second highest, and so on; only when needs of higher priority are satisfied, the consumption of additional products with lower priority is considered.

To capture the idea of a hierarchic structure of needs and wants we specify preferences over the differentiated goods as follows: There is a baseline utility, $v(c(j))$, that captures the utility derived when good j is consumed in quantity $c(j)$. The function $v(.)$ is the same for all differentiated goods, i.e. does not

[2] Other papers which study the impact of inequality on product demand are Eswaran and Kotwal (1993) and Baland and Ray (1991) both of which stick to a static framework. See also Bourguignon (1992).

[3] For models where inequality drives the incentive to improve the quality of products see Glass (1996), Li (1996), and Zweimüller and Brunner (1996, 1998).

depend directly on j and to introduce a hierarchy, we introduce a weighting function $\xi(j)$, with $\xi'(j) < 0$. Because $\xi(.)$ is monotonically decreasing in j we have a hierarchic structure of preferences: low-j goods get a high weight (= get high priority in consumption) whereas high-j goods have comparably low priority.

Before we proceed let us make two further assumptions, the first is primarily for analytical convenience and the second makes sure that the model exhibits a balanced growth path. First, we assume that the choice to buy a certain differentiated product is a take-it or leave-it decision: either a good is consumed in which case one and only one unit is purchased, or is not consumed. This allows us to normalize the baseline utility such that $v(0) = 0$ and $v(1) = 1$. The second assumption concerns the hierarchy: Throughout the analysis we will assume that the weighting function that generates the hierarchy of needs and wants is a power-function, i.e. we assume $\xi(j) = j^{-\gamma}$ with $\gamma \in (0, 1]$.[4]

The preferences over the differentiated products can thus be represented by the following utility function

$$\tilde{u}(\{c(j)\}) = \int_0^\infty \xi(j)v(c(j))dj = \int_0^\infty j^{-\gamma}c(j)dj$$

where we make use of the above normalization $v(c(j)) \equiv c(j)$. Since $c(j)$ can take only two values, 0 or 1. Evidently, $c(j)$ is a dummy variable indicating whether or not good j is consumed. Take the case when a consumer purchases the first n goods in the hierarchy. In this case the above utility is given by $\tilde{u}(\{c(j)\}) = \int_0^\infty j^{-\gamma}c(j)di = \int_0^N j^{-\gamma}di = \frac{N^{1-\gamma}}{1-\gamma}$ and, due to the restriction $\gamma \in [0, 1)$, the integral $\int_0^N j^{-\gamma}di$ does not diverge. While the highest utility arises from consuming all goods in the interval $[0, N]$, it is also evident that the utility integral is finite for any arbitrary bundle of goods with measure N: any arbitrary interval of measure N (or sub-intervals that sum up to measure N) yields instantaneous utility larger than 0 but lower than $\frac{N^{1-\gamma}}{1-\gamma}$.

Apart from the sector of differentiated products, there exists also a second sector that produces a homogeneous good x that can be consumed in continuous amounts. A possible different interpretation of x to which we will frequently refer, is leisure. The total utility flow at any instant is given by the utility received from consuming differentiated products and the utility received from consuming the homogenous good. We assume that the two types of goods are linked by a Cobb-Douglas relationship with parameter ν, where $0 \leq \nu < 1$.[5]

[4] Assumption (i) is made for tractability of the model. In particular, this assumption allows us to calculate the monopoly price of the various goods explicitely in terms of the parameters of the model, and in particular, in terms of income inequality in the model.

Assumption (ii) is essential in the sense that in equilibrium the utility function is CRRA in expenditures which guarantees a balanced growth path. See Foellmi (1999). Provided that firms have the same constant marginal cost and goods are priced either at marginal cost or at the monopoly price, then any baseline utility with $v(0) < \infty$, $v'(c) > 0$ and $v"(c) < 0$ leads to a solution such that the maximized CRRA function.

[5] As it implies constant expenditure shares, the Cobb-Douglas is a fair formulation in the

The instantaneous utility function takes the form

$$u(x,\{c(j)\}) = x^{\nu} \int_0^{\infty} j^{-\gamma} c(j) dj \tag{1}$$

Now consider the decision problem of household i. Households are heterogenous with respect to their available budget E_i but otherwise identical.[6] It is assumed that the first $\overline{N}$ products in the hierarchy are actually available on the market, whereas goods in the interval $(\overline{N}, \infty)$ have not yet been invented.[7] We denote by $p(j)$ the price of the differentiated product j and by p_x the price of one unit of the homogenous good x. Total expenditures of household i are given by $E_i = \int_0^{\overline{N}} c_i(j)p(j)dj + p_x x_i$ where $c_i(j)$ indicates whether household i consumes good j. Hence the static choice problem of consumer i can be written as

$$\max_{x,\{c(j)\}} x_i^{\nu} \int_0^{\overline{N}} j^{-\gamma} c_i(j) dj \qquad s.t. \qquad E_i \geq \int_0^{\overline{N}} c_i(j)\ p(j)dj - p_x x_i \tag{2}$$

Taking expenditures and prices as given, consumer i maximizes his utility by choosing which differentiated products to consume $\{c_i(j)\}_{j\in[0,\overline{N}]}$, and by choosing the optimal amount of the homogenous good x_i. To solve the above problem we can set up the Lagrangian as

$L = x_i^{\nu} \int_0^{\overline{N}} j^{-\gamma} c_i(j) dj + \lambda_i \left(E_i - \int_0^{\overline{N}} c_i(j)\ p(j)dj - p_x x_i \right)$

where λ_i is the Lagrangian mulitplier, which in our context may be interpreted as consumer i's marginal value of wealth. Maximization of the La-

sense that higher inequality does not imply *per se* consumption of the traditional good to rise, as is done in other approaches.

[6] As we assume intertemporal additive separability of the utility function, we can apply two-stage budgeting: The consumers' decision can be split up into two parts: In the first stage, we look at the *intra*temporal decision problem by solving for the optimal structure of consumption at a point of time, given current prices of all goods $p(j)$ and p_x and the currently available budget E_i. In the second stage, we look at the *inter*temporal decision problem and calculate how to allocate the consumers' lifetime resources across time. While the time path of E_i is endogenous, we can take it as given when solving for the optimal structure of expenditures at a given point of time.

[7] We are making a shortcut here: more generally we could assume that a bundle of goods with measure N is available on the market but that this measure does not necessarily coincide with the interval $(0, N)$ in the hierarchy, i.e. there are ' andholes' in the sense that goods with higher priority are still not available whereas goods with low priority have already been invented.

We will abstract from this possibility here as we are primarily interested in the behavior of the economy along the balanced growth path. This means, we start here already with a situation which will prevail in the balanced growth equilibrium, namely when the introduction of new goods follows the prespecified hierarchy of needs and wants. In equilibrium, therefore, there will be no holes as the most recent innovator produces always the goods which has least priority among all goods that are actually available on the market.

grangian with respect to $\{c(j)\}$ and x yields following first order conditions

$$\begin{aligned} c_i(j) &= 1 \quad \text{if} \quad p(j) \leq x_i^{\nu} j^{-\gamma}/\lambda_i \equiv q_i(j) \qquad (3) \\ c_i(j) &= 0 \quad \text{if} \quad p(j) > q_i(j) \\ v x_i^{\nu-1} \int_0^{\overline{N}} j^{-\gamma} c_i(j) dj &= \lambda_i p_x. \end{aligned}$$

The first two conditions say that consumer i will consume good j if its price $p(j)$ is lower than (or equal to) consumer i's willingness to pay $x_i^{\nu} j^{-\gamma}/\lambda_i$ which from now on we denote by $q_i(j)$.[8] The third condition is the familiar condition that says the homogenous good x is consumed up to the point where the marginal utility of consumption of x equals the utility-adjusted price $\lambda_i p_x$.

2.2 The determination of prices and the structure of consumption

Our next step is to discuss the determination of prices. It is assumed that the homogenous good is produced with constant marginal cost and supplied on a competitive market. Hence the price of the *homogenous* good p_x is constant, and exogenously given by the constant marginal cost. (As our aim is to study growth, we will come back to the issue of how p_x changes over time below). The *differentiated* products are also produced at constant marginal costs but supplied on monopolistic markets, as each good has a single supplier. As our aim is to study the implications of hierarchic preferences for distribution and growth, we will assume throughout the paper that all heterogeneity across firms comes only from the demand side (i.e. is the result of the hierarchic preferences) whereas the supply conditions are symmetric. We therefore assume that marginal costs are the same for all *monopolistic* firms. Without loss of generality, we take the marginal cost as the numeraire. (When we discuss intertemporal issues below, we will see that the marginal production cost is also constant *over time*, as input prices grow pari passu with productivity).

Consider a monopolist that supplies some good j. Recall that each household either consumes one unit or does not consume good j. Hence the level of demand depends on how many consumers are willing to purchase good j at a given price $p(j)$. If all households had the same budget (the representative agent case), the demand function of the monopolist is horizontal at the price equal to the representative agent's willingness to pay, and vertical at quantity 1 (the size of the population). In other words, the monopoly price would be equal to $p(j) = q(j) = x^{\nu} j^{-\gamma}/\lambda$, where λ and x are, respectively, the respresentative consumer's marginal value of wealth and optimal consumption of the homogenous good.

Things are somewhat more complicated when consumers are heterogeneous which is the case of our primary interest. The basic point can be illustrated by

[8] This condition comes from comparing the received utility from consuming good j, $x_i^{\nu} j^{-\gamma}$, with its price $p(j)$ times consumer i's marginal utility of wealth λ_i. Rearranging terms yields $p(j) \leq x_i^{\nu} j^{-\gamma}/\lambda_i$.

focusing on the case when there are only two types of households, rich R and poor P. Let us assume a fraction β is poor and $1-\beta$ are rich. The willingsness to pay for the poor and for the rich is, respectively, $q_P(j)$ and $q_R(j)$. Consequently the market demand function is a step function (Figure 1) that starts at quantity 0 and $p(j) = q_R(j)$, is then horizontal up to the kink at quantity $1-\beta$ and is then flat at $p(j) = q_P(j)$ up to the maximum quantity 1.[9] The monopoly price is then either at point A or at point B in figure 1, whichever yields the higher profits.

$figure$ 1

The next step is to look at the equilibrium price structure, that we ask the question how the equilibrium value of $p(j)$ varies with j. Each monopolistic supplier compares the level of profits when charging the richs' willingness to pay $q_R(j)$ and getting demand $1-\beta$ with the level of profits that obtains when charging the poors' willingness to pay $q_P(j)$ and getting maximum demand 1. The corresponding profit levels are, respectively, $[q_R(j) - 1]\,(1-\beta) \equiv \Pi_R(j)$ and $[q_P(j) - 1] \equiv \Pi_{tot}(j)$. (Recall that we have normalized the constant marginal production costs to unity). In other words, good j will be priced at the willingness to pay of the poor if $\Pi_{tot}(j) \geq \Pi_R(j)$ or if $q_P(j) - q_R(j)(1-\beta) \geq \beta > 0$. Note that the latter condition must hold for some j so also $q_P(j)/q_R(j) \geq (1-\beta)$ must hold for some j. But note that $q_P(j)/q_R(j)$ does not depend on j so $q_P(j)/q_R(j) \geq (1-\beta)$ must hold in the static equilibrium.

To solve for the equilibrium price structure in the economy we first note that a situation where only the rich buy goods from monopolistic producers cannot be an equilibrium. If the poor would not buy any differentiated products at all, their willingsness to pay would become infinitely large as their marginal utility of income would become zero. Hence the poor will always buy some goods from the hierarchy so that there exists discrete measure of goods for which $\Pi_{tot}(j) \geq \Pi_R(j)$ holds.

Secondly, a natural conjecture is that the goods are priced such that those with the highest priority (low-j goods) are purchased by both the poor and the rich whereas the goods with lower priority (high-j goods) are priced such that only the rich can afford them. If this conjecture holds in equilibrium there is a 'ciritical' good in the hierarchy, call it N_P, such that for any $j \leq N_P$ we have $\Pi_{tot}(j) \geq \Pi_R(j)$ and for any $j > N_P$ we have $\Pi_{tot}(j) < \Pi_R(j)$. To see that this conjecture holds true in equilibrium we show that, for any j, $\frac{\partial \Pi_R(j)}{\partial j} > \frac{\partial \Pi_{tot}(j)}{\partial j}$, which means that the difference $\Pi_{tot}(j) - \Pi_R(j)$ decreases as we move along the hierarchy. Using the definition $q_i(j) = x_i^{\nu} j^{-\gamma}/\lambda_i$ it is straigtforward to calculate $\frac{\partial \Pi_R(j)}{\partial j} = -\frac{\gamma}{j} q_R(j)(1-\beta)$ and $\frac{\partial \Pi_{tot}(j)}{\partial j} = -\frac{\gamma}{j} q_P(j)$. Hence $\Pi_{tot}(j) - \Pi_R(j)$

[9] Obviously, if there are more types of consumers, there are more such kinks, and in the case of continuous distribution we have a smooth demand function. In any case, under the take-it or leave-it assumption the shape of the demand function reflects the distribution of the consumers' budgets.

decreases in j if $q_P(j)/q_R(j) \geq (1-\beta)$ which must hold in equilibrium (see last paragraph).

We can now state the following

Lemma 1 *a) 'Consumption along the hierarchy'. Prices are set such that for all goods* $j \in [0, N_P]$, *firms charge the price the poor are willing to pay* $p(j) = q_P(j)$, *and for all* $j \in (N_P, N_R]$ *we have* $p(j) = q_R(j)$. *Hence the poor consume all goods* $j \in [0, N_P]$ *and the rich consume all goods* $j \in [0, N_R]$ *where* $0 < N_P \leq N_R \leq \overline{N}$. *This means we have 'consumption along the hierarchy' in the sense that consumer i purchases only the first* N_i *products in the hierarchy and no products* $j > N_i$.

b) 'Market exclusion of the poor'. If the willingnesses to pay of the rich and the poor are sufficiently different we get $N_P < N_R$. *In such a situation the suppliers of low-priority goods set prices too high for the poor. In other words, the poor are excluded from participation in the market for low-priority goods. If* $\nu = 0$, *the poor will always be excluded from some goods in equilibrum.*

Part a) of Lemma 1 follows immediately from the above discussion. To see that Part b) is true, assume $\nu = 0$. If $N_P = N_R = \bar{N}$ were an equilibrium, then, rich and poor would consume exactly the same, since there is no expenditure for x-goods. This would imply that the rich have income left causing their marginal utility of income be zero and thus their marginal willingness to pay be infinity. Hence it would be profitable for a monopolist to deviate, since he could raise profits by selling only to the rich.

At this point several remarks are in place. First, we observe that the equilibrium will be one of the following three scenarios. We could have a situation where (i) only the rich can buy all products that are available on the market, so that $N_P < N_R = \overline{N}$; (ii) both the poor and the rich will buy all products that are available on the market, so $N_P = N_R = \overline{N}$; and (iii) where neither the rich nor the poor can afford all $\overline{N}$ goods, so $N_P < N_R < \overline{N}$. The first scenario will prevail if the willingnesses to pay are sufficiently different between the rich and the poor, (which will be the case if inequality in the available budgets is sufficiently high.) For the second scenario exactly the opposite is true. The third scenario is possible if the inherited measure of producable goods is very large, even relative to willingness to pay of the richest consumer.

Secondly, we note that by setting the price equal to $q_R(j)$, the monopolist sets a price that the poor cannot afford. Thus it appears that it is always at the discretion of the monopolist whether or not the poor can participate in a certain market. But it may also be that even at the lowest possible price (the marginal production cost) the poor are not willing to buy. In this latter case, it is not at the discretion of the monopolist to exclude the poor from that market as selling only to the rich is the only vialbe alternative for the monopolistic supplier.

Thirdly, and more generally, we observe that in our model of hierarchic preferences, the structure of prices is determined by the distribution of of the willingnesses to pay of the various consumers, which themselves reflects the distribution of the households' budgets. This is a result that is absent from the standard monopolistic competition due to the assumption of homothetic preferences: total market demand there is independent of the income distribution and has therefore no effect of the structure of prices. (The same is true for previous attempts to combine a hierarchic structure of demand with market power, where a uniform mark-up is assumed for all products. Murphy, Shleifer, Vishny (1989), Zweimüller (2000)).

Finally, we observe that the distribution of the willingnesses to pay (which reflects the *personal* distribution of income) affects the choice of prices and therefore also the aggregate profits. So, in our model the *personal* distribution of income affects the *functional* distribution.

3 Solving the consumers' problem

We can now characterize the choice problem of the consumers in this economy. As mentioned above, consumers maximize utility over an infinite horizon and due to the additivity of the utility function we can solve the problem by two-stage budgeting. This means we can split the problem into a static one and a dynamic one. In their static choice consumers take expenditures at a given point of time as given and ask how to allocate these expenditures across the homogeneous good and the differentiated products along the hierarchy. The dynamic choice problem is then to ask for the optimal allocation of lifetime ressources over time, taking the structure of consumption at a given point of time as given.[10] We will first look at the static equilibrium before we discuss the dynamic solution of the consumers' problem. Furthermore, in presenting the static and the dynamic solutions we concentrate on the case when the rich but not the poor buy all products that are available on the market ($N_P < N_R = \overline{N}$) and describe the corresponding equilibria in some detail. At the end of this section we will also briefly mention the two remaining scenarios namely, when neither the rich nor the poor can afford all $\overline{N}$ goods ($N_P < N_R < \overline{N}$); and when both the poor and the rich buy all available products ($N_P = N_R = \overline{N}$).

3.1 The static equilibrium

A static equilibrium is a structure of consumption of the differentiated products $\{c_i(j)\}_{j\in[0,\overline{N}]}$, a corresponding structure of prices of these products $\{p(j)\}_{j\in[0,\overline{N}]}$, the consumption levels of the homogeneous product x_i, and the marginal utilities of wealth λ_i, where $i = (R, P)$. When we present the solution we take as predetermined the measure of available products $\overline{N}$ (from the innovation process prior to the point of time we consider), the consumers' budgets E_i (from the first stage of our two-stage budgeting problem), the constant marginal production costs in the monopolistic sector (taken as numeraire) and in the competitive sector (equal to p_x). Exogenous are the utility parameters γ and ν and the population size of poor β.

[10] We will see that the problem is even easier as, along a balanced growth path, the structure of consumption (as measured in efficiency units) does not change over time.

As mentioned above, the equilibrium can take various forms. Here we concentrate on the case when the rich but not the poor buy all products that are available on the market ($N_P < N_R = \overline{N}$) and describe the corresponding static equilibrium in some detail. At the end of this section we will also briefly mention the two remaining scenarios namely, when neither the rich nor the poor can afford all $\overline{N}$ goods ($N_P < N_R < \overline{N}$); and when both the poor and the rich buy all available products ($N_P = N_R = \overline{N}$).

To characterize the static equilibrium in the interesting case $N_P < N_R = \overline{N}$ it will be convenient to introduce two new variables, (i) the fraction of available goods that the poor can afford, $n_P \equiv N_P/\overline{N}$, and (ii) the price of good $\overline{N}$, $\overline{p} \equiv p(\overline{N})$. It turns out that, once the equilibrium values of these two variables are known, the equilibrium structure of consumption $\{c_i(j)\}_{j\in[0,\overline{N}]}$ and the corresponding structure of prices $\{p(j)\}_{j\in[0,\overline{N}]}$ can be derived immediately. Hence when presenting the solution to the static equilibrium we can replace $\{c_i(j)\}_{j\in[0,\overline{N}]}$ and $\{p(j)\}_{j\in[0,\overline{N}]}$ by n_P and $\overline{p}$ and describe this solution in terms of the endogenous variables $n_P, \overline{p}, x_P, x_R, \lambda_P$ and λ_R.

To see how n_P and $\overline{p}$ determine $\{c_i(j)\}_{j\in[0,\overline{N}]}$ and $\{p(j)\}_{j\in[0,\overline{N}]}$, recall from Lemma 1, that equilibrium value of n_P suffices to determine the equilibrium structure of consumption $\{c_i(j)\}_{j\in[0,\overline{N}]}$. The derivation of the equilibrium structure of prices $\{p(j)\}_{j\in[0,\overline{N}]}$ in terms of n_P and $\overline{p}$ requires more steps. Consider first the lower priority goods $j \in (N_P, \overline{N}]$. We recall from Lemma 1 that these goods are priced at the willingness to pay of the rich $p(j) = q_R(j)$, and from equation (3) we know that $q_R(j) = q_R(\overline{N}) \left(j/\overline{N}\right)^{-\gamma}$. Hence for the goods that only the rich buy we have $p(j) = \overline{p}\left(j/\overline{N}\right)^{-\gamma}$. Now consider the goods with high priority $j \in [0, N_P]$. According to Lemma 1 these goods are priced at the willingness to pay of the poor, so $p(j) = q_P(j)$, and from equation (3) we know that $q_P(j) = q_P(N_P)\,(j/N_P)^{-\gamma}$. Hence for the goods that both groups of consumers purchase we have $p(j) = p(N_P)\,(j/N_P)^{-\gamma}$. It remains to determine $p(N_P)$ in terms of n_P and $\overline{p}$. Recall that, for the critical good N_P, the corresponding firm is indifferent between selling only to the rich at price $q_R(N_P) = \overline{p}n_P^{-\gamma}$ or by serving the whole market at price $p(N_P) = q_P(N_P)$. Hence we must have $q_P(N_P) - 1 = (\overline{p}n_P^{-\gamma} - 1)(1 - \beta)$. Assuming the firm supplying good N_P charges the willingness to pay of the poor and serves the whole market, we get $p(N_P) = q_P(N_P)$ and we can express $p(N_P)$ in terms of $\overline{p}$ as: $p(N_P) = \beta + \overline{p}n_P^{-\gamma}(1 - \beta)$. Taken together we can express the equilibrium structure of prices as

$$p(j) = \begin{cases} [\beta n_P^{\gamma} + (1-\beta)\overline{p}] \left(\frac{j}{\overline{N}}\right)^{-\gamma} & j \in [0, N_P], \\ \overline{p}\left(\frac{j}{\overline{N}}\right)^{-\gamma} & j \in (N_P, \overline{N}]. \end{cases} \tag{4}$$

Having determined $\{c_i(j)\}_{j\in[0,\overline{N}]}$ and $\{p(j)\}_{j\in[0,\overline{N}]}$ in terms of n_P and $\overline{p}$ we are now ready to describe the solution to the static equilibrium of the model in terms of the six endogenous variables $n_P, \overline{p}, x_P, x_R, \lambda_P$ and λ_R. The six equations that determine this equilibrium are given by

$$\beta + (1-\beta)\overline{p}n_P^{-\gamma} = \frac{x_P^\nu \left(n_P\overline{N}\right)^{-\gamma}}{\lambda_P} \tag{S1}$$

$$\overline{p} = \frac{x_R^\nu \overline{N}^{-\gamma}}{\lambda_R} \tag{S2}$$

$$\nu x_P^{\nu-1}\frac{\left(n_P\overline{N}\right)^{1-\gamma}}{1-\gamma} = \lambda_P p_x \tag{S3}$$

$$\nu x_R^{\nu-1}\frac{\overline{N}^{1-\gamma}}{1-\gamma} = \lambda_R p_x \tag{S4}$$

$$\frac{E_P}{\overline{N}} = \left[\beta n_P^\gamma + (1-\beta)\overline{p}\right]\frac{n_P^{1-\gamma}}{1-\gamma} + \frac{p_x x_P}{\overline{N}} \tag{S5}$$

$$\frac{E_R}{\overline{N}} = \left[\beta n_P^\gamma + (1-\beta)\overline{p}\right]\frac{n_P^{1-\gamma}}{1-\gamma} + \overline{p}\frac{1-n_P^{1-\gamma}}{1-\gamma} + \frac{p_x x_R}{\overline{N}}. \tag{S6}$$

Equations (S1) and (S2) say that the price of good N_P equals to the willingness to pay of the poor for good N_P, and that the price for good $\overline{N}$ equals the willingness to pay of the rich for good $\overline{N}$. Equations (S3) and (S4) say that, for both types of consumers, the optimal level of x_i is determined such that the marginal utility of x_i equals its utility-adjusted price $\lambda_i p_x$. Finally, equations (S5) and (S6) state that the budget constraints have to be satisfied for both types of consumers.

3.2 Static expeditures and utilities

For further use, it is convenient to reduce this system to two equations in the two unknowns n_P and $\overline{p}$. These two interesting equations are the budget constraints when consumers have made optimal consumption choices, that is their 'expenditure functions'. Moreover, we can also express the maximized utility functions in terms of the endogenous variables in terms of n_P and $\overline{p}$, that is the 'indirect utility functions'.[11]

Combining equations (S1) and (S3) of the above system we can write

$$p_x x_P = \frac{\nu\overline{N}}{1-\gamma}\left[\beta + (1-\beta)\overline{p}n_P^{-\gamma}\right]n_P \tag{5}$$

and

$$p_x x_R = \frac{\nu\overline{N}}{1-\gamma}\overline{p}, \tag{6}$$

[11] We use the terms 'expenditure function' and 'indirect utility function' in the sense that expenditures (utility) evaluated after consumers have made optimal choices. We do not explicitely specify the (minimized) expenditures in terms of utility and prices and the (maximized) utility in terms of prices and expenditures. It should be clear that using the relations derived in the text this can be easily done.

and substitute these relations into equations (S5) and (S6) of the above system. This yields

$$\frac{E_P}{\overline{N}} = [\beta n_P^\gamma + (1-\beta)\overline{p}] \frac{n_P^{1-\gamma}}{1-\gamma} + \left[\beta + (1-\beta)\overline{p} n_P^{-\gamma}\right] \frac{\nu n_P}{1-\gamma} \tag{7}$$

and

$$\frac{E_R}{\overline{N}} = [\beta n_P^\gamma + (1-\beta)\overline{p}] \frac{n_P^{1-\gamma}}{1-\gamma} + \overline{p}\frac{1-n_P^{1-\gamma}}{1-\gamma} + \overline{p}\frac{\nu}{1-\gamma}, \tag{8}$$

we note that, for given values of n_P and $\overline{p}$, E_P and $\overline{N}$ as well as E_R and $\overline{N}$ are proportional. (This is a result of Cobb-Douglas preferences; the power-function for the hierarchy-index; and the constant marginal production cost of the differentiated products).

We proceed to calculate the maximized static utility function in terms of the endogenous variables n_P and $\overline{p}$. Substituting the relations (5) and (6) into the utility flow function (2). This yields for the rich

$$u_R(n_R = 1,\ n_P < 1,\ \overline{p} > 1) = \left(\overline{p}\frac{\nu}{1-\gamma}\frac{\overline{N}}{p_x}\right)^\nu \frac{\overline{N}^{1-\gamma}}{1-\gamma} \tag{9}$$

and for the poor

$$u_P(n_R = 1,\ n_P < 1,\ \overline{p} > 1) = \left(\left[\beta + (1-\beta)\overline{p} n_P^{-\gamma}\right] \frac{\nu}{1-\gamma}\frac{\overline{N}}{p_x}\right)^\nu n_P^{1+\nu-\gamma} \frac{\overline{N}^{1-\gamma}}{1-\gamma} \tag{10}$$

From (7) and (8) we know that, for given values of n_P and $\overline{p}$, the range of available goods $\overline{N}$ and the expenditure levels E_i are proportional for both types of consumers. And we will see below that also the price of the homogeneous goos p_x and $\overline{N}$ are proportional. It follows that the instantaneous utilities can be expressed as

$$u_i(n_P,\ \overline{p}) = \mu_i(n_P, \overline{p}, \frac{\overline{N}}{p_x})\ \frac{E_i^{1-\gamma}}{1-\gamma}. \tag{11}$$

Equation (11) gives the important result that instantaneous utility is of the CRRA-type with hierarchy-parameter γ as the relevant parameter.[12]

[12] An important observation can be made here though the utilities of the rich and the poor are CRRA in their expenditures over time, the ratio of utility between the poor and the rich at a given point of time does not exhibit a CRRA relationship, even if $v = 0$, i.e. $\frac{u(x_R(t), n_R(t))}{u(x_P(t), n_P(t))} \neq \left(\frac{E_R}{E_P}\right)^{1-\gamma}$ in general. The reason is that the expenditure share of a single good is not the same for the rich and the poor, they even do not consume the same goods. Since prices of the various goods are different, rich and poor face a different average price level.

3.3 Intertemporal allocation of expenditures

We now ask the question how consumption expenditures are allocated over time. As we are interested in the balanced growth path of the economy, we will analyze a situation where $\overline{N}$ and E_i grow at the same rate. This means that, along a balanced growth path, (see equations (7) and (8)). To study the intertemporal problem we change notation slightly: as n_p and $\overline{p}$ are constant over time we drop the arguments $(n_P,\ \overline{p})$ in the utility funtion and replace it by the time index s. So $u_i(t)$ is the maximized instanteous utility at date t.

Suppose time is continuous and consumers maximize lifetime utility $U(t)$ over an infinite horizon where lifetime utility is additively separable and the felicity function is given by equation (11). We assume that lifetime utility takes the CRRA-form

$$U_i(t) = \int_t^\infty e^{-\rho(s-t)} \frac{(u_i(s))^{1-\sigma}}{1-\sigma} ds \tag{12}$$

where the parameter $\rho > 0$ denotes the rate of time preference and the parameter $\sigma > 0$ describes the consumers' willingness to shift total consumption (as measures by $u_i(s)$) over time where $1/\sigma$ is the *inter*temporal elasticity of substitution. From (11) we know that also the instanteous utility is of the CRRA-type (in expenditures) with parameter γ and we may interpret $1/\gamma$ as the *intra*temporal elasticity of substitution (among goods along the hierarchy).

The consumers' lifetime resources are given by the discounted value of a labor income flow $\{w(s)l_i\}_{s\in[t,\infty)}$, where l_i denotes the labor endowment of consumuer i, and the value of assets individual i owns at date t, $V_i(t)$. The lifetime budget constraint is then given by

$$\int_t^\infty E_i(s)e^{-r(s-t)}ds \leq \int_t^\infty w(s)l_i e^{-r(s-t)}ds + V_i(t) \tag{13}$$

where r is the interest rate. Since expenditures are proportional to N(t) in equilibrium, the growth rate of expenditures is constant, since we are in steady state. The Euler equation below, which is the solution to the intertemporal problem, then implies that the interest rate must be constant.

$$\frac{\dot{E}(s)}{E(s)} = g = \frac{r-\rho}{\sigma(1-\gamma)+\gamma}. \tag{14}$$

4 The supply side

As mentioned above, the aim of the paper is to analyze the implications of hierarchic preferences for distribution and growth. This means the main focus comes of our analysis is on heterogeneity that comes from the demand side of the economy. The supply side plays a less central role and to keep things simple we assume symmetry of firms as far as production possibilities are concerned. This means that all monopolistic firms have access to the same production technology; and that it is equally costly to design the blueprint and set up the necessary production facilities for a new product, irrespective of the position of this good in the hierarchy.

In this section we describe the supply side of the economy, both the situation in the various sectors at a given point of time and the dynamics of productivity in that sector. Having done that, we can look at the ressource constraint of the economy and can discuss the equilibrium allocation of ressources across sectors. We will confine the analysis to the situation that prevails along a balanced growth path.

When describing the equilibrium allocation of resources across sectors in this Section, we still concentrate on the case where the rich, but not the poor, can afford all goods that are available on the market. That is we focus on the case $N_P < N_R = \overline{N}$. The equilibrium allocation fo ressources remaining two cases, $N_P < N_R < \overline{N}$ (when neither the rich nor the poor can afford all N goods), and $N_P = N_R = \overline{N}$ (all households purchase all N goods) are briefly discussed in Section 7. (The detailed derivations for these cases are presented in the Appendix).

4.1 Production technology and technical progress

To keep things as simple as possible we assume that labor is the only production factor and that the labor market is competitive. The market clearing wage at date t is denoted by $\widetilde{w}(t)$. Consider first the monopolistic sector that produces the differentiated goods along the hierarchy. We assume that the technology in this sector exhibits increasing returns to scale. Before a good can be produced a fixed cost has to be incurred. Having incurred this fixed cost the firm gets access to the blueprint of the new good in the hierarchy and gets a monopoly position on this new market.[13] (This is what we will call an 'innovation' henceforth). This fixed cost consist of a fixed labor input $\widetilde{F}(t)$ and the fixed cost is $\widetilde{w}(t)\widetilde{F}(t)$, equal for all goods. It is assumed that $\widetilde{F}(t)$ *decreases* over time as a result of technical progress. Just like in many recent endogenous growth models, we assume that technical progress is driven by innovations, that is we assume $\widetilde{F}(t)$ is inversely related to the aggregate knowledge stock of knowledge $A(t)$ that reflects the economy-wide productivity at date t. We assume that the knowledge stock of this economy equals the number of known designs, hence we have $A(t) = \overline{N}(t)$. We can thus write $\widetilde{F}(t) = \frac{F}{A(t)} = \frac{F}{\overline{N}(t)}$ where $F > 0$ is an exogenous parameter. Once an innovation has taken place the corresponding output good can be produced with the linear technology

$$l(j,t) = \widetilde{b}(t)y(j,t) \tag{15}$$

where $l(j,t)$ is labor employed to produce good j at date t, $y(j,t)$ is the quantity produced and $b(t)$ is the unit labor requirement. Marginal cost at date t is

[13] By assumption, we rule out that there is no duplication. So when a new good is 'invented' there is one and only one firm that incurs that fixed cost and captures the respective market.

$\widetilde{w}(t)b(t)$, equal for all goods, where $\widetilde{w}(t)$ is the wage rate that applies to the whole economy.

We assume that - as a result of technical progress - not only $\widetilde{F}(t)$ but also $b(t)$ decreases over time. Just like before we assume that technical progress in the production process is a result of innovations that produce new knowledge which leads also to higher productivity of the inputs in the monopolistic sector. Again we model this by assuming that $b(t) = \frac{1/w}{\overline{N}(t)}$, where $w > 0$ is an exogenously given parameter. Along the balanced growth path it must be that wages growth with productivity, so $\widetilde{w}(t)$ must be proportional to $\overline{N}(t)$. Moreover, we have normalized marginal production cost to unity so, for all t, we must have $\widetilde{w}(t)b(t) = 1$. But this can only be the case if wages grow according to $\widetilde{w}(t) = w\overline{N}(t)$.

We also note that our assumptions about technology and technical progress imply that the set-up cost of developing a new good is constant over time and equal to $\widetilde{w}(t)\widetilde{F}(t) = w\overline{N}(t)\frac{F}{\overline{N}(t)} = wF$.

Finally, the marginal cost of a traditional firm which produces good x has to be determined. We assume a linear technology $l_x = b_x x$ where l_x denotes the labor input for good x and b_x is the labor input coefficient which, by assumption, *does not change over time.* Since the wage at date t is given by $\widetilde{w}(t) = w\overline{N}(t)$, the marginal cost and hence the price of good x at date t is given by $p_x(t) = wb_x\overline{N}(t)$. But this also implies that $p_x(t)/\overline{N}(t)$ equals wb_x which is an exogenously given time-invariant constant.

Finally, we denote by g the growth rate of $\overline{N}(t)$. On a balanced growth path we have $g = \frac{\dot{E}(t)}{E(t)} = \frac{\dot{\overline{N}}(t)}{\overline{N}(t)} = \frac{\dot{N}_P(t)}{N_P(t)} = \frac{\dot{N}_R(t)}{N_R(t)}$, this implies that $\overline{N}(t) = \overline{N}(0)e^{gt}$.

4.2 The resource constraint

The economies' resources consist of the stock of knowledge $A(t)$ and homogeneous labor supplied by each household in the economy. The stock of knowlegde is given by the measure of past innovations $\overline{N}(t)$ and the labor supply is normalized to unity. We now proceed by discussing how the labor force is allocated across the various sectors. We denote by L_N the number of workers employed in the sector producing the differentiated hierarchical products, by L_R the number of workers that employed in research to design the blueprints for new such products, and by L_x the number of workers employed in the sector producing the homogenous good. Obviously, in the full employment equilibrium we must have $1 = L_N + L_R + L_x$.

Consider first employment in the production of the differentiated products. Obviously, when $N_P < N_R = \overline{N}$, the resources necessary to produce the differentiated hierarchic products are given by

$$\begin{aligned} \int_0^{\overline{N}(t)} l(j,t)dj &= \beta \int_0^{N_P(t)} \frac{b}{N(t)} di + (1-\beta) \int_0^{\overline{N}(t)} \frac{b}{N(t)} di \\ &= b\left(\beta n_P + (1-\beta)\right) \end{aligned}$$

which means that along a balanced growth path employment in the production of final output of hierarchical products remains constant. Secondly, the level of employment in the research sector is given by the level of innovative activity at date t. Note that, at date t, $\dot{\overline{N}}$ new goods are introduced and each such innovation requires a unit labor input $\frac{F}{N(t)}$. This means that

$$L_R = \dot{\overline{N}}(t)\frac{F}{\overline{N}(t)} = gF$$

workers are employed in the R&D sector at t. Finally, employment in the competitive sector producing the homogenous good is given by

$$\begin{aligned} L_x &= b_x \frac{\beta p_x(t)x_P(t) + (1-\beta)p_x(t)x_R(t)}{wb_x N(t)} = \frac{\beta p_x(t)x_P(t) + (1-\beta)p_x(t)x_R(t)}{wN(t)} \\ &= \frac{\nu}{1-\gamma} b\left(\beta^2 n_P + (1-\beta)\overline{p} n_P^{1-\gamma} + (1-\beta)\overline{p}\right) \end{aligned}$$

where we have used equations (5) and (6).

In sum, when the rich but not the poor can afford all availabe products in the economy, that is in the case $N_P < N_R = \overline{N}$, the resource constraint of the economy is given by the equation

$$1 = gF + b\left(\beta n_P + (1-\beta)\right) + \frac{\nu}{1-\gamma} b\left(\beta^2 n_P + (1-\beta)\overline{p} n_P^{1-\gamma} + (1-\beta)\overline{p}\right)$$

5 The innovation process

To study the impact of hierarchic preferences on distribution and growth we have to specify what determines the level of innovative activities in the economy. An incentive to devote additional resource to innovative activities exists as long as the return to an innovation is larger than the fixed cost to introduce a new good. Hence the equilibrium has to be characterized by a situation where the value of an innovation is less than or equal to the costs of an innovation. Above we have already seen that the innovations costs equal wF.

The value of an innovation depends on the resulting future profit flow. This in turn depends on (i) how the level of demand develops over time, and (ii) on how the prices that innovators can charge for their product change over time. Consider a firm, that at date t, incurs the set-up costs and is granted a patent of infinte length. First of all, it should be intuitively clear that this good is the one with least priority among all the goods actually available; and it is the good with the highest priority among those goods that have not yet been invented. This latter observation come from the fact that, as we have no uncertainty, new innovators will always target their innovation activities towards those goods for which the consumers have the highest willingness to pay. In other words, the R&D process leads to 'innovation along the hierarchy'.

Now consider how the flow profit of the innovator of good $\overline{N}(t)$ develops over time. In the case $N_P < N_R = \overline{N}$, on which we are focusing throughout this section, such a new firm has initially demand $1 - \beta$ as only the rich can initially afford the new product. The price level is initially equal to $\overline{p}$ but changes over time as new innovations take place resulting in productivity increase and corresponding increases in income, which in turn lead to a higher willingness to pay for the existing good allowing previous innovators to charge higher prices. Denote by $\overline{N}(s')$ the good produced by the most recent innovator at date $s' > t$. Obviously the price this firm can charge is given by $\overline{p}$. The firm producing good $\overline{N}(t)$ can charge a higher price as good $\overline{N}(t)$ has a higher priority than good $\overline{N}(s')$, that is we have $\overline{N}(t) < \overline{N}(s')$. From equation (4) we know that, as long as only the rich purchase the product, the corresponding price equals $p(\overline{N}(t)) = \overline{p}\ [\overline{N}(t)/\overline{N}(s')]^{-\gamma}$.

After sufficient time has passed there will be enough growth in incomes that also the poor are willing to purchase good $\overline{N}(t)$. At that date, demand jumps to its maximum level, equal to 1, and stays there forever.[14] At which date does that happen? Denote by Δ the time it takes until the poor can purchase good $\overline{N}(t)$. Obviously, Δ is defined by the equation $N_P(t+\Delta) = \overline{N}(t)$. Along a balanced growth path, all variables grow at rate g, so also $N_P(t)$ will grow at rate g. The equation defining Δ can therefore be rewritten as $N_P(t)e^{g\Delta} = \overline{N}(t)$ from which it follows that $\Delta = -\ln[N_P(t)/\overline{N}(t)]/g = -\ln n_P/g$. Obviously, the duration Δ is long (i) if the poor are very poor (so the fraction of goods the poor can afford, n_P, is small); and (ii) if the growth rate g is low. Using again equation (4), we can determine the prices the innovator of $\overline{N}(t)$ charges after the poor have started to purchase. Denote by $\overline{N}(s'')$ the good introduced by the most recent innovator at date $s'' \geq t + \Delta$. We know that the price for good $\overline{N}(s'')$ equals $\overline{p}$, whereas the price of the good $\overline{N}(t)$, which is now purchased by both the rich and the poor, equals $p(\overline{N}(t)) = [\beta n_P^\gamma + (1-\beta)\overline{p}]\ [\overline{N}(t)/\overline{N}(s'')]^{-\gamma} = [\beta n_P^\gamma + (1-\beta)\overline{p}]\, e^{g\gamma(s''-t)}$

Using the above discussion we may calculate the value of an innovation as

$$\begin{aligned} B &= \int_t^{t+\Delta} (1-\beta)\left(\overline{p}e^{g\gamma(s-t)} - 1\right) e^{-r(s-t)}ds + \int_{t+\Delta}^{\infty} \left([\beta n_P^\gamma + (1-\beta)\overline{p}]\, e^{g\gamma(s-t)} - 1\right) e^{-r(s-t)}ds \quad (16) \\ &= (1-\beta)\left(\overline{p}\frac{1-(n_P)^{\phi/g}}{\phi} - \frac{1-(n_P)^{(\phi+g\gamma)/g}}{\phi+g\gamma}\right) + \left([\beta {n_P}^\gamma + (1-\beta)\overline{p}]\,\frac{(n_P)^{\phi/g}}{\phi} - \frac{(n_P)^{(\phi+g\gamma)/g}}{\phi+g\gamma}\right) \end{aligned}$$

where we used the definition $\phi = r - g\gamma$ and the fact that from (14) $r = \rho + g(\sigma(1-\gamma)+\gamma)$.

[14] That an innovator stays on the market forever is a simplifying assumption. We could introduce, for instance, finite patent protection and assume that the market become competitive once the patent has expired. Our main conclusions would unchanged, as long as patents expire before the poor can afford the good. If patents expire earlier, it is only the willingsness to pay of the rich that counts for the incentive to innovate.

6 The distribution of income and wealth

Until now we have assumed that there are two types of consumers with population size β for the poor and $1-\beta$ for the rich. Furthermore, we have let the consumers' income out of labor and assets be different between households, leading to differences in the optimal budgets $E_i(t)$ between consumers. These differences, in turn, imply certain structures of consumption and prices, and determine the level of aggregate employment in sectors producing, respectively, the homogenous product and the hierarchical products in the economy.

Our analysis led us to conclude that the *personal* distribution of income affects the structure of prices. For instance, in the scenario we are focussing on, $N_P < N_R = \overline{N}$, we must have a sufficiently dispersed distribution of budgets, such that only the rich buy all goods, but the poor cannot afford all of these goods. On the other hand, the scenario, where $N_P = N_R = \overline{N}$ is obviously more likely if inequality in income and wealth is lower (and will be the outcome with perfect equality). We have also found that the structure of prices is determined by the *personal* distribution, which in turn implies that the profit level of each firm and hence also aggregate profits are determined by the personal distribution. Consequently, in this model the *personal* distribution of income affects the distribution of aggregate income between wages and profits, that is the personal distribtuion determines the *functional* distribution.

In general it is obvious, that the chain of causality also goes in the other way. A given distribution of aggregate income leads to a certain distribution of income between households, because in general, households differ in the relative importance of the two income sources. Hence a change in the functional distribution leads to a change in the personal distribution of income.

In order to keep the analysis tractable, we will henceforth assume, that each household has the same composition of income which means that the share of labor income is the same both for poor households and for rich households. The assumption of an identical income composition between the different types of households implies together with CRRA intertemporal utility that the savings rate is equal among individuals. Hence, the personal distribution of income does not change over time. Moreover, changes in the functional distribution do not feed back to the personal distribution, as this just means that the composition of income of each household changes and, in relative terms it changes equally within each household. Hence, the relative incomes are not affected, and the personal distribution is a really exogenous ingredient of the model.

We denote by θ the income level of the poor relative to the average. With constant savings rates we can directly write the expenditures of poor and rich in terms of average expenditures: $E_P(t) = \theta\, \bar{E}(t)$ and $E_R(t) = \frac{1-\beta\theta}{1-\beta}\, \bar{E}(t)$ where the latter expression follows from $\beta E_P + (1-\beta)E_R = \bar{E}$.

It should be clear that this assumption is a simplification that allows us to discuss the impact of income heterogeneity on growth and (the functional) distribution. Clearly, this assumption is not particularly realistic. (It implies, for instance, that the distribution of income and the distribution of wealth are identical, whereas in reality we have a situation where the distribution of wealth is more unequal than the distribution of income.) The main reason why we adopt this assumption is analytical convenience. However, the main mechanisms that drive the results in this model become clear when we use this simplifying assumption.

Using equations (7) and (8), and the fact that our distributional assumption implies $\frac{E_R(t)}{E_P(t)} = \frac{1-\beta\theta}{(1-\beta)\theta}$, we can write relative expenditures as

$$\frac{1-\beta\theta}{(1-\beta)\,\theta} = \frac{[\beta n_P^{\gamma} + (1-\beta)\overline{p}]\,\frac{n_P^{1-\gamma}}{1-\gamma} + \overline{p}\frac{1-n_P^{1-\gamma}}{1-\gamma} + \overline{p}\frac{\nu}{1-\gamma}}{[\beta n_P^{\gamma} + (1-\beta)\overline{p}]\,\frac{n_P^{1-\gamma}}{1-\gamma} + \left[\beta + (1-\beta)\overline{p} n_P^{-\gamma}\right]\frac{\nu n_P}{1-\gamma}}.$$

We note that this equation contains only two unknowns (this is where the distributional assumption makes things analytically tractable). We note that the above equation is linear in $\overline{p}$ which allows us to rewrite this equation as

$$\overline{p} = \frac{\nu\beta n_P + \frac{1-\theta}{(1-\beta)\theta}\beta n_P\,(1+\nu)}{1-n_P^{1-\gamma} + \nu\left(1-(1-\beta)n_P^{1-\gamma}\right) - \frac{1-\theta}{\theta}n_P^{1-\gamma}\,(1+\nu)}. \tag{17}$$

We note that, on the right-hand-side of the above equation, the numerator increases and the denominator decrease in n_P. This implies that $\overline{p}$ is monotonically increasing in n_P. Intuitively, when there is a higher level of income, the poor can afford more goods and the rich are willing to pay more for the existing goods (they can buy all of them). (The distributional assumption guarantees, that the relative income difference remains always constant.)

7 The general equilibrium

The discussion in Sections 3 to 6 has focused on the scenario where the rich, but not the poor buy the product that has least priority among all goods available in the market. In that case we have an equilibrium structure of consumption such that the poor buy all goods in the range $[0, N_P]$ whereas the rich buy the whole menu of goods that is available on the market $\left[0, \overline{N}\right]$. Clearly, these discussion is only relevant if the equilibrium outcome is such that $N_P < N_R = \overline{N}$. However, $N_P(t)$, $N_R(t)$, and $\overline{N}(t)$ are themselves endogenously determined. So, a comprehensive presentation of the general equilibrium of the model has to take account of all possible equilibria that the model may generate. We therefore have also to discuss the cases where the equilibrium outcome is such that no consumer can purchase all $\overline{N}(t)$ available goods (in which case we have $N_P < N_R < \overline{N}$); and the outcome where all consumers can buy all $\overline{N}(t)$ goods (in which case $N_P = N_R = \overline{N}$).

After having described the various possible equilibrium regimes, we proceed by discussing the conditions under which the various outcomes will be established.

7.1 The three possible regimes

The regime $N_P < N_R = \overline{N}$. In the regime when only the rich but not the poor purchase all the monopolistic goods that are supplied on the market we can characterized the equilibrium to the following three equations in the three unknowns $n_P, \overline{p}$, and g (we note that $\phi = \rho + g\sigma(1-\gamma)$).

$$1 = gF + b\left(\beta n_P + (1-\beta)\right) + \frac{\nu}{1-\gamma} b \left(\beta^2 n_P + \beta(1-\beta)\overline{p} n_P^{1-\gamma} + (1-\beta)\overline{p}\right) \quad (18)$$

(resource constraint)

$$\frac{F}{b} = (1-\beta)\left(\overline{p}\frac{1-(n_P)^{\phi/g}}{\phi} - \frac{1-(n_P)^{(\phi+g\gamma)/g}}{\phi+g\gamma}\right) + \left(\left[\beta {n_P}^{\gamma} + (1-\beta)\overline{p}\right]\frac{(n_P)^{\phi/g}}{\phi} - \frac{(n_P)^{(\phi+g\gamma)/g}}{\phi+g\gamma}\right) \quad (19)$$

(zero-profit condition)

$$\overline{p} = \frac{\nu\beta n_P + \frac{1-\theta}{(1-\beta)\theta}\beta n_P\left(1+\nu\right)}{1 - n_P^{1-\gamma} + \nu\left(1-(1-\beta)n_P^{1-\gamma}\right) - \frac{1-\theta}{\theta} n_P^{1-\gamma}\left(1+\nu\right)} \quad (20)$$

(static equilibrium condition)

.

It is obvious that this system can easily be reduced to two equations in the two unknowns, by substituting the last equation into, respectively, the zero-profit condition and the resource constraint. Therefore, the most convenient presentation of the equilibrium in the regime $N_P < N_R = \overline{N}$ is in terms of the growth rate, g, and the fraction of monopolistic goods that the poor can afford, n_P.

The regime $N_P < N_R < \overline{N}$. In this case where neither the poor nor the rich can afford all products that are available on the market, the general equilibrium differs form the above regime in two respects. First, in this scenario good $\overline{N}$ has no demand and hence the price of this good, $\overline{p}$, is not defined in this case. However, the structure of price can be expressed similarly as before in terms of the price of the good with least priority that is actually purchased. This good is now N_R and we can express all other prices in terms of $p(N_R)$. It is easy to see that the price of good N_R must equal the marginal cost, that is $p(N_R) = 1$. If $p(N_R) > 1$ it would be profitable for a firm $j > N_R$ to start production since the willingness to pay of the rich would be above marginal costs.

The second crucial difference between the regime $N_P < N_R < \overline{N}$ and the regime $N_P < N_R = \overline{N}$ is that fraction of goods that the rich can afford is now an additional endogenous variable. It turns out convenient to express the new endogenous variables in term of the *waiting time of the innovator.* Obviously, a new innovator has no demand at the date when the innovation takes place.

The reason is that not even the rich can afford this product, and the innovator has to wait until the rich become willing to pay at least a price that covers the firm's cost of production which equal to unity. Nevertheless, the firm has an incentive to make the innovation, and to patent it. This innovation has to be made in time, and in order to prevent other innovators from capturing this market the innovation has to be made 'in time', i.e. before there is demand for this product. How long is the waiting time? Suppose we are on a balanced growth path with rate g, and the rich can afford $N_R(t) < \overline{N}(t)$ products. The waiting time which we denote by δ is defined by the equation $N_R(t)e^{g\delta} = \overline{N}(t)$, or equivalently, $\delta = -\frac{1}{g}\ln\left(\frac{N_R(t)}{\overline{N}(t)}\right)$. Obviously, the waiting time δ is short when growth is high and/or when the rich can afford a high fraction of the available products.

In the appendix we show that the general equilibrium in the regime $N_P < N_R < \overline{N}$ boils down to three equations in the three unknowns, $\widetilde{n}_P, \delta$, and g, where we now have $n_P \equiv N_P(t)/N_R(t)$ as the fraction of goods purchased by the rich, that the poor can afford. (Note that this is not a change in the definition of n_P, as in the regime $N_P < N_R = \overline{N}$, n_P is also the fraction of goods purchased by the rich that the poor can afford as we have $N_R = \overline{N}$; we also note that the mass of goods the poor consume at date t, $N_P(t)$, is now given by $N_P(t) = e^{-\delta g} n_P \overline{N}(t)$). The three equations are the resource constraint, the zero-profit condition, and the relation of the relative expenditures between the rich and the poor:

$$1 = gF + be^{-\delta g}\left(\beta n_P + 1 - \beta\right) + \frac{\nu}{1-\gamma} be^{-\delta g}\left[\beta^2 n_P + \beta(1-\beta)n_P^{1-\gamma} + (1-\beta)^2\right] \quad (21)$$

(resource constraint)

$$\frac{F}{b} = \begin{pmatrix} (1-\beta)\left(\frac{1-n_P}{\phi} - \frac{1-(n_P)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right) \\ +\left(\left[\beta n_P^{\gamma} + (1-\beta)\right]\frac{(n_P)^{\frac{\phi}{g}}}{\phi} - \frac{(n_P)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right) \end{pmatrix} \cdot e^{-\delta[\phi+g\gamma]} \quad (22)$$

(zero-profit condition)

$$\frac{1-\theta}{(1-\beta)\,\theta} = \frac{\left(1+\nu-n_P^{1-\gamma}\right) - \nu\left(\beta n_P + (1-\beta)n_P^{1-\gamma}\right)}{\left(\beta n_P + (1-\beta)n_P^{1-\gamma}\right)(1+\nu)} \quad (23)$$

(static equilibrium condition)

It should be clear that this system of equaitons reduces conveniently to two equations in the two unknowns, the growth rate g and the waiting time δ. To see this, note that the only endogenous variable that shows up in the third equation is n_P. Moreover, the numerator is decreasing and the denominator is increasing in n_P, meaning there is a unique value[15] of $n_P \equiv \tilde{n}_P$ that satisfies

[15] Note that this value is strictly smaller than 1, since $n_P = 1$ implies the right hand side of the static equilibrium condition to be zero.

the third equation. $\tilde{n}_P$ depends on the primitive parameters of the model γ, β, θ, and ν.[16] Once n_P is determined, we are left with the resource constraint and the zero-profit condition as the remaining equaitons and with g and δ as the remaining endogenous variables.

We also note that at the point where the switch from the regime $N_P < N_R = \overline{N}$ to the regime $N_P < N_R < \overline{N}$ takes place we have $\overline{p} = 1$ and $\delta = 0$. It is straightforward to check from both resource constraints and the zero-profit conditions in both regimes, that these two respective equations become identical for $\overline{p} = 1$ (in regime $N_P < N_R = \overline{N}$) and $\delta = 0$ (in regime $N_P < N_R < \overline{N}$). This means that at the switch of the regimes there is no discrete jump in the grwoth rate g.

The regime $N_P = N_R = \overline{N}$. Finally, it remains to describe the static equilibrium when we have a situation where both types of consumers purchase all differentiated goods that are available on the market, the case $N_P = N_R = \overline{N}$. There is one crucial difference to the former two cases: in both of those cases we had a situation such that the good that has least priority for consumer i, N_i has a price that is equal to consumer i's willingness to pay for that good, $q_i(N_i)$. Now, as N_i is identical for both types of consumers, we have a situation where the good that has least priority for the rich, is priced at the willingness to pay for the poor. But this means that we have a situation where the richs' willingness to pay for good $\overline{N}$ is higher than the price $\overline{p}$. This is important as it implies that the rich spend relatively more of their budget on the homogeneous good than they would if the firm could get the willingness to pay from the rich.

The system becomes easier than in the former two cases as we have now a situation where all consumers buy all goods, so $n_P = 1$ and $\delta = 0$. Compared to the previous regime ($N_P < N_R < \overline{N}$) we now get rid of two variables, but have only one additional variable, the price of good $\overline{N}$, which, just like before, we denote by $\overline{p}$. In the Appendix we solve the system step by step, and show that the general equilibrium can be reduced to two equations, the resource constraint and the zero-profit condition in two unknowns: the growth rate g, and the price of the good with least priority $\overline{p}$. These equations are

$$1 = gF + b + b\overline{p}\frac{1+v-\theta}{\theta(1-\gamma)} \quad (24)$$

(resource constraint)

$$\frac{F}{b} = \overline{p}\frac{1}{\phi} - \frac{1}{\phi + g\gamma} \quad (25)$$

(zero-profit condition)

Also here we consider the point where the switch from the regime $N_P < N_R = \overline{N}$ to the regime $N_P = N_R = \overline{N}$ takes place. At the switch we have $n_P = 1$. From the general equilibrium conditions for regime $N_P < N_R = \overline{N}$

[16] In particular, $\frac{\partial \tilde{n}_P}{\partial \theta} > 0$ and $\frac{\partial \tilde{n}_P}{\partial \beta} < 0$.

we immediately see that, when $n_P = 1$ the resource constraint and the zero-profit condition become identical to the above two equilibrium conditions for the regime $N_P = N_R = \overline{N}$.

7.2 A graphical representation of the equilibrium

In the following we will show under which conditions an equilbrium exists and when it is unique. Furthermore, we will discuss when one of the three regimes actually occurs. In particular, the analysis will allow us to discuss parameter constellations that make certain regimes more likely. From the discussion above we know that the interesting variables are: the growth rate g; the fraction of goods the poor can afford n_P; the waiting time of the most recent innovator δ; and the price charged by the most recent innovator $\overline{p}$.

In this section we discuss the general equilibrium of the model by using a graphical representation. From the discussion in the last subsection it has become clear that the equilibrium conditions in each regime can be conveniently reduced to two equations in two unknown. In all three regimes, one of the endogenous variables is the growth rate g. However, the second relevant endogenous variable is different across regimes: it is the fraction of goods purchased by the poor n_P in regime $N_p < N_R = \bar{N}$; it is the innovator's waiting time δ in regime $N_P < N_R < \bar{N}$; and it is price charged by the most recent innovator $\overline{p}$ in regime $N_p = N_R = \bar{N}$. In what follows we will represent the general equilibrium by looking at the zero-profit condition and the resource constraint (substituting out the static equilibrium condition) for all three regimes (Figures 3 and 4). In (g, δ) space for the regime $N_P < N_R < \bar{N}$; in (g, n_P)-space for the regime $N_P < N_R = \bar{N}$; and in $(g, \overline{p})$ space for the regime $N_P = N_R = \bar{N}$.

Before we start to discuss, respectively, the shapes of the resource constraint and the zero-profit condition in the various regimes, it is useful to look at the relevant ranges of the endogenous variables (other than g) in the various regimes. The problem is easy in regime $N_P < N_R < \bar{N}$ where we focus on the waiting time δ as the second endogenous variable. δ starts at zero which is the case when the rich are indifferent between purchasing and not purchasing the good of the most recent innovator. It is also evident that δ may become very large, infinitely large in the case of stagnation. The problem is less obvious in the two remaining regimes. $N_P < N_R = \bar{N}$ and $N_P = N_R = \bar{N}$. The following Lemma discusses the relevant ranges of the two respective second endogenous variables, the fraction of goods purchased by the poor n_P (in regime $N_P < N_R = \bar{N}$) and the price charged by the most recent innovator $\overline{p}$ (in regime $N_P = N_R = \bar{N}$). We note that these ranges refer to endogenous variables, and the Lemma discusses how the relevant limits for these endogenous variables depend on the exogenous parameters of the model. We will say 'a regime becomes more likely' if the relevant range of the respective endogenous variable becomes broader.

Lemma 2 (Regime Switches).

a) Consider a switch from the regime $N_p < N_R < \bar{N}$ to the regime $N_p < N_R = \bar{N}$. At the switch the most recent innovator charges price $\bar{p} = 1$ and has *no waiting time, so $\delta = 0$. The switch occurs at consumption level of the poor $n_P = \tilde{n}_P$ where $\tilde{n}_P$ is the value of n_P that satisfies equation (23). The regime $N_p < N_R = \bar{N}$ therefore starts at $n_P = \tilde{n}_P$ (here the most recent innovator is indifferent between not selling to the rich and earning no flow profit and selling to the rich at a price equal the marginal production cost and earning a zero flow profit), and ends as $n_P = 1$ (when it becomes optimal for the poor consumers to consume all available varieties).*

b) Consider a switch from the regime $N_p < N_R = \bar{N}$ to the regime $N_p = N_R = \bar{N}$. At the switch the poor start to buy all available varieties, so n_P becomes unity. The most recent innovator becomes indifferent between selling only to the rich and selling to the entire market. When this firm sells to only to the rich, it charges a price equal to $\bar{p} = \hat{p} = \frac{\nu\beta + \frac{1-\theta}{(1-\beta)\theta}\beta(1+\nu)}{\nu\beta - \frac{1-\theta}{\theta}(1+\nu)}$. When this firm sells to the entire market, the price is $\frac{\hat{p}-\beta}{1-\beta}$. Consequently, when we are in the regime $N_p = N_R = \bar{N}$, the lower limit for the price of the most recent innovator becomes $\frac{\hat{p}-\beta}{1-\beta}$. (See Figure 2).

Proof. Part b. Insert $n_P = 1$ into (20) and the formula for $\hat{p}$ follows directly. ■

We proceed by discussing the general equilibrium of the model by looking at the resource constraint RC and the zero-profit condition Z. We will discuss the general equilibrium using a graphical representation which is done in Figures 3 and Figures 4 below. Before doing so we study the shape of the two curves in more detail. This is done in the following two Lemmas.

Lemma 3 *(zero profit condition Z)*

a) The value of innovation monotonically falls in the growth rate if $\gamma \leq \frac{\sigma \frac{F\rho}{b}}{1+\sigma\frac{F\rho}{b}}$.

b) The zero profit condition crosses the n_p-axis at n_p^z where $\bar{p} = 1 + \frac{1}{1-\beta}\frac{F\rho}{b}$ given that $1 + \frac{1}{1-\beta}\frac{F\rho}{b} \leq \hat{p}$.

c) If $1 + \frac{1}{1-\beta}\frac{F\rho}{b} > \hat{p}$ the zero profit condition crosses the $\bar{p}$-axis at $\bar{p} = 1 + \frac{F\rho}{b}$

Proof. See appendix. ■

In the $N_p < N_R = \bar{N}$ regime the value of innovations increases in n_P as can be seen by direct inspection of the zero-profit condition. Though the limits of the integral depend on n_P, they do not affect the derivative's value because $\Pi_R(N_P) = \Pi_{tot}(N_P)$. From the above Lemma we know that with flat hierarchy the value of innovations always falls in the growth rate g. Combining these two elements it follows that the zero profit condition is monotonically increasing in (g, n_p)-space starting at n_P^z. In the regime $N_P = N_R = \bar{N}$ the value of innovations increasing in the price of the most recent innovator $\bar{p}$, and falls in the growth rate g (again under the conditions of Lemma 1). Hence in this regime the zero profit constraint monotonically increases in the $(g, \bar{p})$-space.It remains to discuss the regime $N_P < N_R < \bar{N}$. From the zero-profit condition it is straightforward to verify that a longer waiting time reduces the value of innovations.

Hence we conclude that, with a 'flat hierarchy' (see above Lemma), the zero profit constraint does not 'bend to the left' and the zero profit constraint is monotonically increasing, starting in the regime $N_P < N_R = \bar{N}$ at $n_P^{\tilde{z}} > \tilde{n}_P$ and then continues to switch to the regime $N_P = N_R = \bar{N}$. The regime $N_P < N_R < \bar{N}$ is never reached. Hence with a flat hiearchy the zero-profit locus looks like in Figure 3. With a steep hierarchy things are different. In that case the zero-profit condition is not monotonic. It still starts at $n_P^{\tilde{z}}$ but then has a negative slope, and may reach the regime $N_P < N_R < \bar{N}$. As the growth rate g becomes larger, the slope becomes positive again and reaches again the regime $N_P < N_R = \bar{N}$. Finally, for sufficiently high g, the zero-profit curve switches to the regime $N_P = N_R = \bar{N}$.

Lemma 4 (resource constraint RC)

a. The resource constraint crosses the n_P-axis at $n_P^{RC} \geq \tilde{n}_P$ if $1 \geq b\,(\beta\tilde{n}_P + 1 - \beta) + \frac{\nu}{1-\gamma}b[\beta^2\tilde{n}_P + \beta(1-\beta)\tilde{n}_P^{1-\gamma} + 1 - \beta]$.

b. The resource constraint crosses the n_P-axis at $n_P^{RC} \leq 1$ if $\frac{1-b}{b}(1-\gamma) \leq \frac{1+\nu-\theta}{\theta - \frac{1-\theta}{\beta}\frac{1+\nu}{\nu}}$.

Proof. Part a. The right hand side of the resource constraint (18) increases in n_P. We get the condition directly by inserting $g = 0$ into (18).

Part b. From (18) we get a condition that $n_P^{RC} \leq 1$, namely by noting that with $n_P = 1$ and $g = 0$ the right hand side would exceed one: $1 \leq b + \frac{\nu}{1-\gamma}b\left(\beta^2 + \beta(1-\beta)\hat{p} + (1-\beta)\hat{p}\right)$. Inserting the value $\hat{p}$ from Lemma 2 and rearranging terms, gives us the required result. ■

Part a of the above Lemma states a necessary condititon for the region $N_p < N_R = \bar{N}$ to exist. If this condition does not hold the resource constraint can only be fulfilled in the $N_p < N_R < \bar{N}$ regime. Conversely, only if the condition in Part b of the above Lemma is violated, the equilibrium can be in the regime $N_P = N_R = \bar{N}$. This condition says that the differentiated sector has to be sufficiently productive, so that a situation where all consumers buy all available differentiated products is feasible. If this is the case, the resource constraint crosses the $\bar{p}$-axis at $\bar{p} = \frac{1-b}{b}\frac{\theta(1-\gamma)}{1+\nu-\theta}$ (see equation (24).

In the $N_p < N_R = \bar{N}$ regime the resource constraint (18) is falling in the (g, n_P)-space. We see this from (18): more resources are needed when the growth rate g is higher because there are more researchers, and when the share of the products consumed by all consumers n_P is higher (note that $\partial\bar{p}/\partial n_P > 0$). The curve crosses the n_P-axis at n_P^{RC} which is implicitely defined by (18) and $g = 0$. But Lemma 4 above exactly states the conditions on the parameter values such that $\tilde{n}_P < n_P^{RC} < 1$.

In the $N_p = N_R = \bar{N}$ regime the resource constraint is a linear function in g and $\bar{p}$. The intuition is that the first order conditions of consumer optimization suggest the expenditures on traditional goods to rise if $\bar{p}$ rises, thus more resources are needed when $\bar{p}$ increases. Finally, as higher growth rate needs more researchers, we conclude that the resource constraint is a falling line in the $(g, \bar{p})$-space.

In the $N_p < N_R < \bar{N}$ regime the resource constraint (34) is a function of the growth rate g and the waiting time δ since $\tilde{n}_P$ is given by the static equilibrium condition. Looking at (34) we see directly that, if δ rises, less resources in production of innovative and traditional goods are needed. If g rises more researchers are needed. With δ kept fixed resources needed in goods production fall but under the condition of Lemma 3a. the first effect dominates: higher g leads to an increase in needed resources. We conclude that the resource constraint is a falling curve in the (g, δ)-space.

figure 2

Having discussed that shape of the zero profit conditions and the resource constraint we can now talk about the existence and uniqueness of the general equilibrium in our model. We have seen that, if the hierarchy is flat enough, the zero-profit condition is monotonic, and that the resource constraint is always monotonic. This allows us to state the following proposition.

Proposition 5 (existence and uniqueness of equilibrium).

a. If the hierarchy is flat enough, so that the zero-profit condition is monotonic, and if the resource constraint crosses the n_P- or $\bar{p}$-axis to the right of the zero profit condition, there exists a unique general equilibrium with a positive growth rate. If the resource constraint crosses the n_P- or $\bar{p}$-axis to the left of the zero profit condition, the unique equilibrium is stagnation.

b. If the hierarchy is flat enough, so that the zero-profit condition is monotonic, both the regimes $N_P < N_R = \bar{N}$ and the regime $N_P < N_R < \bar{N}$ can be equilibrium outcomes. The regime $N_P < N_R < \bar{N}$ can only arise, if the hierarchy is steep enough.

7.3 Steeper Hierarchy

We finally discuss shortly the case when $\gamma > \frac{\frac{\sigma}{1-\beta}\frac{F\rho}{b}}{1+\frac{\sigma}{1-\beta}\frac{F\rho}{b}}$, i.e. when the zero profit constraint "bends to the left" at $n_P^{\approx}$. Note first, that the whole discussion concerning the behavior of the resource constraint still holds because Lemma 1 was not needed there. Only the zero profit constraint changes significantly. Figure 3 below shows a case where a positive growth equilibrium with waiting time exists.

figure 3

8 Comparative Statics

We have developed a model which allows us to discuss the effects inequality has on the demand structure and, in particular, on the demand for innovators.

Thus, it is natural to ask how the growth rate g, the poors' share n_P/n_R,and the share of the traditional sector x are affected if the inequality parameters β and θ change.

8.1 No traditional sector ($\nu = 0$)

To gain intuition, it is instructive to look at the baseline case where $\nu = 0$, i.e. no traditional sector exists. Note that in this case the regime $N_P = N_R = \bar{N}$ can not exist, because the necessary condition for existence $\theta - \frac{1-\theta}{\beta}\frac{1+\nu}{\nu} > 0$ is violated if $\nu = 0$ (see Lemma 4). But the intuition is very clear indeed: If $N_P = N_R = \bar{N}$ were an equilibrium, rich and poor would consume exactly the same, since there is no expenditure for x-goods. This would imply that the rich have income left causing their marginal utility of income be zero and thus their marginal willingness to pay be infinity. Hence it would be profitable for a monopolist to deviate, since he could raise profits by selling only to the rich. We can state the following proposition

Proposition 6 *If $\nu = 0$ and hierarchy is flat, the growth rate g increases and the share of the poor n_P/n_R decreases if θ decreases or β increases.*

Proof. See appendix. ■

The proposition states that increases in inequality in the Lorenz-sense (as captured by an increase in θ or a decrease in β) unambigously increase growth. The intuition can be grasped by looking either at the allocation of labor or at the resulting incentives for innovations.

With a higher θ the poor become relatively richer, thus their consumption share increases, but this needs more labor in final good production what means that less researchers can be employed, this reduces growth. On the other hand, if β increases there are less people in the economy who consume all goods, hence more labor is left for research and growth rises. To get some intuition by looking at the innovation incentives note that the research expenditures equal profits in this economy. Since higher inequality rises growth, as is suggested by the proposition, it is equivalent to say that the profit share increases with inequality. But this simply means that the average markups are higher in this economy. The reason is that monopolists may charge higher markups from the rich and more product are sold at higher markups (since the consumption of the poor falls). The lower markups on products which both buy cannot dominate the first two effects.

8.2 The general case $v > 0$

With $v > 0$, we have to refer to simulations. However, we can draw general conclusions for the now possible regime $N_P = N_R = \bar{N}$.

Proposition 7 *In the $N_P = N_R = \bar{N}$ regime a rise in θ, i.e. decreasing inequality, unambigously increases growth. A change in β leaves the growth rate unaffected.*

Proof. The equilibrium is characterized by equations (24) and (25). A rise in θ decreases the right hand side of (24), hence higher growth for given $\bar{p}$ is feasible (RC-curve shifts up). The parameter β does not appear in (24) and (25). ■

Since the monopolists always sell to both groups, the prices they can set are determined only by the marginal willingness to pay of the poor (θ increases), the poor want to pay more whath allows the innovators to raise prices and increase profits. This increases in innovation incentive raises the growth rate. On the other hand, a change in the groups size β can have no effect on the growth. With θ held constant the marginal willingness to pay of the relevant consumers is unaffected and their innovation incentives remain the same.

In the $N_p < N_R = \bar{N}$ regime we have to refer to simulations. We got the result, as can be seen from figure 4 below, that increases in θ (decreasing inequality) always decreases growth, i.e. the result from the $\nu = 0$ case still holds. The figure shows further that for $\theta > 0.926$ decreasing inequality raises the growth rate, because for $\theta > 0.926$ the $N_P = N_R = \bar{N}$ regime arises in equilibrium.

figure 4

Instead, for some parameter constellations where inequality is not too high we found that an increase in β (which increases inequality since θ is fixed) decreases the growth rate, an example is plotted in the second graph of figure 4. This is an important result: For the demand structure "higher inequality" is not always the same. Changes in θ affect the willingness to pay, whereas changes in β affect the market size. The simulation result now says that higher inequality due to higher β has a negative effect on growth. The reason is not the rise of inequality itself, because a lower θ would rise both inequality and growth, but the smaller market size $1 - \beta$ for the newest goods, sold only to the rich.

9 Conclusions

The relationship between inequality and demand structure in an endogenous growth model where consumers have non-homothetic preferences was explored. Preliminarily, we repeat the main results. (i) Changes in inequality affect the aggregate price structure and there may be market exclusion of the poor due to high prices. (ii) If there is exclusion, higher inequality tends to increase growth because the profit share increases. However, higher inequality due to a bigger group of poor people may reduce growth. (iii) If the innovators always sell to the whole population, inequality has an unambigously negative impact on growth. Prices are then determined by the willingness to pay of the poor. An even more egalitarian distribution allows the monopolist to set higher prices and earn higher profits as the poor are the 'critical' consumers that determine demand at the extensive margin.

10 Appendix

10.1 The case $N_P < N_R < \overline{N}$

10.1.1 Maximized static utilities and expenditures ($N_P < N_R < \overline{N}$)

We have now an additional variable, $\delta = -\frac{1}{g}\ln(N_R/\overline{N})$ that says how long an innovator has to wait until his product has positive demand. At the same time the price of the good that has least priority among all goods that are acutally sold, i.e. the price of good N_R must equal the marginal cost, that is $p(N_R) = 1$. (If $p(N_R) > 1$ it would be profitable for a firm $j > N_R$ to start production since the willingness to pay of the rich would be above marginal costs.) The prices of the goods $j > N_R$ are not defined. Using analogous arguments as in Section 3 (for the regime $N_P < N_R = \overline{N}$), in the case when we have $N_P < N_R < \overline{N}$ it is straighforward to calculate the equilibrium price structure as

$$p(j) = \begin{cases} [\beta n_P^\gamma + (1-\beta)] \left(\frac{j}{N_R}\right)^{-\gamma} & j \in [0, N_P], \\ \left(\frac{j}{N_R}\right)^{-\gamma} & j \in (N_P, N_R]. \end{cases} \tag{26}$$

where we note that now $n_P = N_P(t)/\overline{N}(t)$. The static equilibrium is then the solution to the following six equations in the six unknowns $n_P, n_R, x_P, x_R, \lambda_P$ and λ_R. (note that $N_P(t) = n_P e^{-\delta g}\overline{N}$ and $N_R(t) = e^{-\delta g}\overline{N}$).

$$\beta + (1-\beta) n_P^{-\gamma} = \frac{x_P^\nu \left(n_P e^{-\delta g}\overline{N}\right)^{-\gamma}}{\lambda_P} \tag{R1}$$

$$1 = \frac{x_R^\nu \left(e^{-\delta g}\overline{N}\right)^{-\gamma}}{\lambda_R} \tag{R2}$$

$$\nu x_P^{\nu-1} \frac{\left(n_P e^{-\delta g}\overline{N}\right)^{1-\gamma}}{1-\gamma} = \lambda_P p_x \tag{R3}$$

$$\nu x_R^{\nu-1} \frac{\left(e^{-\delta g}\overline{N}\right)^{1-\gamma}}{1-\gamma} = \lambda_R p_x \tag{R4}$$

$$\frac{E_P}{\overline{N}} = [\beta n_P^\gamma + (1-\beta)] \frac{n_P^{1-\gamma}}{1-\gamma} + \frac{p_x x_P}{\overline{N}} \tag{R5}$$

$$\frac{E_R}{\overline{N}} = [\beta n_P^\gamma + (1-\beta)] \frac{n_P^{1-\gamma}}{1-\gamma} + \frac{1-n_P^{1-\gamma}}{1-\gamma} + \frac{p_x x_R}{\overline{N}} \tag{R6}$$

Again the first two equations equate the willingnesses to pay to the respective good with least priority for both types of consumers, the equations (R3) and (R4) determine the equilibrium amount of the homogeneous good and the final two equations (R5) and (R6) say that the budget constraints have to be satisfied for both types of consumers.

We proceed similarly as before and reduce this system to two equations in the two unknowns n_P and n_R. Combining, respectively, equations (R1) and

(R3), and equations (R2) and (R4), we can write

$$p_x x_P = \frac{\nu \overline{N}}{1-\gamma} \left[\beta + (1-\beta) n_P^{-\gamma}\right] n_P \tag{27}$$

and

$$p_x x_R = \frac{\nu \overline{N}}{1-\gamma} n_R, \tag{28}$$

and substitute these relations into equations (S5) and (S6) of the above system. This yields

$$\frac{E_P}{\overline{N}} = \left[\beta n_P^{\gamma} + (1-\beta)\right] \frac{n_P^{1-\gamma}}{1-\gamma} n_P + \left[\beta + (1-\beta) n_P^{-\gamma}\right] \frac{\nu n_P}{1-\gamma}, \tag{29}$$

and

$$\frac{E_R}{\overline{N}} = \left[\beta n_P^{\gamma} + (1-\beta)\right] \frac{n_P^{1-\gamma}}{1-\gamma} n_P + \frac{1-n_P^{1-\gamma}}{1-\gamma} + \frac{\nu}{1-\gamma}, \tag{30}$$

and we note that both E_P and $\overline{N}$ as well as E_R and $\overline{N}$ are proportional.

Finally, we calculate the maximized static utilities in terms of the endogenous variables n_P and n_R. Substituting the relations (27) and (28) into the utility flow function (2). This yields for the rich

$$u_R(n_R < 1,\ n_P < 1) = \left(\frac{\nu}{1-\gamma} \frac{1}{w b_x}\right)^{\nu} e^{-\delta g(1+\nu-\gamma)} \frac{\overline{N}^{1-\gamma}}{1-\gamma} \tag{31}$$

and for the poor

$$u_P(n_R < 1,\ n_P < 1) = \left(\left[\beta n_P^{\gamma} + (1-\beta)\right] \frac{\nu}{1-\gamma} \frac{1}{w b_x}\right)^{\nu} \left(n_P e^{-\delta g}\right)^{1+\nu-\gamma} \frac{\overline{N}^{1-\gamma}}{1-\gamma} \tag{32}$$

From (7) and (8) we know that, for given values of n_P and $\overline{p}$, the range of available goods $\overline{N}$ and the expenditure levels E_i are proportional for both types of consumers. Moreover, we will see in the next section that also $\frac{\overline{N}}{p_x}$ is a constant. It follows that the instantaneous utilities can be written as

$$u_i(n_P, \delta) = \mu_i(n_P, \delta) \frac{E_i^{1-\gamma}}{1-\gamma}. \tag{33}$$

Equation (11) gives the important result that instantaneous utility is of the CRRA-type with hierarchy-parameter γ as the relevant parameter.[17]

[17] An important observation can be made here though the utilities of the rich and the poor are CRRA in their expenditures over time, the ratio of utility between the poor and the rich at a given point of time does not exhibit a CRRA relationship, even if $v = 0$, i.e. $\frac{u(x_R(t), n_R(t))}{u(x_P(t), n_P(t))} \neq \left(\frac{E_R}{E_P}\right)^{1-\gamma}$ in general. The reason is that the expenditure share of a single good is not the same for the rich and the poor, they even do not consume the same goods. Since prices of the various goods are different, rich and poor face a different average price level.

10.1.2 The resource constraint ($N_P < N_R < \overline{N}$)

The resource constraint when $N_P < N_R < \overline{N}$ is given by the sum of employment in R&D L_R, manufacturing of the differentiated hierarchical products L_N; and production of the homogenous good L_x. Just like before, we have again a level of R&D employment equal to $L_R = gF$; a level of employment to produce output in the monopolistic sector given by $L_N = (1-\beta)\widetilde{b}(t)N_R(t) + \beta\widetilde{b}(t)N_P(t) = b\left[(1-\beta)e^{-\delta g} + \beta n_P\right]$; and using (27) and (28), a level of employment necessary to produce the output of the homogeneous good, $L_x = \frac{v}{1-\gamma}b\left(\beta\left[\beta n_P^{-\gamma} + (1-\beta)\right]n_P + (1-\beta)e^{-\delta g}\right)$. The resource constraint is therefore given by

$$1 = gF + b\left(\beta n_P + (1-\beta)e^{-\delta g}\right) + \frac{\nu}{1-\gamma}b\left[[\beta^2 + \beta(1-\beta)n_P^{-\gamma}]n_P + (1-\beta)e^{-\delta g}\right] \tag{34}$$

10.1.3 The zero-profit condition ($N_P < N_R < \overline{N}$)

In equilibrium we must have that the innovation cost equals the value of an innovation. In the equilibrium where $N_P < N_R < \overline{N}$ no household can afford good $\overline{N}$ when the innovator incurs the fixed cost so that good $\overline{N}$ is actually available on the market. This means that innovator have a waiting time δ during which nobody will buy the new product. At date $t+\delta$, the waiting time ends, and the rich consumers start to buy the good $\overline{N}(t)$ (which has been invented at date t). From date $t+\delta$ until date $t+\delta+\Delta$ only the rich can afford this good, as monopolists set prices equal to the willingness to pay of the rich. But at date $t+\delta+\Delta$ also the poor have a sufficiently high willingness to pay so the monopolist will cut prices and charge the willingsness to pay of the poor, so the monopolistic firm gets the entire market. The waiting time δ until the rich start to buy consumers in the economy is given by $N_R(t+\delta) = \overline{N}(t)$ and using the fact that both $\overline{N}(t)$ and $N_R(t)$ grow at the same rate, δ can be calculated as $\delta = -(1/g)\ln n_R$. Moreover, the additional waiting period Δ until also the poor buy (i.e. until it becomes profitable to cut prices and sell to the entire market) is given by $\Delta = -(1/g)\ln\frac{n_P}{n_R}$.

By these arguments, we can calculate the zero-profit condition in the case Proceeding similarly as in the case $N_P < N_R < \overline{N}$ as (we proceed similarly as in the case $N_P < N_R < \overline{N}$ described in detail in the main text)

$$\frac{F}{b} = \int_{t+\delta}^{t+\delta+\Delta}(1-\beta)\left(\left(\frac{\overline{N}(s)}{\overline{N}(t)}\right)^{\gamma}-1\right)e^{-r(s-t)}ds$$
$$+\int_{t+\delta+\Delta}^{\infty}\left([\beta n_P^{\gamma}+(1-\beta)]\left(\frac{\overline{N}(s)}{\overline{N}(t)}\right)^{\gamma}-1\right)e^{-r(s-t)}ds$$
$$= \left(\begin{array}{c}(1-\beta)\left(\frac{1-n_P}{\phi}-\frac{1-(n_P)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right)\\ +\left([\beta n_P^{\gamma}+(1-\beta)]\frac{(n_P)^{\frac{\phi}{g}}}{\phi}-\frac{(n_P)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right)\end{array}\right)\cdot e^{-\delta[\phi+g\gamma]}$$

where we have again used the definition $\phi = r - g\gamma$ and the fact that from (14)

10.1.4 The equilibrium conditions ($N_P < N_R < \overline{N}$)

The general equilibrium consists of the following 3 equations in the 3 unknowns n_P, n_R, and g: (i) the zero-profit conditions, (ii) the resource constraint, and (iii) the condition on relative expenditures

$$\frac{F}{b} = \left(\begin{array}{c}(1-\beta)\left(\frac{1-\left(\frac{n_P}{n_R}\right)^{\frac{\phi}{g}}}{\phi}-\frac{1-\left(\frac{n_P}{n_R}\right)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right)\\ +\left(\left[\beta\left(\frac{n_P}{n_R}\right)^{\gamma}+(1-\beta)\right]\frac{\left(\frac{n_P}{n_R}\right)^{\frac{\phi}{g}}}{\phi}-\frac{\left(\frac{n_P}{n_R}\right)^{\frac{\phi+g\gamma}{g}}}{\phi+g\gamma}\right)\end{array}\right)\cdot n_R^{\frac{\phi+g\gamma}{g}} \qquad (E\prime a))$$

$$1 = gF + b\left(\beta n_P + (1-\beta)n_R\right) + \frac{\nu}{1-\gamma}b\left[[\beta^2+\beta(1-\beta)\left(\frac{n_P}{n_R}\right)^{-\gamma}]n_P + (1-\beta)n_R\right] \qquad (E'b))$$

and, by deviding both sides of equation (30) by equation (29), using $E_R/E_P - 1 = \frac{1-\theta}{(1-\beta)\theta}$, we get

$$\frac{1-\theta}{(1-\beta)\,\theta} = \frac{\left(1+\nu-n_P^{1-\gamma}\right)-\nu\left(\beta n_P+(1-\beta)n_P^{1-\gamma}\right)}{\left(\beta n_P+(1-\beta)n_P^{1-\gamma}\right)(1+\nu)}. \qquad (E\prime c))$$

10.2 The case $N_P = N_R = \overline{N}$

10.2.1 Maximized static utilities and expenditures ($N_P = N_R = \overline{N}$)

Finally, it remains to describe the static equilibrium when we have a situation where both types of consumers purchase all differentiated goods that are available on the market, the case $N_P = N_R = \overline{N}$. There is one crucial difference to the former two cases: in both of those cases we had a situation such that the good that has least priority for consumer i, N_i has a price that is equal to consumer i's willingness to pay for that good, $q_i(N_i)$. Now, as N_i is identical for both types of consumers, we have a situation where the good that has least priority for the rich, is priced at the willingness to pay for the poor. But this means that we have a situation where the richs' willingness to pay for good $\overline{N}$ is higher than the price $\overline{p}$. This is important as it implies that the rich spend relatively more of their budget on the homogeneous good than they would if the firm could get the willingness to pay from the rich. It terms of our equilibrium equations, it implies that we have only five equations (plus one inequality) in the following five variables, $\overline{p}, x_P, x_R, \lambda_P$ and λ_R

$$\overline{p} = \frac{x_P^{\nu}\overline{N}^{-\gamma}}{\lambda_P} \tag{Q1}$$

$$\overline{p} < \frac{x_R^{\nu}\overline{N}^{-\gamma}}{\lambda_R} \tag{Q2}$$

$$\nu x_P^{\nu-1}\frac{\overline{N}^{1-\gamma}}{1-\gamma} = \lambda_P p_x \tag{Q3}$$

$$\nu x_R^{\nu-1}\frac{\overline{N}^{1-\gamma}}{1-\gamma} = \lambda_R p_x \tag{Q4}$$

$$\frac{E_P}{\overline{N}} = \frac{\overline{p}}{1-\gamma} + \frac{p_x x_P}{\overline{N}} \tag{Q5}$$

$$\frac{E_R}{\overline{N}} = \frac{\overline{p}}{1-\gamma} + \frac{p_x x_R}{\overline{N}}. \tag{Q6}$$

Relations (Q1) and (Q2) say that the price of good $\overline{N}$, $\overline{p}$, is equal to the willingness to pay of the poor, but lower than the willingness to pay of the rich. The remaining equations are the conditions for the equilibrium quantities of the homogeneous good x_i ((Q3) and (Q4)) and the budget constraints for both types of consumers ((Q5) and (Q6)).

We can reduce this system to one equation in $\overline{p}$. Using (Q1) and (Q3), solving for $p_x x_P$ yields

$$p_x x_P = \frac{\nu\overline{N}}{1-\gamma}\overline{p}, \tag{35}$$

and substituting this into into (Q5) yields the expenditure function for the poor (which solves for $\overline{p}$)

$$\frac{E_P}{\overline{N}} = \overline{p}\frac{1+\nu}{1-\gamma}. \tag{36}$$

The expenditure function of the rich can now be easily expressed in terms of the endogenous variables $\overline{p}$ and x_R as From the budget constraint $E_R = \frac{\overline{p}\overline{N}}{1-\gamma} + p_x x_R$

and the previous equation (36) it is straightforward to calculate

$$\frac{E_R}{\overline{N}} = \frac{\overline{p}}{1-\gamma} + \frac{p_x x_R}{\overline{N}} \tag{37}$$

The maximized static utility for the poor can be using that x_P is given by (35). Substituting this into the static utility (2) yields

$$u_P(n_R = n_P = 1,\ \overline{p} > 1) = \left(\overline{p}\frac{\nu}{1-\gamma}\frac{\overline{N}}{p_x}\right)^{\nu}\frac{\overline{N}(t)^{1-\gamma}}{1-\gamma} \tag{38}$$

To get the utility value of the rich, we use (36) and (37) to calculate $x_R = \frac{N}{p_x}\left(\frac{E_R - E_P}{\overline{N}} + \frac{\overline{p}\nu}{1-\gamma}\right)$ and substitute this into the utility function (2) to get

$$u_R(n_R = n_P = 1,\ \overline{p} > 1) = \left(\frac{N}{p_x}\left(\frac{E_R - E_N}{N} + \frac{\overline{p}\nu}{1-\gamma}\right)\right)^{\nu}\frac{\overline{N}(t)^{1-\gamma}}{1-\gamma}.$$

Provided that $\frac{N}{p_x}$ and $\frac{E_R - E_N}{N}$ are constants (which will be the case along the balanced growth path), we can write the maximized static utilities as

$$u_i(n_P,\ \overline{p}) = \mu_i(n_P, \overline{p}, \frac{\overline{N}}{p_x})\ \frac{E_i^{1-\gamma}}{1-\gamma}.$$

10.2.2 The resource constraint ($N_P = N_R = \overline{N}$)

The resource constraint in the case when $N_P = N_R = \overline{N}$ is given by the sum of employment in R&D L_R, manufacturing of the differentiated hierarchical products L_N; and production of the homogenous good L_x. Just like before, we have again a level of R&D employment equal to $L_R = gF$; a level of employment to produce output in the monopolistic sector given by $L_N = (1-\beta)\widetilde{b}(t)N_R(t) + \beta\widetilde{b}(t)N_P(t) = b$; and using (35) and (??), a level of employment necessary to produce the output of the homogeneous good, $L_x = b_x\left[\beta\frac{\overline{N}}{p_x}\frac{\nu\overline{p}}{1-\gamma} + (1-\beta)\frac{\overline{N}}{p_x}\left(\frac{E_R - E_P}{\overline{N}} + \frac{\overline{p}\nu}{1-\gamma}\right)\right] = \frac{1}{w}\left[\beta\frac{\nu\overline{p}}{1-\gamma} + (1-\beta)\left(\frac{E_R - E_P}{\overline{N}} + \frac{\overline{p}\nu}{1-\gamma}\right)\right]$ (because $b_x\frac{\overline{N}}{p_x} = b_x\frac{\overline{N}(t)}{w(t)b_x} = \frac{1}{w}$). Now we use $\frac{E_P}{\overline{N}} = \overline{p}\frac{1+\nu}{1-\gamma}$ and $E_R - E_P = E_P\left[\frac{1-\beta\theta}{(1-\beta)\theta} - 1\right] = \frac{1-\theta}{(1-\beta)\theta}$. So we get $\frac{E_R - E_P}{\overline{N}} = \frac{E_P}{\overline{N}}\frac{1-\theta}{(1-\beta)\theta} = \overline{p}\frac{1+\nu}{1-\gamma}\frac{1-\theta}{(1-\beta)\theta}$ and $L_x = b\left[\beta\frac{\nu\overline{p}}{1-\gamma} + (1-\beta)\left(\overline{p}\frac{1+\nu}{1-\gamma}\frac{1-\theta}{(1-\beta)\theta} + \frac{\overline{p}\nu}{1-\gamma}\right)\right] = b\left(\overline{p}\frac{1+\nu}{1-\gamma}\frac{1-\theta}{\theta} + \frac{\overline{p}\nu}{1-\gamma}\right) = b\overline{p}\frac{(1+\nu)(1-\theta)+\theta\nu}{(1-\gamma)\theta} = b\overline{p}\frac{1+\nu-\theta}{(1-\gamma)\theta}$. This yields e resource constraint is therefore given by

$$1 = gF + b + b\overline{p}\frac{1+\nu-\theta}{\theta\,(1-\gamma)}.$$

10.2.3 The zero-profit condition ($N_P = N_R = \overline{N}$)

In equilibrium we must have that the innovation cost equals the value of an innovation. In the equilibrium where $N_P = N_R = \overline{N}$ all households buy all $\overline{N}$ goods already from the point of time when the innovator incurs the fixed cost and enters the market. The price the firm charges equals the willingness to pay of the poor already from the beginning. Hence the zero-profit condition in this case simplifies to

$$\begin{aligned} \frac{F}{b} &= \int_t^{\infty} \left(\overline{p} e^{g\gamma(s-t)} - 1 \right) e^{-r(s-t)} ds \\ &= \overline{p} \frac{1}{\phi} - \frac{1}{\phi + g\gamma}. \end{aligned} \tag{39}$$

10.2.4 The equilibrium conditions ($N_P = N_R = \overline{N}$)

When $N_P = N_R = \overline{N}$ the equilibrium conditions consist of two equations in the two unknowns g and $\overline{p}$

$$\frac{F}{b} = \overline{p} \frac{1}{\phi} - \frac{1}{\phi + g\gamma} \qquad (E''a))$$

and

$$1 = gF + b + b\overline{p} \frac{1 + \nu - \theta}{\theta (1 - \gamma)}. \qquad (E''b))$$

(We observe that the 'third' condition, relative budgets, is now redundant, as $n_P = n_R = 1$.)

10.3 Proof of Lemma 3

We first prove part b. and c. of the Lemma. To calculate the value of $\overline{p}$ where Z crosses the horizontal axis, we have to solve the zero profit condition for $\overline{p}$ where $g = 0$.

Proof. In the $N_P < N_R = \overline{N}$ regime we get, using formula (16) and noting that $\Delta \to \infty$ as $g \to 0$.

$$B|_{g=0} = \int_t^{\infty} (1 - \beta) (\overline{p} - 1) e^{-\rho(s-t)} ds = \frac{1 - \beta}{\rho} (\overline{p} - 1) = \frac{F}{b}$$

We solve for $\overline{p}$ and Lemma 3b. follows immediately. ■

Proof. In the $N_P = N_R = \overline{N}$ regime we use $(E''a))$ and get

$$\frac{1}{\rho} (\overline{p} - 1) = \frac{F}{b}$$

Solving again for $\overline{p}$ yields Lemma 3c. ■

Proof. We derive the value of an innovation in the $N_P < N_R = \overline{N}$ regime (16) with respect to g and we get

$$\begin{aligned}\frac{\partial B}{\partial g} &= \int_t^{t+\Delta} (1-\beta)\left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\overline{p}e^{g\gamma(s-t)}\right] e^{-r(s-t)}(s-t)ds \\ &\quad -\frac{\ln n_P}{g^2}\Pi_R(\bar{N}(t+\Delta)) + \frac{\ln n_P}{g^2}\Pi_{tot}(\bar{N}(t+\Delta)) \\ &\quad + \int_{t+\Delta}^{\infty} \left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\left[\beta n_P^{\gamma} + (1-\beta)\overline{p}\right] e^{g\gamma(s-t)}\right] e^{-r(s-t)}(s-t)ds\end{aligned}$$

Note first that $\Pi_R(\bar{N}(t+\Delta) = \Pi_{tot}(\bar{N}(t+\Delta))$. We give first a sufficient condition for the second integral above has to be negative. The integral starts at $t+\Delta$, this implies that $e^{g\gamma(s-t)} \geq e^{g\Delta\gamma} = n_P^{-\gamma}$. We note that

$$\left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\left[\beta n_P^{\gamma} + (1-\beta)\overline{p}\right] e^{g\gamma(s-t)}\right] \leq (\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\left[\beta + (1-\beta)\overline{p}n_P^{-\gamma}\right] \leq 0$$

if $\overline{p}n_P^{-\gamma} > \overline{p} \geq 1 + \frac{\gamma}{\sigma(1-\gamma)(1-\beta)}$. This condition also implies the first integral to be negative. We directly see that $(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\overline{p}e^{g\gamma(s-t)} \leq 0$ if $\overline{p} \geq 1 + \frac{\gamma}{\sigma(1-\gamma)}$, which is a weaker condition.

If $\frac{\partial B}{\partial g} < 0$ the zero profit condition has to be positively sloped. Together with Lemma 3b. this implies that $\bar{p} \geq 1 + \frac{1}{1-\beta}\frac{F\rho}{b}$. Combining we get the final condition

$$\frac{1}{1-\beta}\frac{F\rho}{b} \geq \frac{\gamma}{\sigma(1-\gamma)(1-\beta)}$$

If we solve this expression for γ, Lemma 3a. follows for the $N_P < N_R = \overline{N}$ regime.

For the $N_P = N_R = \overline{N}$ regime the result is much easier to prove. In that regime, the derivative of the value of innovation with respect to g reads

$$\frac{\partial B}{\partial g} = \int_t^{\infty} \left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\overline{p}e^{g\gamma(s-t)}\right] e^{-r(s-t)}(s-t)ds$$

Using the same argument as above, the integral is negative if $\overline{p} \geq 1 + \frac{\gamma}{\sigma(1-\gamma)}$. Again, the zero profit condition has a positive slope under the conditions of the Lemma. From Lemma 3c. we hence note that $\overline{p} \geq 1 + \frac{F\rho}{b}$. Combining we get the same final condition and the same solution for γ as above. ■

10.4 Proof of Proposition 7

With flat hierarchy the $N_P < N_R = \overline{N}$ regime is the only outcome in equilibrium. The static equilibrium condition (20) reads, when $\nu = 0$.

$$\bar{p} = \frac{\beta}{1-\beta}\frac{(1-\theta)\,n_P}{\theta - n_P^{1-\gamma}}$$

This implies directly that $\frac{\partial \bar{p}}{\partial \theta} < 0$ and $\frac{\partial \bar{p}}{\partial \beta} > 0$.

How are the equilibrium curves defined by (18) and (19) affected? A rise in θ does not affect RC, since this parameter does not appear if $\nu = 0$. A rise in β, however, implies that less resources are needed, RC shifts up. To discuss the shifts of Z note that $\Pi_{tot}(j) = [\beta n_P^{\gamma} + (1-\beta)\overline{p}] \left(\frac{j}{N}\right)^{-\gamma} - 1$ and $\Pi_R(j) = \left[\overline{p}\left(\frac{j}{N}\right)^{-\gamma} - 1\right](1-\beta) = \overline{p}(1-\beta)\left(\frac{j}{N}\right)^{-\gamma} + \beta$. Using the formula for $\bar{p}$ above we get the expression $\overline{p}(1-\beta) = \beta\frac{(1-\theta)n_P}{\theta - n_P^{1-\gamma}}$. Hence, $\overline{p}(1-\beta)$ falls in θ and increases in β. With n_P fixed, we directly get the result that $\frac{\partial \Pi_{tot}(j)}{\partial \theta} < 0$, $\frac{\partial \Pi_R(j)}{\partial \theta} < 0$ and $\frac{\partial \Pi_{tot}(j)}{\partial \beta} > 0$, $\frac{\partial \Pi_{tot}(j)}{\partial \beta} > 0$. Obviously, the Z-curve shifts to the right when θ increases and it shifts to left when β increases.

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Figure 1: Aggregate Demand for Good j and Decision Problem of the Monopolist

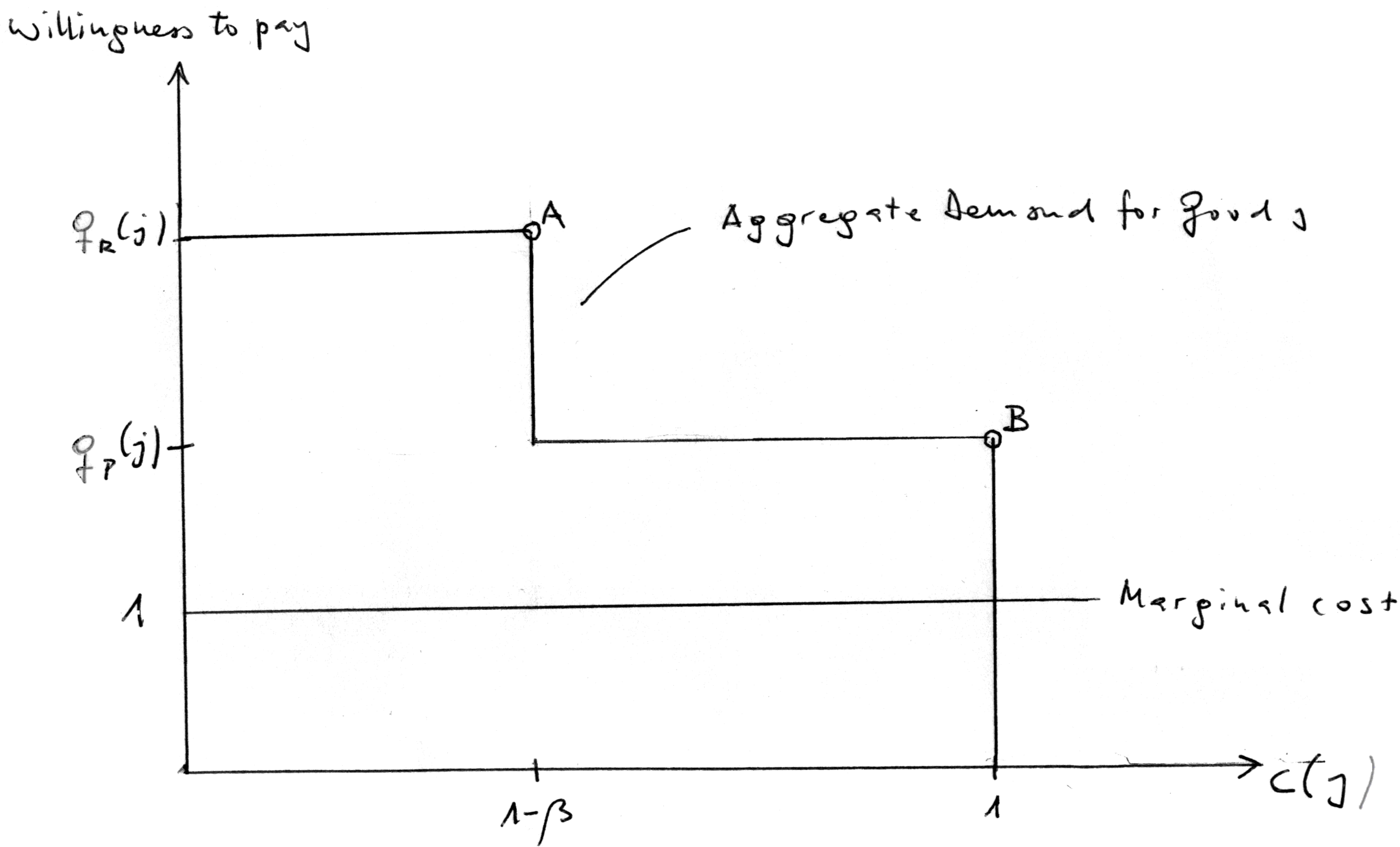


Figure 2 A Unique Positive Growth Equilibrium with Flat Hierarchy

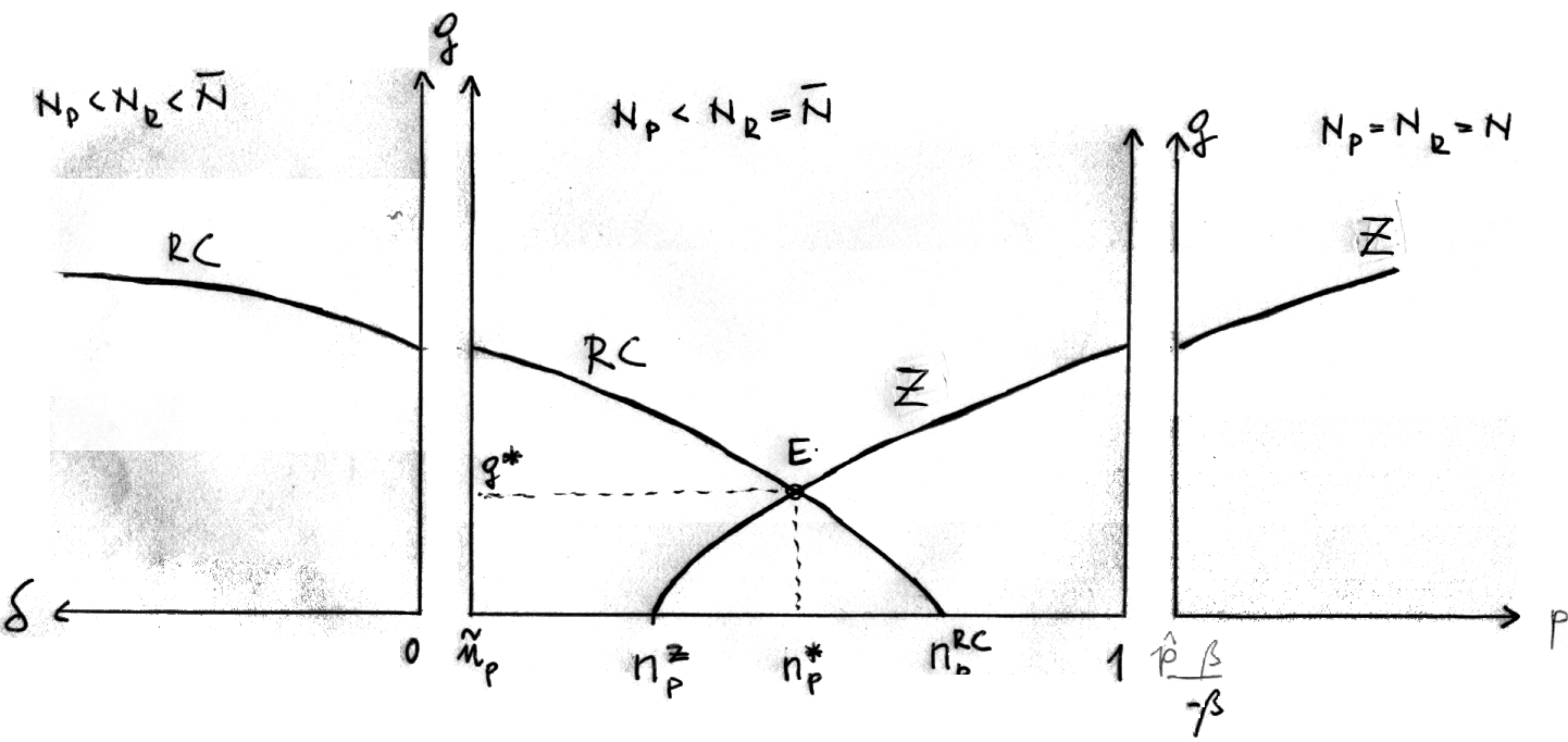


Figure 3: An Equilibrium With Positive Waiting Time δ (Steep Hierarchy)

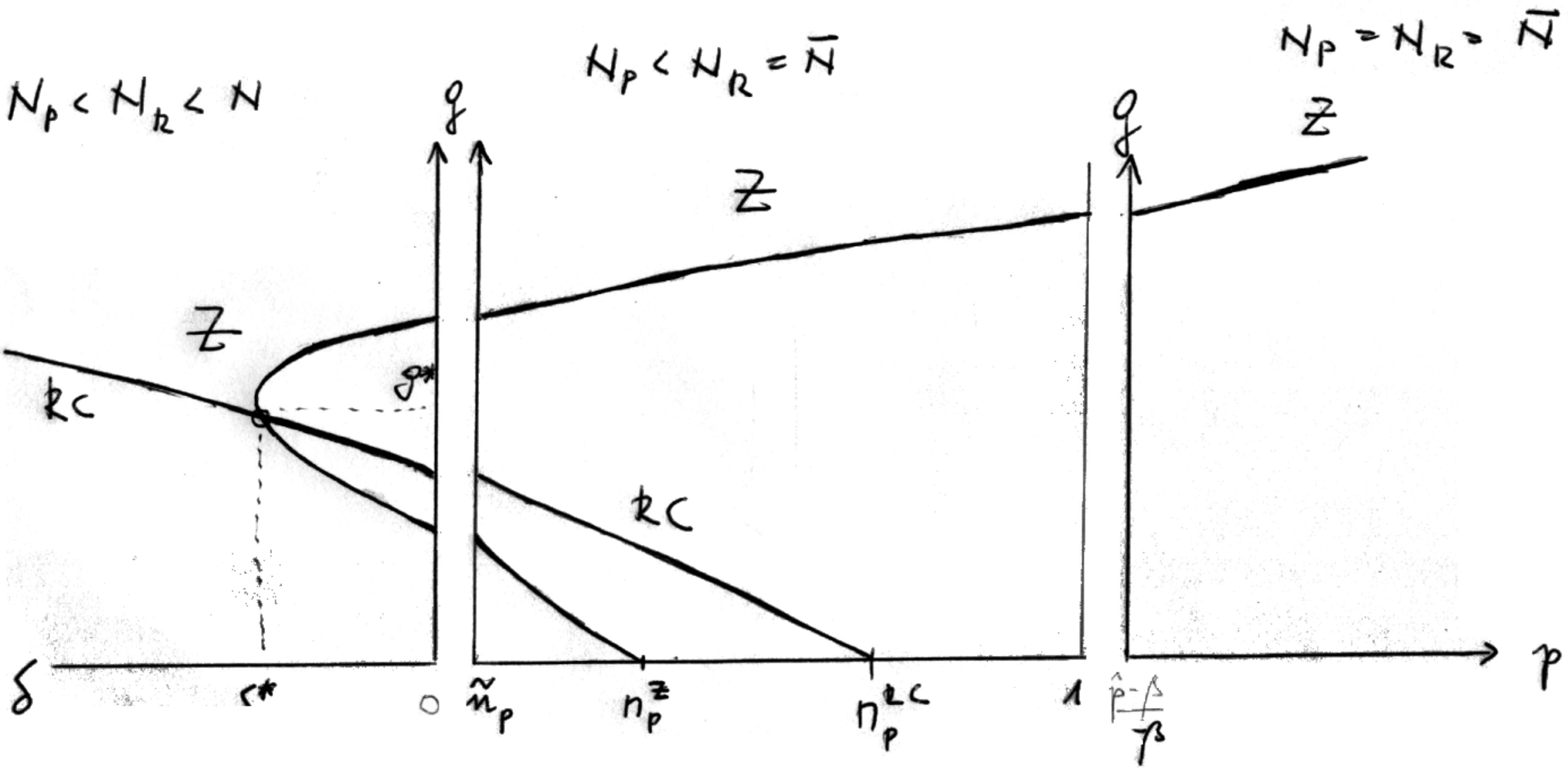
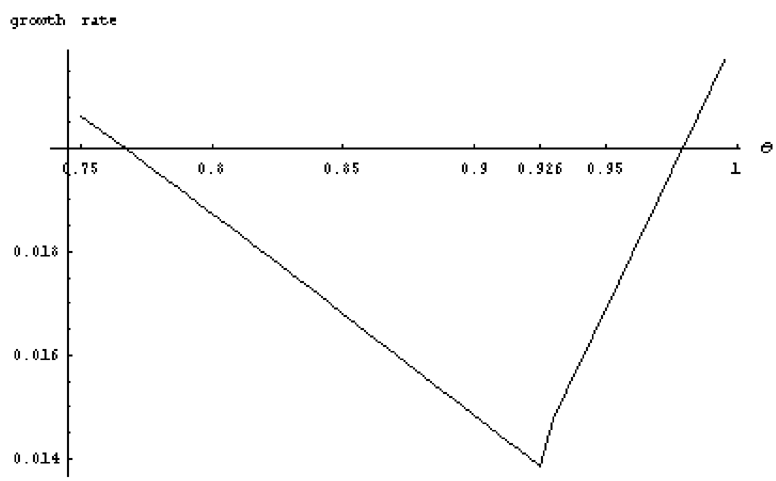


Figure 4: Simulations

Default values:
$\theta = 0.8$, $\beta = 0.5$, $F = 5$, $b = 0.3$, $\sigma = 2$, $\rho = 0.02$, $\gamma = 0.3$, $\nu = 0.8$

The growth rate in dependence of θ

The regime switch, where $n_P = 1$, arises at $\theta = 0.926$



The growth rate in dependence of β