The structure of growth models:

A comparative survey

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Abstract: A huge literature has stemmed from Harrod and Domar's pioneering model of economic growth. Thus, the reader may be disoriented by the technical details and the specific problems considered by each work. The aim of this paper is to provide a survey of the most important models explaining economic growth, from the Classical economists to the more recent endogenous growth theory. The survey mainly intends to convey the intuition of the logic behind each model by emphasising the decision structure and the logical relationships within each model, rather than trying to provide a rigorous and complete description of the models and problems considered.

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1. Introduction

The basic problem of growth theory is to describe the behaviour of an expanding economy over time. The more traditional way to conceive growth is to consider it as due to the accumulation of capital. In its attempt to construct a theory of growth (see e.g. Solow (1956)), Neoclassical economics has tried to extend its static theory of distribution to a dynamic context, and in order to be successful in this attempt it had to assume decreasing returns with respect to the accumulated factor (see e.g. Bertola (1993), (1994)). This assumption has the consequence that the accumulation process has only transitory effects on the rate of growth, whose long run behaviour remains therefore unexplained within the model and it is characterised by the constancy of the capital/labour and product/labour ratios. As a consequence, empirically relevant examples of permanent growth, like sustained increase in the per capita stock of capital, are attributed to the 'compensating influence of residual factors that have been assumed away in the model' (Kaldor (1961, p. 177)).

The properties of the Neoclassical growth theory have been always questioned not only on empirical ground but also on the theoretical one. One of the main criticisms has been that the rate of growth of economies should depend upon the thriftness of the economy and that technical change should be the outcome of intentional decisions of economic agents. The recent literature on endogenous growth theory has been successful in dealing with these criticisms and it has been able to construct a variety of models in which the rate of growth depends upon the saving decision of

households and/or technological change is the intentional or unintentional outcome of the maximising behaviour of agents.

Kurz and Salvadori (1998, Ch. 5; 1999) and other authors have pointed out that the endogenous growth theory represents a substantial break with respect to the Neoclassical theory by recovering various aspects of the Classical view of the economic process. In fact, while the Neoclassical theory of factor-income distribution is hardly consistent with endogenous growth (Frankel (1962)), Classical economists, basing their distribution theory on non economic elements and assuming full reproducibility of means of production and constant returns, constructed a theory of growth which was able to ensure the possibility of persistent (endogenous) growth.

While the work of Kurz and Salvadori is helpful in highlighting the contiguity of the endogenous growth theory with the Classical tradition, its scope is too restricted to allow a full appreciation of the features of each theory in terms of common and idiosyncratic elements. On the other hand, we believe that having a complete idea of the basic structure of each theory is important for possible cross-fertilisation between different streams of analysis.

The aim of this paper is to provide a first attempt in this direction by examining in a very simple way the structure of the most important growth models. More specifically, we shall analyse the Classical theory of growth, the Neoclassical, the Keynesian, the Lucas and the 1990 Romer models

of endogenous growth. While we accept the Kurz and Salvadori's idea that the endogenous growth theory can be considered as retrieving the Classical view as far as distribution and the sources of growth are concerned, we shall try to point out the existence of common aspects and specific features of the models considered. To be more specific we shall emphasise that there is a contiguity between Classical growth theory, Neoclassical growth theory and endogenous growth theory as far as the relationship between savings and investment is concerned, in that all these models conceive the investment decision as always co-ordinated with the saving decision, while, as it is well known, the Keynesian models propose a radically different view in which investment determine savings and in this way the theory of growth turns out to be intrinsically connected with the theory of business cycle. We shall emphasise, by contrast, that there are elements of contiguity between the Classical tradition, the Keynesian one and the endogenous growth theory as for the view concerning the adaptation of the rate of growth of population and the rate of growth of investment.

The paper is organised as follows. In the next section the early contributions to growth theory by Classical economists and von Neumann are illustrated. In Section 3 the Keynesian tradition will be considered. Section 4 deals with various streams of Neoclassical growth theory, from the Solow model to the discounted Ramsey models of Cass (1965) and Koopmans (1965). In Section 5 we shall review the Lucas and the 1990 Romer models of endogenous growth. In Section 6 we shall summarize the main aspects of each model. We want to emphasize that our analysis cannot be

considered exhaustive at all, and several models are not considered. Among others, we exclude from analysis all overlapping generations models and models with altruism.

2. Accumulation oriented models

2.1. Classical models of economic growth

Classical economists centred their attention on the economic growth of nations and explained this phenomenon through a theory based upon the class structure of the capitalist economy. They individuate three classes, workers, capitalists and rentiers, which have their own specific role in the economic process. Workers own labour and sell it on the labour market for the subsistence wage. Rentiers own land and rent it to capitalists in order to get a rent. Capitalists own produced means of production and organise production by employing labour and renting land. Profits are their income. The behaviour of all these agents is governed by the attempt to get as much as possible from the resources they own. As far as the employment of the income is concerned, the usual interpretation of Classical economists is that they conceive each class as characterised by a specific behaviour (see, for example, Kaldor (1961, p. 180))[1]: while workers and rentiers substantially consume all the income they get, capitalists save and invest essentially the entire amount of profits. However, while workers buy essentially subsistence goods, due to the low level of wages, rentiers buy essentially luxury goods. An important aspect of the Classical view on the

consumption decision is that allocation of income is not determined by preferences or by the type of income earned but mainly by the social group to which who receives the income belongs. The sociological analysis of the rise of capitalism carried out by Smith in Book III of *The Wealth of Nations*, makes this position very clear, by pointing out that capitalism arises by the emergence of a class, the 'merchants and artificers', which 'acted merely from a view to their own interest', by contrast '[T]o gratify the most childish vanity was the sole motive of the great proprietors' (Smith (1976, III, Ch. IV, p. 440)). [2] Capitalists are the class which use their riches in order to improve their condition by accumulating them. The mechanism by means of which savings are transformed into investment can be direct, if the savers are also entrepreneurs, or indirect, through the capital market, if savers are not entrepreneurs. In any case Classical economists accept the view that all savings are transformed into investment (see e.g. Hagemann (1998)). However, while Classical economists accept the idea that savings can be equalised to investments through the capital market, it can be hardly said that they share the Neoclassical view according to which this equalisation is due to the adjustment of the interest rate. In fact, for Classical economists the interest rate is determined by the rate of profits (Smith (1976, I, p. 99), Ricardo (1951, vol. I, pp. 363-364)), while the equalisation between savings and investment is ensured by the fact that 'the demand for capital is infinite' at the current rate of interest (Ricardo (1951, vol. VI, p. 301)). Moreover, the

[1] Smith, for example, while allowing that wages and rents could be employed in savings, maintains that capitalists are 'naturally the most disposed to accumulate' (Smith (1976, ii, p. 128).

[2] For a more complete elaboration on this point see Rosenberg (1975).

rate of profit is determined by the wage rate, which in turn depends upon the conditions of the labour market. Classical economists held that the rate of growth of population depends upon the wage rate: a wage rate higher than the natural one, i.e. that one which maintains constant population (Ricardo (1951, vol. I, p. 91)), yields a population increase (Smith (1976, I, p. 90), Ricardo (*Ibidem*.)). More specifically the higher the wage rate, the higher the rate of growth of population: 'It is not the actual greatness of national wealth, …., which occasions a rise in the wages of labour. It is not accordingly, in the richest countries, but in the most thriving, or in those which are growing rich the fastest, that the wages of labour are highest' (Smith (1976, I, p. 78), see also Hollander (1973, p. 158), Eltis (1984, p. 88)). Hence Smith, anticipating Malthus, conceives the demographic law as a technological rule for producing labour as any good: '… the demand for men, like that for any other commodity, necessarily regulates the production of men; quickens it when it goes on too slowly, and stops it when it advances too fast' (Smith (1976, I, p. 89)).

Within the Classical framework it is possible to consider two kinds of models. The first is the Ricardian model, which emphasises the tendency towards the stationary state due to the existence of scarce natural resources, the second model is rooted in the Smithian and Marxian tradition which emphasises the progressive nature of economic growth. Both models are commonly interpreted as being characterised by the assumptions that capital and labour are employed in fixed proportions, capitalists anticipate wages, that are entirely consumed by workers, landowners consume all rents and capitalists/entrepreneurs invest all profits. While the existing amount(s) of land(s) is (are) constant, labour supply is constant only in the short run, whereas in the long run it

is infinitely elastic at the natural wage rate w*. This means that the supply of labour can be eventually increased (or decreased) indefinitely at the wage rate w*, although the Malthusian law mentioned above regulates the rate of increase of population during any transitional phase.

In Figure 1 the basic Classical model is described. Following Smith we assume that the economy is growing and that the wage rate is at a level such to ensure a growth of population equal to the growth of demand for labour.[3] In the top the time axes is drawn, and we consider a generic period t beginning at time t and ending at time t+1. Continuous arrows indicate decision nodes, while dotted arrows indicate functional relationships where the variable(s) at the end of the arrow is (are) determined by the variable(s) at the beginning of it. At the beginning of period t, the economy is endowed with a set of available methods of production, M_t, with a given amount of physical capital, X_t, with a given amount of labour, N_t, and with a given amount of natural resources (land), T (T is a vector if land is of different quality). Set M_t must be interpreted as the set of methods describing the output per worker that can be obtained on the different pieces of land or on the same piece of land and that are available in the economy. The wage w_t is given at the level defined above. The amount of land is assumed to be constant, while the other magnitudes can change over time. For a given amount of capital X_t, the wage w_t determines the set of cost-minimising methods of production, M_t*. Capitalists employ the whole capital X_t, and, together with the wage rate w_t, determine the allocation of total capital between physical capital (K_t) and wage goods anticipated to workers (W_t),

[3] This assumption does not implies that there is full employment of labour. An “industrial reserve army” is compatible with the case considered.

and determine also the decisions concerning the demand for labor (L_t) and for land (R_t). The magnitudes K_t, L_t and R_t determine the net production Y_t - X_t which is devoted to pay rents and profits. Rents are entirely spent in consumption while profits are entirely saved and reinvested. At the beginning of time t+1, the investment I_t determines the stock of capital X_{t+1}, the available technology, M_{t+1}, may be different from the technology available the preceding period[4], the supply of labour is changed according to an exogenously given Malthusian rule, while the amount of natural resources remains unchanged, by assumption.

The Ricardian model can be considered as a particular version of the basic Classical model, and it is characterised by the further assumptions that the set of available methods is constant over time and that land scarcity that goes with capital accumulation pushes output per worker down below what it is needed for reproduction. The Ricardian model is illustrated in Figure 2, that is a diagram adapted from one by Kaldor (1955-1956) (see also Kurz and Salvadori (1998)).

The assumption that the marginal productivity of labour-cum-capital is decreasing as the accumulation of capital goes on is described by curve MP in Figure 2. By the further assumption that the marginal productivity decreases below the subsistence wage rate w*, one obtains that there

[4] It seems reasonable to say that Marx considered explicitly investment to improve technology. In the basic Classical model here considered we shall neglect this fact and we consider technical improvement due to non-deliberate economic decisions, like in Smith's view (see McNulty (1968)). Investment in R&D will be taken up in details later, in dealing with the endogenous growth theory.

must exist a level of capital stock that makes the rate of profit equal to zero, or sufficiently low, with the consequence that the accumulation process will stop (Ricardo (1951, vol. I, Chapter 5)).

By contrast, the Smithian-Marxian model is characterised by constant productivity of labour and capital. This is justified by the fact that Smith and Marx do not put any emphasis on the role of natural resources as a factor limiting growth. Figure 2 illustrates also this case with the marginal productivity of labour represented by the curve MP'. In this case, as it is immediate to see, there is no limit to growth. The reason is immediate to understand: the constancy of returns ensures that the rate of returns on investment is constant over time, hence the accumulation can be carried out indefinitely. In Figure 2, even if the economy is growing, the wage rate is still fixed at the natural level w*. This however is inconsistent with the Malthusian rule, which for a growing system prescribes a wage rate above the subsistence level. The fact that a positive rate of growth is inconsistent with a wage rate fixed at the 'natural' level w* can be handled by noting that under the Classical saving assumption the working in time of the labour market 'provides a mechanism for keeping the rate of accumulation of capital in step with the rate of increase in the labour supply' (Kaldor, (1961), p. 188).

The logic of the short run adjustment towards the steady state path with full employment in the Smithian-Marxian theory can be intuitively grasped by means of Figure 3.[5] Let us assume that there is only one good produced by itself as circulating capital and by labour under constant returns, and that a single method of production is available. Without loss of generality we can now suppose that the given method uses v units of capital and 1 unit of labour per unit of output. Let Y_t be the production level of period t. Then $Y_t = \min (K_t/v, L_t)$, where K_t is the amount of circulating capital employed at time t and L_t is labour employed at time t ($W_t = w_t L_t$, where w_t is the wage rate at time t, is therefore the wages capital anticipated to workers at time t). It is assumed that the fraction s of profits is saved. Time is discrete. Indicating with X_t the total capital employed at time t, if the wage rate is positive, then from the equilibrium conditions $X_t = K_t + w_t L_t$, $Y_t = K_t/v = L_t$ one obtains $Y_t(w_t) = L_t(w_t) = X_t/(v+w_t)$, $K_t(w_t) = vX_t/(v+w_t)$, $W_t(w_t) = w_t X_t/(v+w_t)$. The first relation associates to each stock of total capital the demand of labour (or, equivalently, the level of production), the second and the third associate to each level of total capital the circulating capital and the wage goods anticipated to workers, respectively. The first function is illustrated in Figure 3, where it is indicated by $Y(w_t)$.

Notice that $Y(w_t)$ rotates clockwise as the wage rate increases. Suppose that the population rate of growth is $G_n(w_t)$. Since the capital/labour ratio is a constant, if the stock of capital at time t is X_t,

[5] Figure 3 apparently does not reflect some important features of the Marxian view of the accumulation process; in particular, the full employment condition. However, this figure is able to make the classical approach comparable to the more recent models as far as the adjustments on the capital market is concerned.

then in order to maintain full employment at the given wage rate in the next period, it is necessary to supply $(1+ G_n(w_t))X_t$ at the end of period t as capital. This requirement is illustrated in Figure 3 by the curve $X'(w_t)$. This curve rotates anticlockwise as w_t increases. The vertical distance between the curve $Y(w_t)$ and the 45° curve indicates the surplus for each level of initial total capital. Since there is no rent, this surplus makes up net profits. Hence, curve $s(Y(w_t) - X_t) + X_t$, giving the amount of savings out of profits plus the total capital, indicates the capital stock that capitalists will employ at time t + 1. In the case illustrated in figure, for example, for the level of total capital OX, the segment AB indicates the amount of net saving (i.e. net of total capital) necessary to ensure continuous full employment over time at the given wage rate (the segment XB indicates the amount of total resources which must be saved in order to ensure an increase of demand of labour equal to the increase in supply). This amount can be interpreted as the *net demand for capital* for steady state full-employment growth. The segment AC indicates the amount of net savings (i.e. net of total capital) available for employment in productive activities. This magnitude can be interpreted as the *net supply of capital*.

For the sake of simplicity we assume that at time t there is full employment of labour, i.e. $L_t = N_t$. In Figure 3, if total capital at time t is OX, then at time t+1 the demand of labour for full employment equilibrium is given by the segment X”F while the supply of labour is X’E. Hence the supply of capital is greater than the demand of capital for steady state full-employment, wages will increase and also the rate of population growth will increase. The former change yields a clockwise rotation of curve $Y(w_t)$ and, therefore, of curve $s(Y(w_t) - X_t) + X_t$, and an anticlockwise

rotation of curve $X'(w_t)$. The equilibrium between net demand and supply of capital for steady state equilibrium will be reached when wages reach a value w^e such that demand and supply of capital for steady state equalises: $(1+ G_n(w^e)) = s/(v + w^e)$.

From this intuitive description, it can be seen that for Classical economists adjustment on the capital market between savings and investments is not the same as in the Neoclassical economics, since it occurs mainly through adjustments in the labour market. Moreover, unlike Neoclassical growth theory as we shall see, the rate of growth of the economy is determined by the interplay between savings and population growth rate, the former being completely employed in investment and the latter being endogenously given as an increasing function of the real wage rate.

2.2. The von Neumann model

Von Neumann studied a multisector version of the Classical Smithian-Marxian model. From the formal point of view, this model is a multisector linear model with only labour as the non-produced means of production and possibly with joint production. Von Neumann looks for an activity level vector yielding the maximum rate of growth of the system and the associate competitive price system. He deals with the labour in exactly the same way as Classical economists did that is labour does not appear explicitly in his model because it is "produced" by a linear technology by means of wage goods (Champernowne (1945-46), see also Kurz and Salvadori (1993)). It follows that production is carried out by means of physical capital and wage good capital and the aim of the productive activity is exclusively the accumulation of capital. With reference to Figure 1, the model

is characterised by a zero value of R_t and a set of production techniques M_t which is stationary and is represented by a finite set of linear processes. Finally, X_t and Y_t are vectors.

The equilibrium concept considered by von Neumann is the balanced growth equilibrium in which the economy is growing at the maximum technical rate. He shows that under very general conditions there exists a semi-positive equilibrium vector of activity levels, a semi-positive equilibrium price vector and a non negative rate of interest which is equal to the (maximum) rate of growth.

The multisector von Neumann model can be considered the first complete economic model in which the rate of growth is endogenously determined. Some of the elements that characterizes this model, as the assumption that all factors of production are producible, have been recently used in the modern endogenous growth literature to obtain persistent growth in absence of technological change (e.g. Lucas (1988), Jones and Manuelli (1990), Rebelo (1991)). This approach to generating endogenous growth will be discussed in Section 5.

3 The Keynesian tradition

3.1. Harrod-Domar model

Harrod-Domar model (see Harrod (1939), (1948), Domar (1946)) is the first economic model analysing in a formal way the problem of growth. For this reason, particular attention is paid to make explicit the relationship between the consumption-saving decision as an optimal decision by

households and the investment decision as an optimal decision by entrepreneurs, although these behaviours are not theoretically developed. In fact, the consumption-saving decision is defined, following the Keynesian approach, by an exogenously given propensity to consume, while the investment decision is defined by the accelerator principle. In this model, production is obtained only by means of physical capital and labour. Given the usual Keynesian assumption of fixed prices, firms choose the best technique at the given prices. Then generically there is only one cost-minimizing technique, which implies that the capital/labour ratio and the capital/production ratio are uniquely determined (By using both the normalization and the notation of the previous section, we have $K_t/L_t = K_t/Y_t = v$). Since Harrod and Domar, following Keynes, believe that the market mechanism is not able to attain the full employment of labour, they focus their attention only on the equilibrium on the goods market - which holds when the savings are equal to the desired investment - rather than the general equilibrium on the goods and labour markets.

An economy growing along a path with equilibrium on the goods market is said on its warranted growth path. Along this path one obtains that $G_W = s/v$ where G_W is called the warranted rate of growth of income, s is the rate of saving and v is the capital/production ratio. The behavioural hypothesis on producers and the Keynesian multiplier yield that the warranted growth path is unstable. If the warranted growth path ensures also the full employment of labour - a possibility which is just accidental in this model - the economy is said to be on the golden age growth path.

Figure 4 illustrates the structure of the Harrod-Domar model. At time t the economy is endowed with a technology M_t, which is assumed to be subject to exogenous technical progress, a stock of

capital X_t, and a given amount of labour N_t. Entrepreneurs decide the level of investment at time t on the basis of the acceleration principle, according to which the optimal level of investment depend upon the expected desired level of capital employed at time t+1, X^e_{t+1}, and the current level of capital stock. In equilibrium the former must be equal to the actual one. By the multiplier principle, the investment at time t determines the equilibrium level of production at this time. Given the production function and the hypothesis on prices, the cost minimising technique, M^*_t, is chosen and the desired amounts of capital and labour at time t are obtained, $K_t/v = L_t.= Y_t$. Assuming to be on a warranted path, the current amount of capital X_t must be equal to the desired capital stock K_t. In this model there is nothing ensuring the equilibrium on the labour market, i.e. it is not necessary that $N_t = L_t$.

Aggregate income Y_t determines the aggregate amount of consumption C_t and hence the aggregate amount of saving S_t through the Keynesian assumption of constant propensity to consume; i.e. $C_t = (1-s)Y_t$ and $S_t = sY_t$. By the equilibrium condition on the goods market one obtains that $sY_t = I_t$, i.e. the saving decision of households is coherent with the investment decision of producers.

Figure 5 describes the process of accumulation in the Harrod-Domar model. The level of production that ensures full employment of the capital stock is described by curve Y_t, while the saving function is function sY_t. By assuming that the decay rate of capital is δ, $0 \leq \delta \leq 1$, the curve δK_t indicates the amount of resources which must be devoted to reintegrate the decayed capital, while curve $(n+ \delta)K_t$ indicates the amount of resources necessary to ensure the reintegration of capital and an increase of the demand for labour equal to the rate of growth of population. According to Harrod and Domar's

view, investment determines the level of income, Y_t, which determines in turn the net savings, i.e. the net supply of capital for full employment of capital steady state, which is indicated by the segment AC. The current stock of capital, which is optimal by the accelerator principle, OK, and the rate of growth of population determine the net demand of capital for full employment of capital and labour steady state, indicated by segment AB. In the case illustrated in Figure 5, the supply of capital yields a demand for labour at time t+1, DE, greater than the supply of labour available at that time, FG. Harrod and Domar, being concerned mainly with steady state conditions, do not consider in details what happens in this case, although it is suggested that an increase in wages and a subsequent inflationary process could be generated.

3.2. The Kaldor and Pasinetti models

Kaldor (1954) (1961) holds a different view concerning the causes of growth with respect to the view held by most of contemporary growth theorists. In fact, he holds that it is not saving, investment, technical progress and population growth that are the causes of growth - these being just features of growth - but the attitude of investing by society and in particular of entrepreneurs. In this he follows the Keynesian approach in conceiving the expansion of the economy as driven by psychological and social factors like "human attitude to risk-taking and money-making" (Kaldor (1954, p. 228)).

In looking for a growth theory which explains the real dynamics of economies, he criticises Harrod's model on the ground that it explains only the growth of a cycle-less economy with full-

employment of savings rather than the *actual* rate of growth of a system that does not mantain a moving equilibrium. Indeed he sustained that in a system in which growth results from successive booms and slumps the actual trend is determined by the 'natural rate' of growth (Kaldor (1954)). Because of the sociological factors underlying the phenomenon of growth, he maintains, following here the fundamental Schumpeterian intuition, that a satisfactory growth theory cannot be constructed without a business cycle theory (Kaldor (1954) (1957)). However, he never develops in a formal way his position on economic growth, and his major contribution consists in solving in an original way the stability problem of the Harrod-Domar model. This is accomplished by allowing the possibility that the economy can growth along a natural growth path through adjustments in the rate of saving due to changes in the distributive shares between wages and profits and assuming that the population rate of growth is constant. The latter assumption is adopted only in his formal works, while in several non-technical articles (see, e.g. Kaldor (1960) he and other Post-Keynesian economists like Joan Robinson (see e.g. Robinson (1963) share the classical view that the rate of growth of population depends upon the wage rate and, therefore, upon the rate of growth of the economy.

Kaldor's contribution can be seen in Figure 6, where the usual production function (Y_t), supply of capital ($s(w_t)Y_t$), demand of capital for steady state with full employment $(n+\delta)K_t$ and demand of capital for reintegration (δK_t) and demand for labour curve (L_t) are depicted (we still assume that the there is only one method of production). Notice that now the average saving rate depends inversely upon the wage rate due to the relation $s(w_t) = s_w Y_t + (s_p - s_w)\Pi_t(w_t)$, where s_p is the saving

rate out of profits, s_w the saving rate out of wages and $\Pi_t(w_t)$ the profits (see, e.g. Kaldor (1956)), which are inversely related to the wages. If the stock of capital is OK, then the supply of capital, KC, is higher with respect to the demand of capital which ensures full employment and constant capital/labour ratio, KB. At time t+1 the demand of labour, FG, is higher than the supply of labour, DE, therefore wages increase. This yields a clockwise rotation of the curve $s(w_t)Y_t$, and L_t tends to N_t. The equilibrium is attained when w_t is such that $s(w_t)Y_t = (n+\delta)K_t$; in this case, the supply and the demand of capital are equalised at the level which ensures full employment. Although the equilibrating mechanism is described in term of forces operating in the labour market, Kaldor suggested (e. g. Kaldor (1961, pp. 196-7)) that the process could equally takes place in the goods market through changes in prices.

Kaldor's approach has been developed and pursued further by Pasinetti (1962) and by a quite huge literature which, however, in emphasising the study of equilibrium paths have shifted attention from the original attempt to construct a growth theory out of a business cycle theory to the more traditional view of constructing a theory of the economic growth at the natural rate.

4. Neoclassical growth models

4.1. The Solow model

A different attempt to solve the stability problem of full-employment steady state is that of Solow (1956). He accomplishes this task by assuming a Neoclassical production function that allows for

flexible coefficients of production. By adopting a Neoclassical framework, Solow changes the object of analysis with respect to the Keynesian growth theory; in fact, according to him the major problem is to construct a theory of general full employment growth, and the most important concern is to ensure the convergence of the economy towards the natural growth path. Hence, growth theory has to explain the *potential* growth of economies (Solow (1999, 2000)), without paying attention, therefore, to cyclical trends of the economy and their possible effects on the long run trend of the economy.

Solow assumes that there is only capital and labour as factors of production. The technology is represented by means of a Neoclassical production function with constant returns to scale, decreasing productivity with respect to physical capital and possibly labour-augmenting technical progress. In order to construct a model that conciliates full employment of resources with growth, Solow assumes that prices are flexible and therefore all markets are cleared. In particular, the equilibrium on the capital market yields that investments are equal to savings while the equilibrium on the labour market yields that there is always full employment of labour.

Production is distributed between savings and consumption on the basis of a Keynesian saving rule. If savings are equal to the level of investment which ensures the constancy of the per capita capital with full employment, then the economy is in steady state. Otherwise price adjustment on the capital market yields the equalisation between savings and investments and appropriate changes of the per capita capital until the steady state is attained. The convergence process towards the steady state is ensured by the assumption of decreasing productivity of capital.

Figure 7 illustrates the structure of Solow model. In this figure at time t the economy is endowed with a technology set, a given amount of capital and labour. Producers choose the best technique according to the ruling prices, and full employment of resources is ensured by the fully flexibility of them. The resulting income is employed as consumption or savings according to the usual, exogenously given, propensity to save, s. Savings are completely transformed into investments by the flexibility of the interest rate on capital market. Investment and the initial stock of capital determine the amount of capital available at time t+1. The technology set and the amount of labour available at this time are determined exogenously by the values of these variables at time t, the former by assuming a positive rate of technical progress, the latter by a positive rate of population growth.

Figure 8 illustrates the accumulation process in Solow model. In this figure, all variables are expressed in per capita terms. Curve $(n+\delta)k$ indicates the (pro capite) demand of capital for a full employment steady state with constant pro capite capital, while curve $sf(k)$ indicates the (pro capite) supply of capital. If the capital pro capite is Ok', the supply of pro capite capital, k'A, exceeds the pro capite demand of capital for a full employment steady state with constant pro capite capital, k'B. Hence, in order to maintain full employment on the labour market the rate of interest will decrease and entrepreneurs find it profitable to increase the demand for capital. The adjustment of price and the consequent increase of demand is such to equalise demand with supply. At the new, higher level of capital pro capite production will generate a new supply of capital and a new

demand of capital for a full employment steady state with constant pro capite capital will arise. If the former is equal to the latter, a steady state is attained, if the former is still greater than the latter a further decrease of the interest rate and of the pro capite capital occur. And so on.

From what has been said, Solow's view of growth theory is very different from Kaldor's one. In fact, as it appear from the preceding section, while Kaldor intends to construct a growth theory which explain the *actual* rate of growth, Solow wants to construct a theory which explains the "evolution of potential output" (Solow (1999, p. 639)). In order to carry on their research program, therefore, Neoclassical economists have paid no attention to the problem of cyclic behaviour of economy by assuming full employment of resources along the growth path.

4.2. The growth model à la Ramsey

Inspired by the article by Ramsey (1928), several growth models have been constructed in order to improve the Solow's model by endogenising the rate of saving of households. The early models following this approach are characterised by a *normative* interpretation of the accumulation process in that the economic decisions concerning production and saving are taken by a planner choosing over an infinite horizon. An important peculiarity of these early contributions is that the production side of the economy and the income employment side are not separated since a unique agent, the planner, has control both on production and saving decisions. Later on, the literature has emphasised that the optimal path chosen by the planner coincides with the path chosen by a

perfectly competitive economy with many agents. This step is important in order to ensure that Ramsey's approach is able to provide a *positive* theory of growth like the traditional models, as Harrod-Domar' and Solow's ones. The decentralisation problem, moreover, has opened the way to the construction of growth models in which the equilibrium path is analysed independently of the optimal, centralised path. This frees the theory from the hypothesis of perfectly competitive markets or from the assumption of absence of external effects, and it is one of the most important contributions of the recent endogenous growth theory.

In the traditional growth models *à la* Ramsey (Cass (1965), Koopmans (1965)), the assumptions concerning the production function are the usual Neoclassical ones and the planner is endowed with a separable and stationary utility function u() and a constant discount rate ρ. The optimal path, which dynamically has the structure of a saddle point, is unique and converges towards the steady state path.

In Figure 9 the basic structure of the normative version of Ramsey model is illustrated. At time t the economy is endowed with a technology set M_t and given amounts of capital X_t and labour N_t. The planner chooses the best method of production M^*_t and the entire amount of labour and capital to produce the output, Y_t. The planner then decides how allocate the amount Y_t between consumption and savings which are immediately transformed into investment, I_t. The stream of consumption levels $(C_t)_{t=0}^{\infty}$ is chosen in such a way to maximise the sum $\Sigma_0^{\infty}\rho^t u(C_t)$ subject to the constraint $X_{t+1} = Y_t - (1-\delta)X_t - C_t$, where δ is the capital decay rate. Investment at time t and the initial stock of

capital X_t determine the stock of capital available at time t+1, X_{t+1}. The amount of labour available at time t+1, N_{t+1}, is determined by a demographic rule that is given exogenously.

The accumulation process in Ramsey' model is similar to the Solow's one described by Figure 8. Now, however, along the optimal path the rate of saving changes over time and converges towards the long period level associated with the steady state.

5. Endogenous growth models

The aim of the endogenous growth theory is twofold: first, to overcome the shortcoming of Solow and Ramsey models which are not able to explain sustained growth, second, to provide a rigorous model in which all variables which are crucial for growth, in particular savings, investment, and technical knowledge, are the outcome of rational decisions. For this reason, the endogenous growth theory has adopted as reference theoretical structure Ramsey's model in which saving is the outcome of a maximising agent and the equilibrium growth path is seen as the consumption/saving trajectory chosen by rational agents by solving an intertemporal optimization problem.

As for the former, the endogenous growth literature points out that a necessary condition for perpetual growth is that from the household's point of view the rate of interest should never be driven too low, and this is ensured if the productivity of accumulated factors does not decrease to zero as accumulation goes on (see for example, Jones and Manuelli (1997)). On the contrary, if this case occurs, savings will be driven to a level that is not enough for fuelling sustained growth. In this perspective, the main object of the endogenous growth theory has been to develop economically

meaningful ways of ensuring non-decreasing returns to scale with respect to the accumulated factors. This has been accomplished either by removing the scarcity of natural resources or by introducing technical progress. As far as the former is concerned, for example, labour has been straightforwardly transformed into a fully reproducible resource, human capital. As for technical progress, one the main feature of the endogenous growth theory is the capacity of endogenise the investment decision yielding technological progress which consists mainly in the introduction of new intermediate and/or final goods (Romer (1990) Grossman and Helpman (1991), Aghion and Howitt (1992) (1998)).

5.1. Accumulation of physical and human capital

The simplest endogenous growth model is the so-called AK model (see e.g. Rebelo (1991)). It can be considered as a Ramsey model with the assumption that the production function is linear with respect to the physical capital and that scarce resources are not considered explicitly. Within the AK model it is possible to obtain a constant positive rate of growth of the per capita consumption along the optimal path. This path, moreover, can be rationalised as the outcome of a decentralised perfectly competitive economy.

A second approach to obtain sustained growth is to introduce human capital (see e.g. Lucas (1988)). By following this approach, the persistent growth is obtained by transforming labour from a scarce resource to a fully reproducible factor by interpreting it as human capital. However, now an additional decision has been taken by households concerning how much resources to employ in the accumulation of physical and human capital.

Figure 10 illustrates the structure of Lucas model. At time t the economy is endowed with a technical set for producing the output M_t, a technical set for the production of human capital MH, given stocks of physical capital X_t and human capital H_t. Set MH is made up by only one linear method of production and does not change over time. On the basis of the ruling prices, households choose the amount of human capital to employ in productive activities, uH_t , and in the further accumulation of human capital $(1-u)H_t$. Firms, on the other hand, choose the best technique M^*_t, the amount of capital K_t and the amount of human capital to employ in production. Flexibility of prices ensures full employment of factors. Households allocate the production Y_t between consumption and savings according to their utility function, and savings are, as usual, transformed into investment through the capital market. At time t+1 the economy will be endowed with a technical set M_{t+1}, which is different from the one at time t because of exogenous technical change and externalities from the accumulation of human capital, a technical set MH, a capital stock X_{t+1} determined by the initial capital stock X_t and the investment I_t, and, finally, a stock of human capital H_{t+1} determined by the investment $(1-u)H_t$ and the production process in MH.

The accumulation process in the endogenous models here considered are similar to that one illustrated in Figure 5 for the Harrod-Domar model. Similarly to this model, along the optimal path the rate of saving is constant, however, unlike this model it is determined endogenously by the maximizing behaviour of the planner or of households. Moreover, the AK model, unlike the Keynesian models, follows the Classical and Neoclassical tradition in conceiving investment as induced by savings, rather then the other way round.

5.2. Technical progress

In the Lucas' model externalities are present but they are not essential to ensure sustained growth. An alternative approach to the accumulation of factors to ensure it is the introduction of technical progress. Technical progress can be considered as improvement in technological knowledge incorporated in new production function -usually due to unintentional effects like externalities - (see e.g. Romer (1986), or introduction of new products which can be either intermediate goods (Romer (1990), Aghion and Howitt (1992)) or final goods (Grossman and Helpman (1991). This class of models is important in the development of a theory of growth since, because of externalities or because of an explicit introduction of imperfect competitive markets, it makes a substantial break with respect to the growth models *à la* Ramsey given that on the equilibrium path there is no longer an optimal allocation of resources. This implies that the analysis of the optimal path cannot be any longer obtained by means of the normative approach but has to be carried on through a decentralised mechanism which requires a detailed description of the behaviour of agents and of the working of market mechanisms.

The structure of the model with externalities is very similar to the Ramsey model's one, except that it is assumed the existence of a finite number of firms whose production and accumulation decision affects positively and unintentionally the technology of all firms. This effect ensures the constancy of the productivity of the accumulated factors, removing in this way the tendency towards a growth path in which the rate of growth is determined only by exogenous factors like growth of population.

Models with new products are usually three sector models: a final good sector, an intermediate sector and an invention sector which produces "designs" of new intermediate products. Competition is perfect in the final and invention sectors, while the intermediate goods one is imperfectly competitive because there is a fixed cost associated to the purchase of new design. The imperfect competition in the intermediate sector is necessary to ensure the existence of a profit in this sector which, through the fixed cost, will be transferred to the invention sector. These profits in turn are the incentive for the inventive activity to be furtherly carried on. These models endogenise the production of new ideas and introduce the Schumpeterian idea that technical progress is linked with imperfect competition. However, it is not possible to consider this class of model a truly Schumpeterian one since the production of new ideas is here conceived as a smooth process, while in Schumpeter invention is strictly linked, like in Kaldor's view, to business cycle.

Figure 11 illustrates the basic model taken from Romer (1990). There exists only one final good, which can be employed also as physical capital, infinite intermediate goods and potentially producible "ideas". Each intermediate good is produced by means of physical capital and only one idea. The technology set for producing the consumption good is assumed to be stationary, while it is assumed that the production sets for producing ideas and intermediate goods are made up by only one method of production. The method of production to produce ideas is linear and requires only labour, while the method of production for the production of intermediate requires physical capital and ideas. It is linear with respect to physical capital, however it exhibits fixed cost due to the use of the corresponding idea. At time t the economy is endowed with technology sets to produce new

"ideas", intermediate goods and the consumption good indicated respectively by MI_t, MIN_t and MC, a given stock of physical capital X_t and a given amount of labour N_t. Households choose how much labour to allocate to produce consumption goods and to produce ideas. The amount of researcher's labour, together with the technology for producing ideas, obtains new ideas affecting the technology set for producing intermediate goods available next period. Intermediate goods are produced by employing the whole physical capital and the relevant technology. Intermediate goods, in turns, together with labour for production of final goods and the relevant technology produce the final good. This is allocated by households into savings and consumption. The former, through the capital market, are transformed completely into investment, which determine the stock of physical capital available next period, X_{t+1}. The production set for producing the final goods at time t is assumed to be the same as that one at time t, while the set for producing ideas at time t+1, MI_{t+1}, is different from that one at time t because of externalities from existing amount of ideas. The latter is interpreted as learning by doing or as public nature of researchers' skills. Unlike externalities in Lucas' model, here the existence of externalities is important in guaranteeing sustained growth since it ensures the linearity of production function in research for new ideas.

6. Concluding remarks

In this paper we have surveyed in a simple and intuitive way different schemes for the analysis of accumulation process conceived in various economic traditions, from Classical economists to some of endogenous growth theorists. Our main aim has been to highlight similarities and differences

among these alternative theories. As a major point we have pointed out that there is a continuity from Classical to endogenous growth theory, through, partially, Keynesian (see also Hahn and Matthews (1965), pp. 8-9), concerning the fact that the steady state is conceived as endogenously determined by the model, by contrast Neoclassical economists see it as exogenously determined by factors considered not in the realm of economic explanation. On the other hand, we have emphasized a contiguity between Classical, Neoclassical and endogenous growth theory, as opposed to the Keynesian theory, as far as the saving/investment relationship is concerned. In fact, while the former theories conceive saving as wholly transformed in investment, and therefore, the growth being determined by saving themselves, Keynesian theory conceived investment as the source of growth and no relationship between the former and the latter variable exists necessarily.

Under the ever increasing awareness that explaining the differential rates of growth of nation is still an open problem quite a few recent works try to construct models of growth which, following the Keynesian approach, try to explain the actual – rather than the maximum – rate of growth. These works point out mainly the role of entrepreneurship or the working of institutions in explaining the actual rate of growth of the economies (see e.g. (Murphy, Shleifer and Vishny (1991), Acemoglu (1995), Mauro (1995)). It is interesting that these models are substantially following Kaldor's early insight according to which this analysis 'will require, …, some kind of integration of economics and sociology' (Kaldor (1954, p. 238)).

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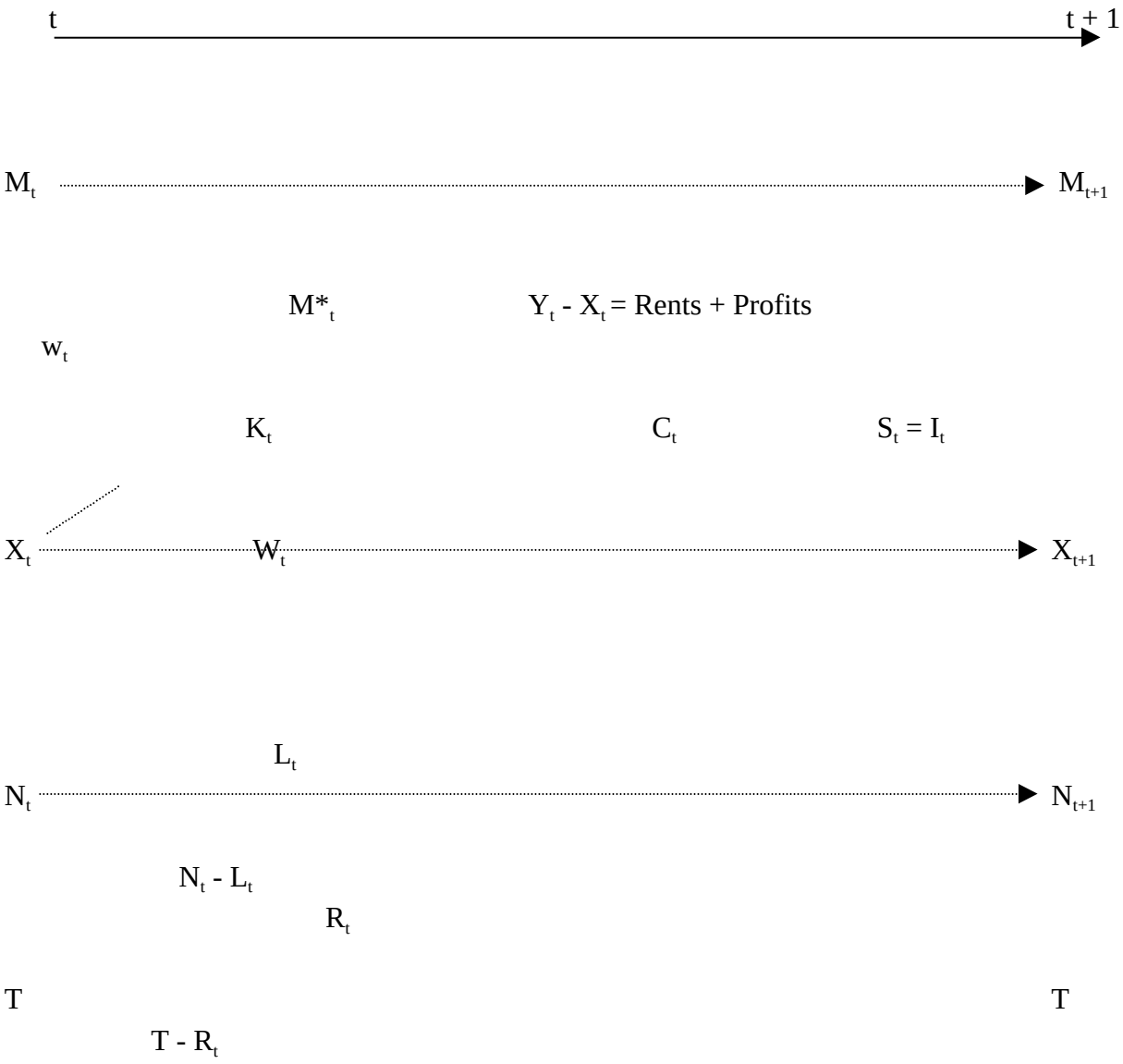

Figure 1. The basic classical model. Continuous arrows indicates decision nodes, dotted arrows indicate functional relationships, where the variable at the end of the arrow depends upon the variable at the beginning. The following relations holds true: $W_t = w_t L_t = X_t - K_t$, $X_{t+1} = X_t + I_t$,. If $w_t = w^*$, i. e. the wage rate is at the natural level, then $N_t = N_{t+1}$. In general, $N_{t+1} - N_t$ has the same sign of $w_t - w^*$.

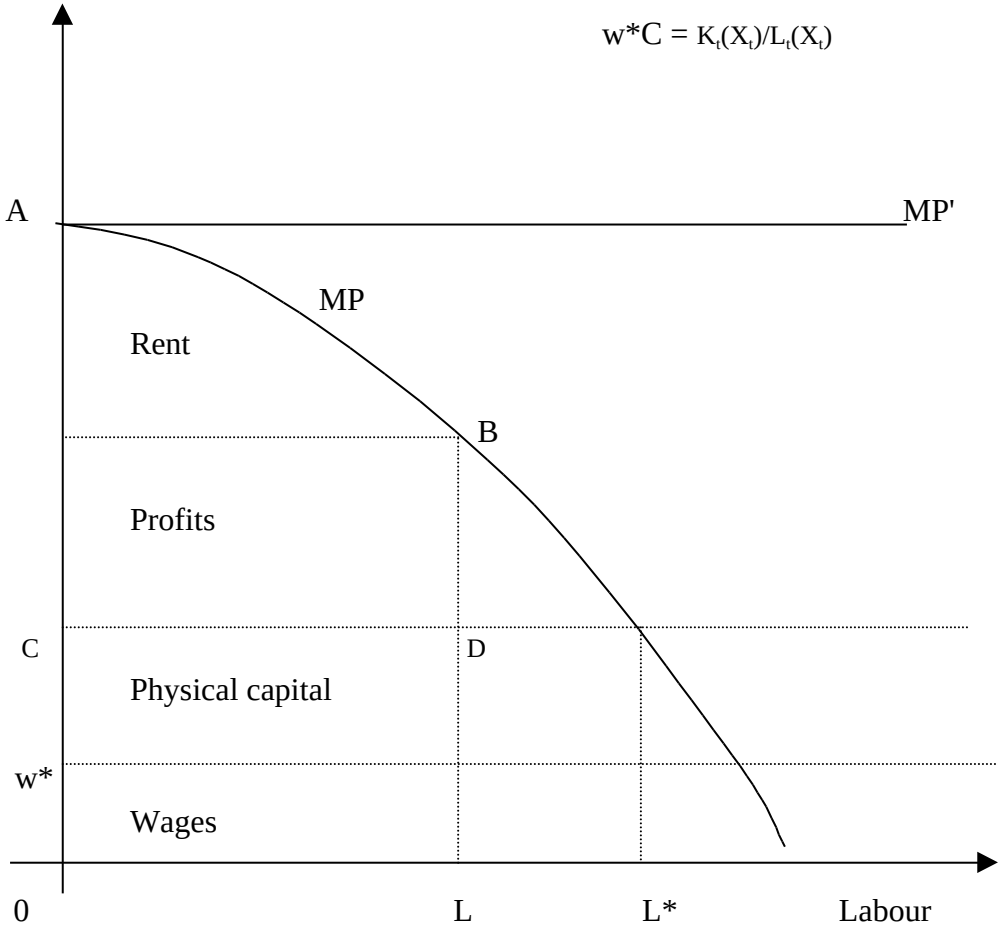

Figure 2: The Ricardian and Smithian-Marxian models. In the Ricardian model the productivity of labour is described by curve MP. Given the amount L of labour employed, the total production is the area OLBA, while net production is given by area CDBA. The latter is divided into rents and profits as in figure. Profits profis yields accumulation of capital and the amount of labour employed increases over time. When L* is employed, profits are zero and the economy stagnates.
In the Smithian model the productivity of labour is described by the curve MP'. Profits are never equal to zero.

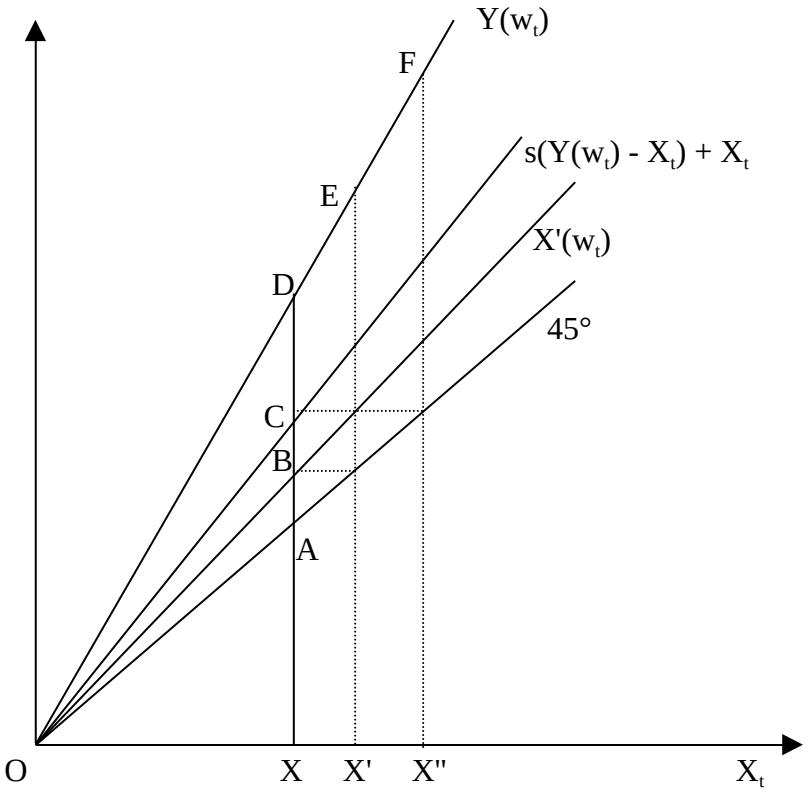

Figure 3. The Smithian-Marxian model. When the total stock of capital is OX the net supply of capital (lenght AC) is greater than the net demand of capital to ensure steady growth with full employment (lenght AB). Provided there is full employment at time t, the total stock of capital at time t+1 ensuring the latter condition is XB. However, capitalists will supply at time t+1 a total stock of capital equal to XC. Then, the resulting excess demand of labour, X"F - X'E, will cause an increase of wages. The latter yields a clockwise rotation of curves $Y(w_t)$ and $s(Y(w_t) - X_t) + X_t$ and an anticlockwise rotation of curve $X'(w_t)$. The equilibrium is attained when the wage rate w_t satisfies the relation: $(1+ G_n(w_t)) = s/(v + w_t)$.

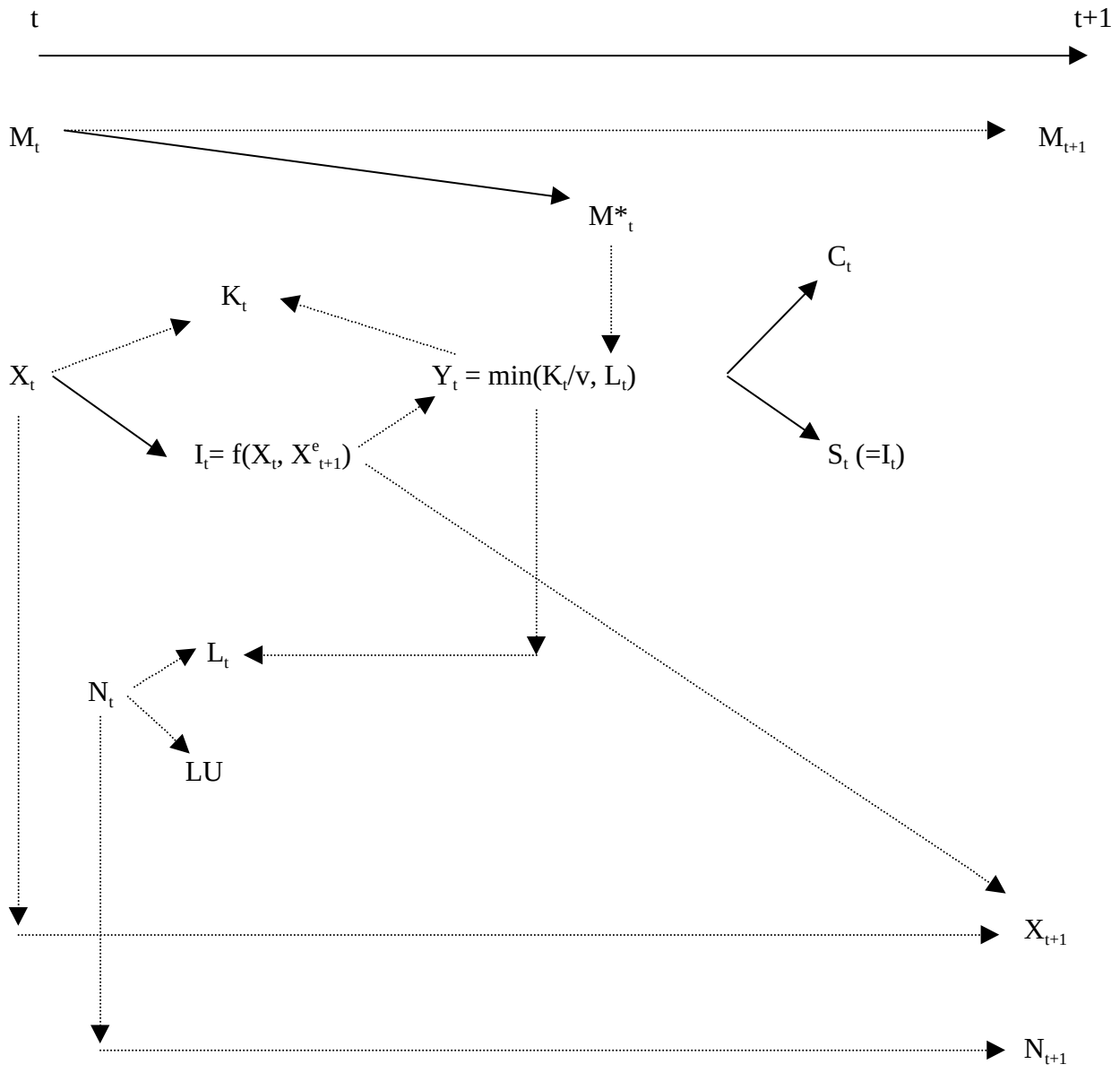

Figure 4. Harrod-Domar model. Superscript e indicates expected values. The economy is on a warranted path, hence the expected values are also true values, and the equilibrium on the goods market implies that $S_t = I_t$.

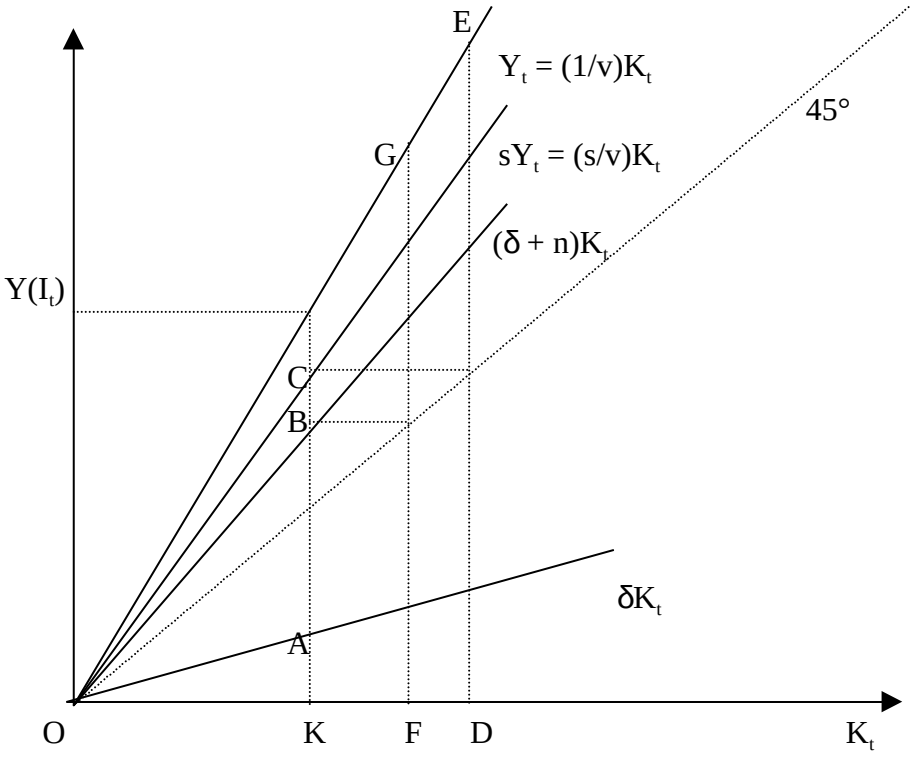

Figure 5. The accumulation process in the Harrod-Domar model. Suppose that at time t there is full employment of labour, the investment I_t determines the production level $Y(I_t)$, which, in turn, determines the supply of capital, KC, and the demand of capital for full employment growth, KB. The demand for labour at time t+1, DE, is greater than the supply of labour, FG, and the economy cannot growth along the warranted path.

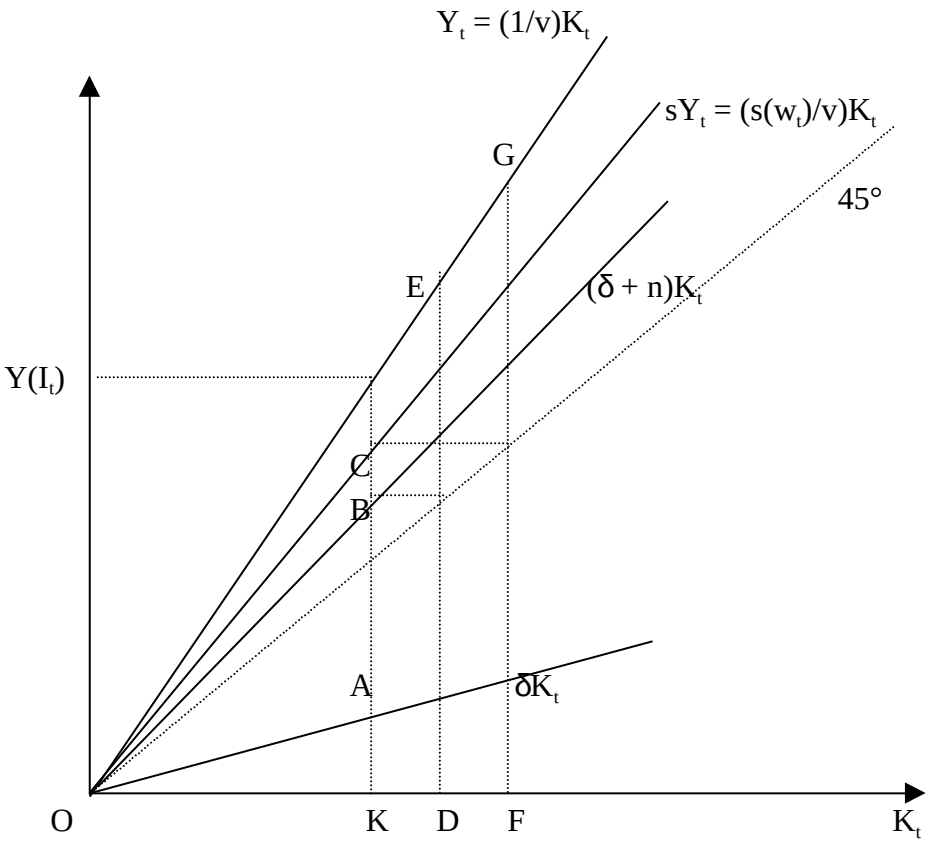

Figure 6. The accumulation process in the Kaldor model. When the total stock of capital is OK and there is full employment of labour, the net supply of capital, AC, is greater than the net demand of capital to ensure steady growth with full employment AB. At time t + 1 the demand of labour, FG, is greater than the supply of labour, DE. There will be excess demand of labour and hence a higher level of real wages. Therefore, a clockwise rotation of curve $s(w_t)Y$ occurs. The equilibrium is attained when the wage rate is such that $s(w^e)/v = (\delta + n)$.

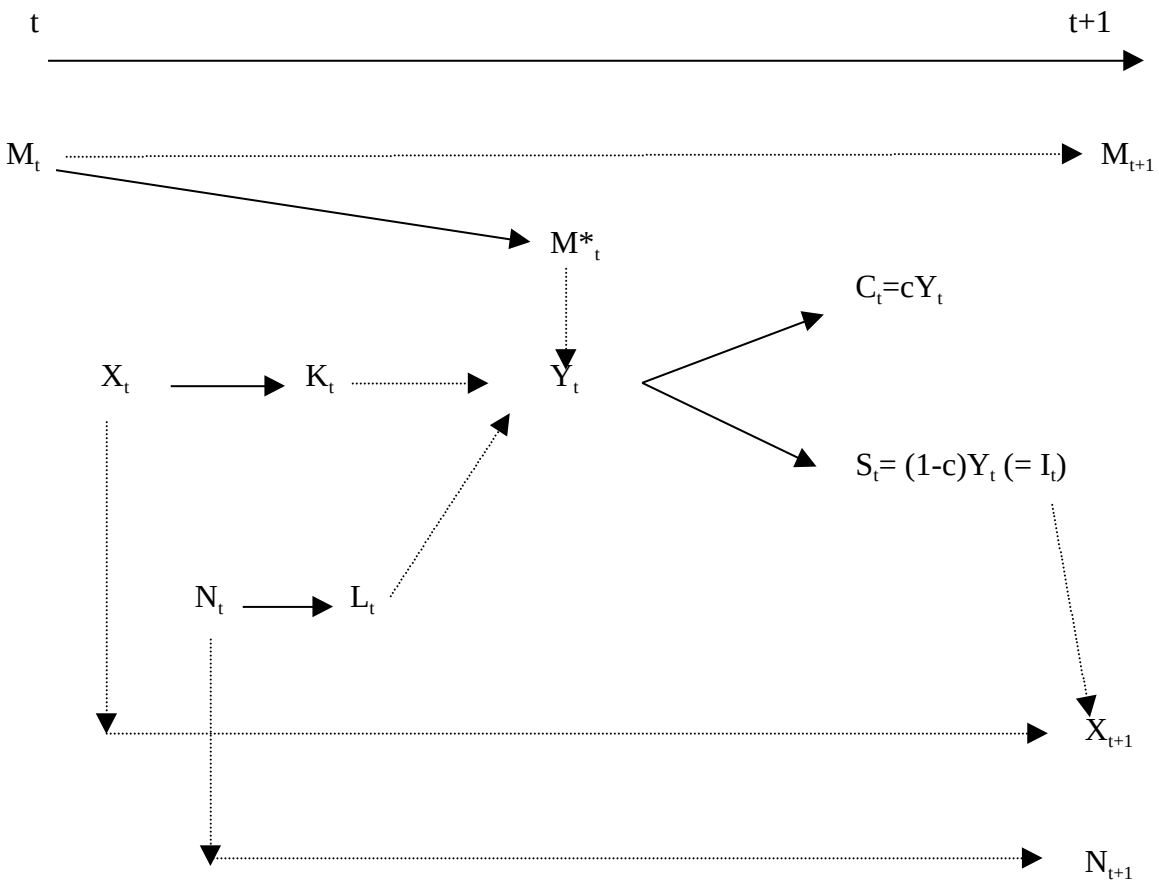

Figure 7: The Solow model. Price flexibility in all markets ensures that all factors are fully employed and that $S_t = I_t$.

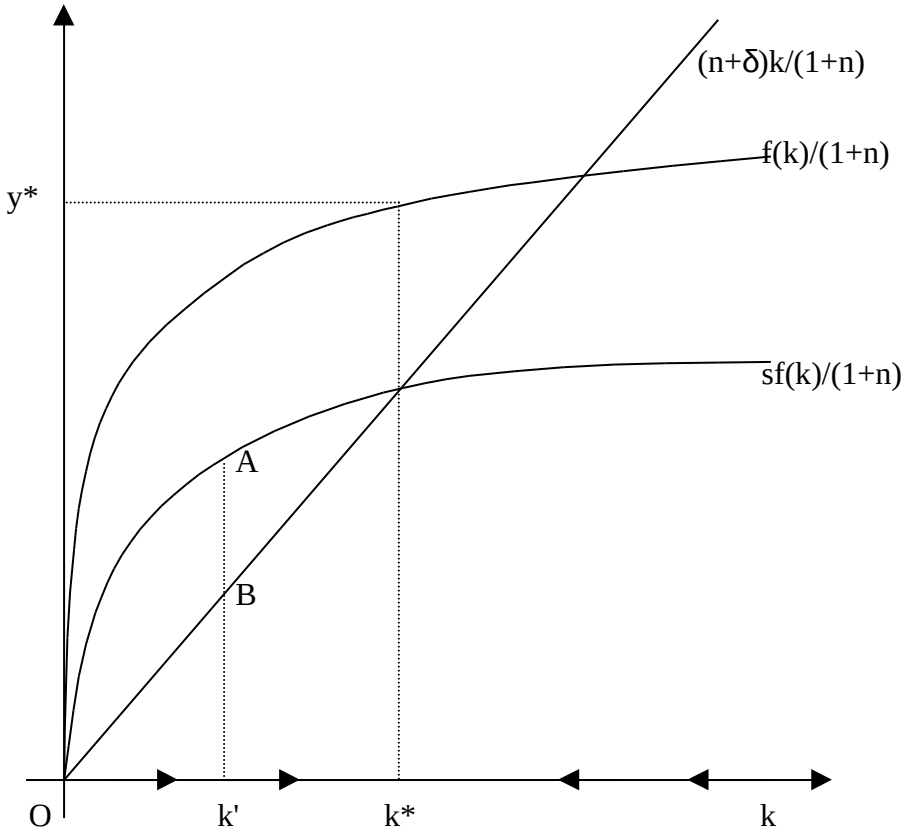

Figure 8. The Solow model in discrete time and without technical progress. The exogenous rate of population growth is n, δ ($0 \leq \delta \leq 1$) is the decay rate of capital. The magnitudes k* and y* are the steady state level of per capita capital and income.

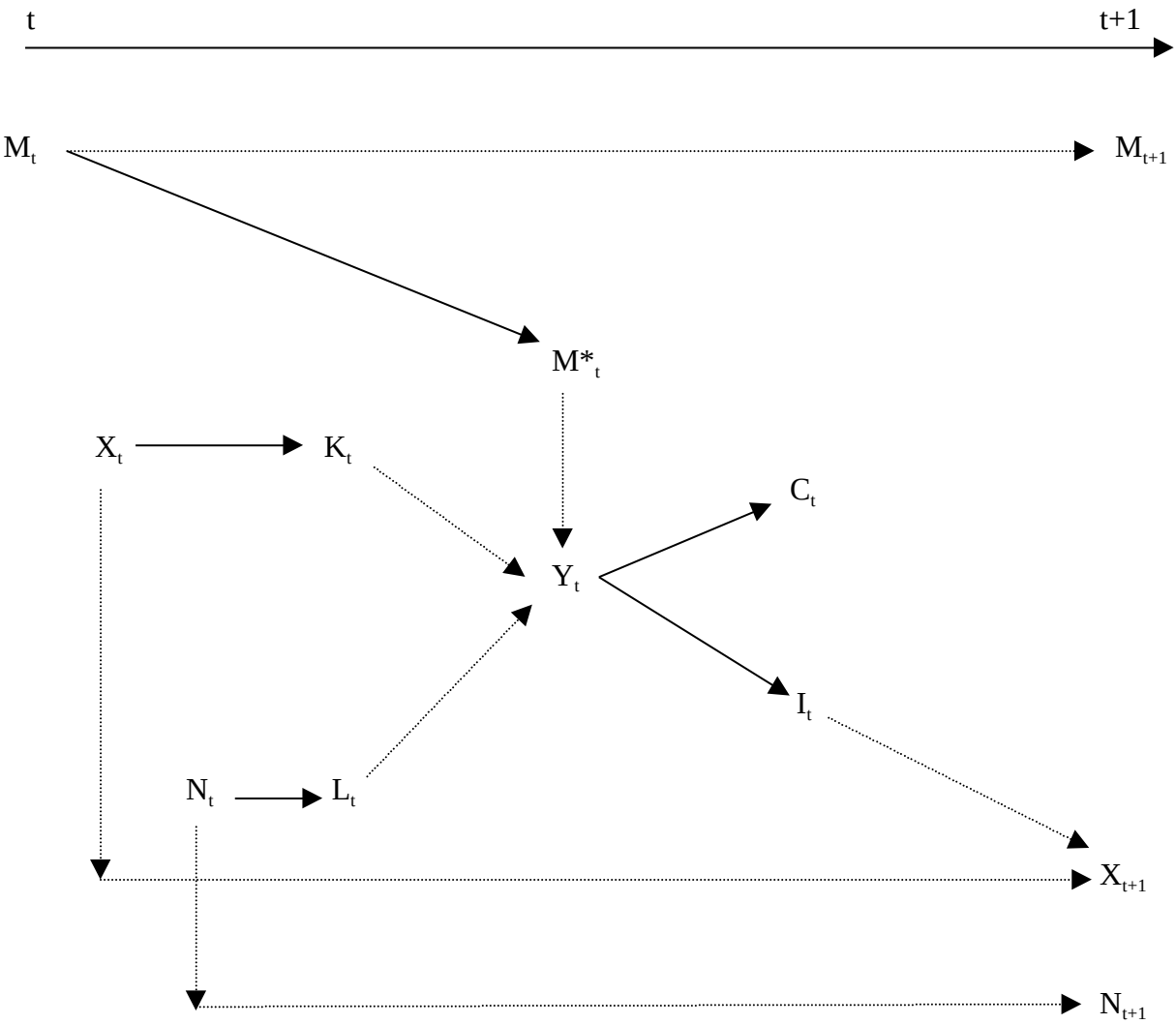

Figure 9. The basic normative model *à la* Ramsey. Any decision of saving is a decision of investment.

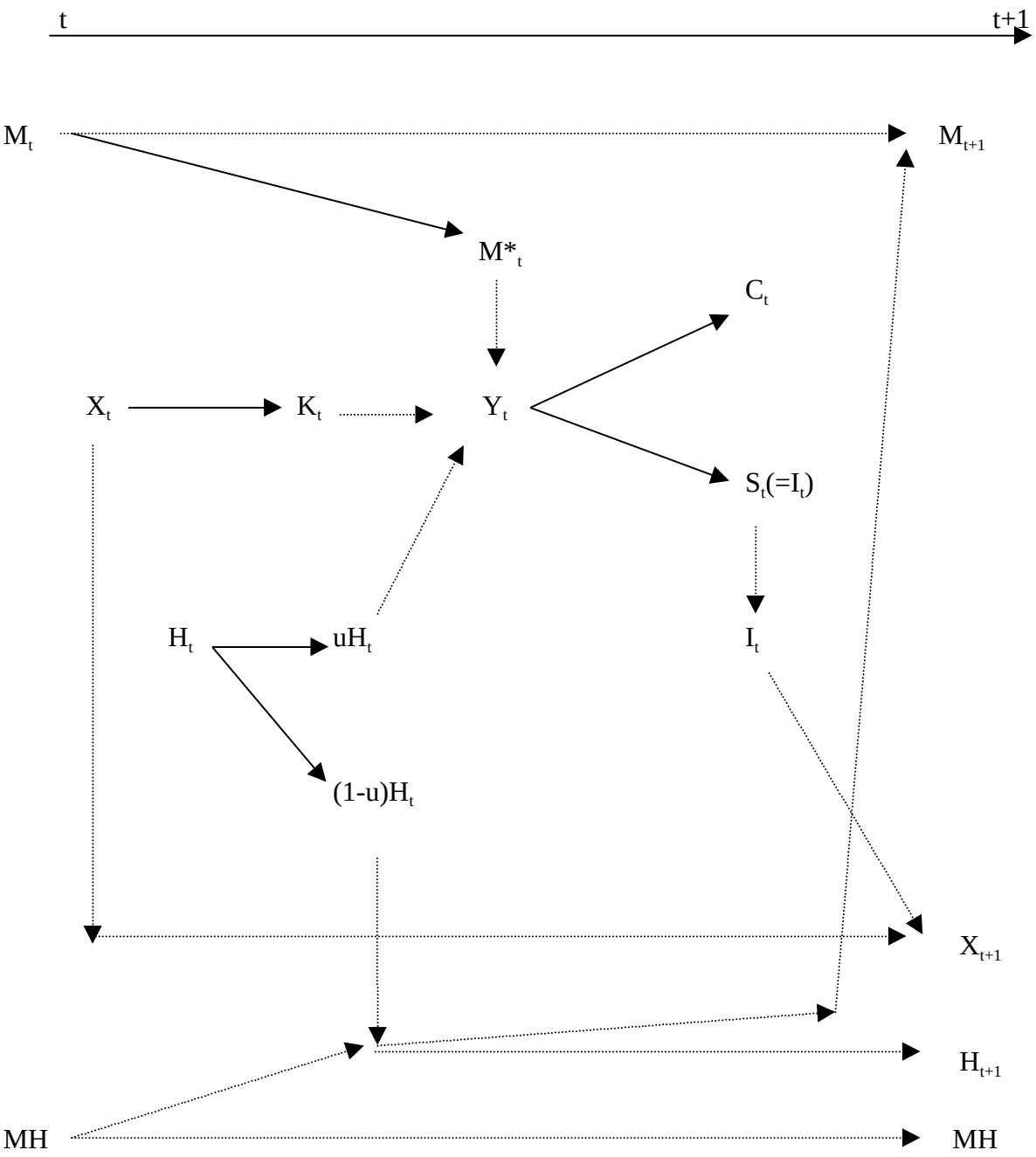

Figure 10: The Lucas model.

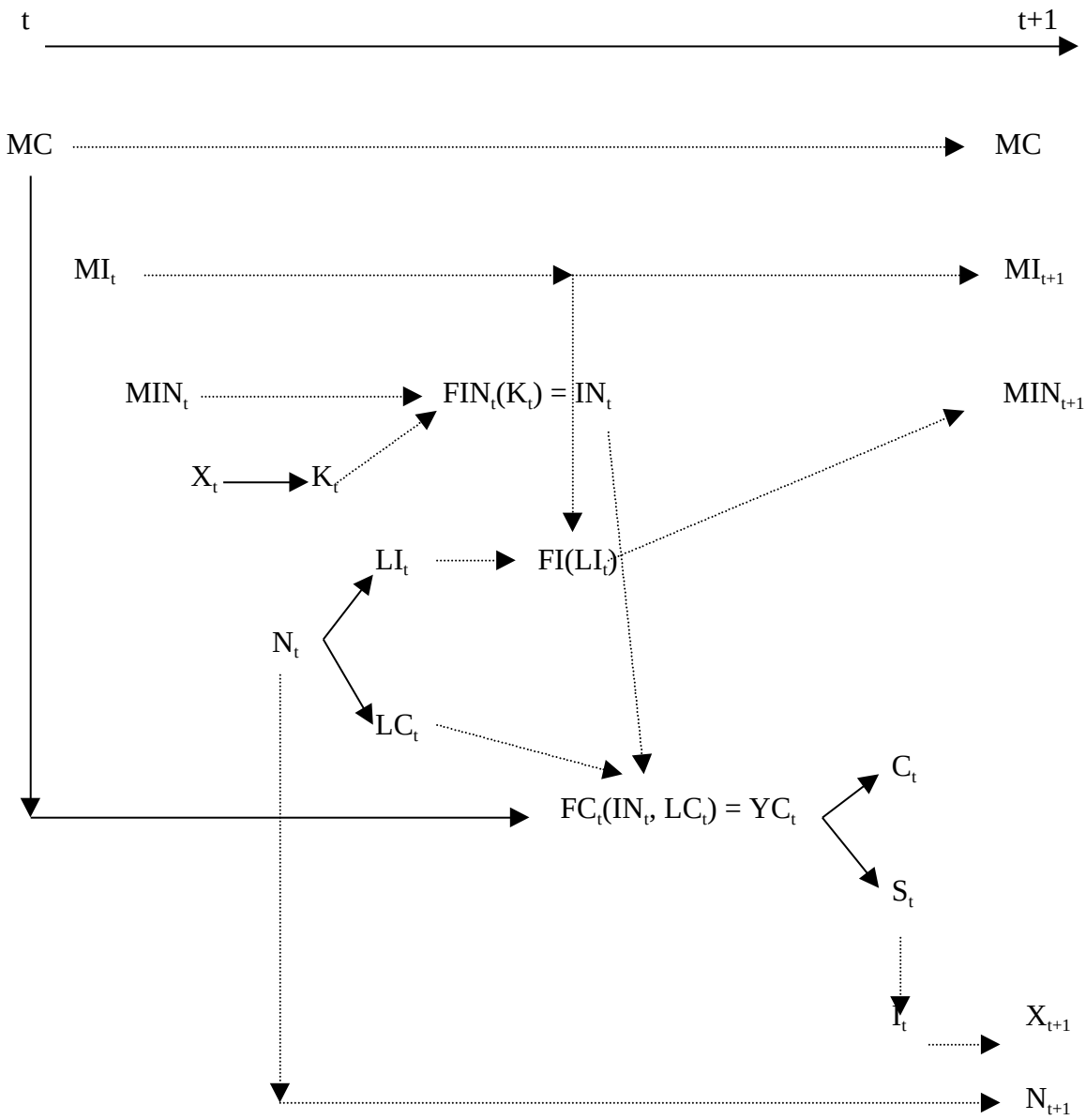

Figure 11: The Romer (1990) model