Human Capital Production: Incentives and Social Policy Issues.

Stephane Ciriani, Université Paris II Pantheon Sorbonne ERMES.

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Abstract

This paper studies the effects of public policies an social interactions on the accumulation of human capital.We first consider an infinitely lived heterogenous agent dynamic model, where each new generation inherits human capital, then we lay out a majority voting model, with a median voter setting the tax rate.The human capital production model incorporates recursive preferences a la Epstein and Zin non-additively separable dynastic utility function, in an overlapping generations setting without credit and insurance market. The second framework studies the case of a poor economy where government involvment would reduce the number of agents who get education, then decrease the rate of growth, as agents' income may be too low to raise enough funds to finance education.We then compare the redistributive properties of the two frameworks and the growth enhancing effects provided by private and public incentives. It appears that public education system may be socially preferred while private education investment setting is still more efficient. In the long run, the degree of agents' heterogeneity is lower in the public regime as without a majority-voting system setting the tax rate, income inequality is perpetuated in the private regime where greater funds are invested on more efficient agents. Private selfish incentives to increase personal funds are then stronger than incentives concerning publicly provided goods and services, leading to a greater degree of heterogeneity among social groups.

Keywords: Education Finance, Incentives, Income Distribution, Inequality, Optimal Tax Rate, Political Regimes.

JEL Categories: O,I,H,D.

Introduction.

This paper analyses two distinct frameworks where individual decision rules along with social policies in a given period have a strong effect on the welfare of individual living in the successive periods.We compare the effects of individual, public and private optimal decision rules on the distribution of income and on the process of individual human capital accumulation, then we focus on the persistence of inequalities and its effects on the growth rate and welfare of the economy. In the first framework, altruistic parents take decisions concerning schooling time and education expenditures of their offsprings and the number of children under both private and public egalitarian regime: the externality provided by the public education regime is responsible for a loss in efficiency, while there is lower heterogeneity in this majority voting setting. The agents have a dynastic utility function and the human capital production function depends, as in Glomm and Ravikumar (1992), on bequest to the offspring, private or public education expenditures, time allocated to the rearing of children, and the inner ability of the young. The results depends on the right incentives provided by the public services, as the public education regime can dominate the private one in terms of welfare and heterogeneity among agents .While all the young agents have specific, innate learning abilities, the dynamics of income, adult human capital and redistribution are affected by this initial, random endowment: all adult altruistic agents voting for the same tax rate because of the log-linear preferences take into account the future human capital of students. As a result, they take into account the average rate of human capital of the society, which allows to determine a threshold level for human capital of young agents when adult. The determination of a threshold value to select an education regime, along with the analytical solutions for the policy rules, allows this comparison between the two education system : as incentives are provided by anticipations on wages in the long run and welfare evaluation, it is possible to state that government envolvment taking the form of a majority-voting process allows all parents to raise more children, spend less time working and devote a lower share of income to education. In the private case, the coefficient of intergenerational transmission of human capital is higher, and such gap lowers the marginal benefit corresponding to the higher human capital and future earnings of the offsprings, (Sheik Rahim 2000). We then study the case of human capital production in a poor economy, where the total amount of resources available for private consumption and education expenditures depends on child labor, given the constraints of missing credit and insurance markets, and the absence of government. The budget constraint of adult agents are, as in the previous model, affected by a change in regime: in this framework, a rise in education cost of a child requires higher wage to help the household to continue to raise enough education funds. There is no government to provide right incentives or legal restrictions to avoid child labor, and altruistic parents cannot do without child earnings. As the human capital of the offspring when adult depends on the parent's wage in the first period, there is also intergenerational transmission of inequalities, but in this case, both household's budget constraint and the properties of human capital production function prevent these differences in initial endowment to have effects on equilibrium decision rules (Hazan and Berdugo 2001). The abscence of a public sector or social planner leads adult agents to choose between two distinct regimes: the lack of legal restrictions allows child labor, then a young agent can work in two different sectors, as each provides one specific levels of wage. This choice will affect private consumption and education expenditures of the household: we can show formally that an increase in the education cost of the young leads leads children to choose the higher wage sector, which allows their parents to relax their liquidity constraint. As a result, children will spend less time studying, and will earn higher wages in low skilled sector as the amount of time devoted to work is greater. The distribution of income among parents in the first model, and the wage differential between young and old agents, along with the gap between child earnings and the cost of education in the second setting, affects the incentives of the young to acquire education. In such a poor economy the prospect of future low-skilled jobs may also decrease the incentives to remain in the education system, instead of spending more time working.This economy remains in a poverty trap as there are no knowledge or technical spillovers to increase the wage gape between generations, then to provide the right incentives to restrict child labor and excluding the agents from the higher wage sector.

1. The Human Capital Production.

Theoretical foundations .

The first model analysed in this paper incorporates random ability distribution, endogenous fertility and dynastic utility function, as adult agents determine their optimal rate of fertility in both private and public education regimes. They are endowed with a single unit of time divided between the rearing of children, and their own work. In turn, the human capital function of the offspring depends on these bequests, the inherited human capital, the private or public expenditures in education, and a random distributed variable capturing the learning abilities effect on the individual level of human capital of the offspring when adult. The main assumption in this model concerns the form of the utility function. Following Becker and Barro (1988), the utility of an altruistic parent is given by :

$$U_0 = u(c_0, n_0) + \alpha(n_0)n_0U_1$$

where $\alpha(n_0)$ is a measure of the parent's degree of altruism towards their children. It is assumed that the parent's current utility is not subject to the size of the family (i.e. the number of children), and that the altruism coefficient, which is endogenous, has constant elasticity with respect to the number of children. The altruism coefficient is then written as $\alpha(n_i) = \psi(n_i)^{-\varepsilon}$ where η is the degree of intertemporal altruism and ε is the degree of intratemporal altruism . If $0 < \eta < 1$, the parents are selfish, as the marginal utility of their own consumption is greater than the marginal utility of their child's consumption when the size of the family is normalized to one. The dynastic recursive utility function is expressed as :

$$U_o = \sum_{i=0} \psi^i N_i^{1-\varepsilon} u(c_i) \ .$$

where N_i is the size of the family . An increase in the coefficient α or in decrease in the parameter ε means that the parents care more about the utility of their children, that is to say, there is an increse in the degree of generational altruism . Following Becker and Barro (1989), an increase in both kinds of altruism leads to higher fertility . However, Becker (1991), shows that altruistic parents invest more in the education quality and the human capital accumulation of their chidren because their own utility will benefit from such investment return, (even if they do not have more children). This investment in chidren's education does not only concerns bequests left by parents but takes account of the possibility to increase parental funds devoted to the rearing and the schooling effort of children, as purchased educational inputs (teacher time, classroms, books, computers, nutrition) .The cost of rearing children and financing their education and schooling time is supported by the parents, under the credit and the insurance constraints; (in our model, it is assumed that there are no insurance or credit market available). The lower the rate of time preference, the higher is the parents altruism toward the next generation . In this model, we focus on the intergenerational altruism . We assume that the absence of credit and insurance markets leads adult agents to use only a fraction $\delta^r$of their total income for the education of their children. They devote an amount of time ω to the rearing of children, which has a fixed cost x. Our analyse is based on the study of the dynamical properties of the model as the distribution of welfare and the evolution of inequalities among individuals of each generation. The agents maximise intertemporal recursive utility function and choose the optimal tax rate to finance public education expenditure, in a majority-voting setting, then the endogenous rate of fertility, the time devoted to the education of children, the level of labour supply. The parents invest time costly education in their offspring .This make children more costly to rear, which may induce a fall in birth rate, unless education is partly provided by public services. Our model is similar to Sheik Rahim (2000), and Benabou (1996), while our analyses of the overlapping generation model focuses on the role of education in the determination of the interactions between growth, income and inequality: in such model, parents care about the quality of education, which is linked to their own level of human capital. The interaction between both levels of human capital determines the level of human capital of children, while in the other framework, the interaction between parental income, education cost and children's wage have a strong influence on parental liquidity constraint and the acquisition of knowledge by children .

We assume that the human capital production function is expressed as(1):

$$h_{it+1} = \kappa\epsilon_{it+1}\omega_{it}^{\eta}E_{it}^{\alpha}h_{it}^{\gamma}$$

It is similar to the function found in Glomm and Ravikumar(1992). Thus human capital is the product of the five following inputs : a fixed, exogenous productivity parameter, κ, innate ability of the young individual, which we suppose to be uninsurable, log–normally distributed and common to all members of a given family : $log\ \epsilon_{it} \sim N(\frac{-s^2}{2}, s^2)$, parental human capital,quality of educational inputs that are received, E_{it}, then, the amount of time spent by the members of the previous generation in providing education to the young . We assume also that human capital is log–normally distributed, with mean m_0 and variance Δ_0^2 : $log\ h_0 \sim N(m_0, \Delta_0^2)$. The same goes for the distribution of $log\ h_{it}$: $log\ h_{it} \sim N(m_{it}, \Delta_{it}^2)$. The assumption: $\eta,\ \alpha,\ \gamma \in [0,1]$ ensures that each factor has diminishing returns, except when $\eta = 1$, which induce constant returns to schooling time(1).The absence of credit and insurance markets prevents all parents from choosing an investment level that equates the marginal return on human capital and the marginal return on bequests . Obviously, less endowed families will have to opt for lower levels of investment in the human capital of their children, which allows the transfer of inequalities from the old generation to the new one . The missing insurance market prevent parents from insuring against the risk linked to the unknown level of inner ability ε_{it+1}which is supposed to be log–normally distributed, with mean $(\frac{-s^2}{2})$ and standard deviation s^2. Each old Agent allocate her unitary time endowment between education of children, ω_{it}, working, l_{it}, children rearing, n_{it}, at the exogenous cost of x for each child. Old Agent's income is used to finance private consumption and private education of the young agents. Individual human capital h_{it} and income $y_{it=}h_{it}l_{it}$ remain log-normally distributed over time. If $log\ h_{it} \sim N(m_t, \Delta_t^2)$ the distribution of income at time t will be $log\ y_{it} \sim N(m_t + log\ l_t,\ \Delta_t^2)$ where the motion of the moments m_t and Δ_t^2 is determined further. The introduction of uncertainty about the evolution of the innate ability ε_{it} allows us to use a Kreps-porteus utility function, in order to differentiate attitude towards risk from intertemporal preferences. We use an additively-separable dynastic utility function, following Becker and Barro (1988), in a recursive representation a la Epstein and Zin (1989). We obtain the log-linear specification of Agent's utility after three steps : from the recursive dynastic type utility, we get:

$$U_{it} = \left\{ c_{it}^{1-\sigma} + \rho[\, n_{it}^{1-\varepsilon}\ (E_t(U_{it+1}^{1-\nu}))^{\frac{1}{1-\nu}}]^{\,1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

Where the Arrow-Pratt measure of relative risk-aversion is given by ν and the intertemporal elasticity of substitution for consumption is given by $\frac{1}{\sigma}$. We take the log-linear form of the equation and we obtain :

$$log\ U_{it} = log\ c_{it} + \rho(1-\varepsilon)logn_{it}\ + \frac{\rho}{1-\nu}\ log\ [E_t(U_{it+1}^{1-\nu})]$$

where $\rho \in (0,1)$ is the discount rate factor .

Because of the log-linear preferences, all households, under a given education regime, will have identical decision rules, and will opt for a same fertility rate, a same labor supply, a same amount of time allocated to the education of children and a same fraction of income devoted to education quality of the offspring. In the egalitarian public regime, all individuals vote for the same tax rate, as in the public regime, all young agents are supposed to receive the same level of education, (or education quality, following Benabou 1996 and Glomm and Ravikumar 1992). It is assumed that all agents choose their equilibrium fertility rate, labor supply, time allocation given the average human capital, income, and number of children in the economy. Given the random distribution properties of the level of individual students'abilities, it is assumed that a less gifted student will suppose higher education cost to increase the marginal benefit of education.

The Solution of the Model .

An adult agent chooses optimally the values of the number of children, the share of time devoted to the rearing of children, and the fraction of income allocated to education. The private consumption of a household is therefore expressed as $c_t = y_t - \widetilde{y_t} = (1-\delta^r)y_t$ in the private education regime and expressed as $c_t = y_t - \widetilde{y_t} = (1-\tau)y_t$ in the public education regime where the tax rate $\tau \in (0,1)$ is set by a majority-voting decision rule. The budget constraints is given by the equality: $nE_t = y_t^{p,g}$ where E_t stands for the level of education expenditures. Under the public regime, the policy variables $\{\overline{n},\ \overline{y}_t^g\}_{t=0}^{\infty}$ are taken as given by the adult agent while under the private regime, the policy sequence$\{n,\ y_t^p\}_{t=0}^{\infty}$ is chosen optimally by the agent. Therefore, The Bellman equation for family i during period t under private and public budget constraints are expressed as :

Under the private regime:

$$log\ U_t(h) = \underset{n,\tau,\omega}{max} \quad \left\{log\ c + \rho(1-\varepsilon)logn\ + \tfrac{\rho}{1-\nu}\ log\ [E_t(U_{t+1}^{1-\nu})]\right\}$$

$$s.t\ : c_t = y_t - y_t^p = (1-\delta^r)y_t,\ \ y_t^p = \delta^r y_t$$

$$h_{it+1} = \kappa\epsilon_{it+1}\omega_{it}^{\eta}E_{it}^{\alpha}h_{it}^{\gamma}$$

$$n = \tfrac{1-l}{x+\omega}$$

$$nE_t = y_t^p\ .$$

Under the public regime:

$$log\ U_t(h) = \underset{n,\tau,\omega}{max} \quad \left\{log\ c + \rho(1-\varepsilon)logn\ + \frac{\rho}{1-\nu}\ log\ [E_t(U_{t+1}^{1-\nu})]\right\}$$

$$s.t\ :\ c_t = y_t - \overline{y}_t^g = (1-\tau)y_t,\ \overline{y}_t^g = \tau y_t$$

$$h_{it+1} = \kappa\epsilon_{it+1}\omega_{it}^{\eta}E_{it}^{\alpha}h_{it}^{\gamma}$$

$$n = \frac{1-l}{x+\omega}$$

$$\overline{n}E_t = \ \overline{y}_t^g.$$

Proposition 1:

The decision of the households are then, under the private education regime:

$$l^{*p} = \frac{1-\rho+\rho\alpha}{(1-\rho)[1+\rho(1-\varepsilon)]}$$
$$\tau^{*p} = \frac{\rho\alpha}{1-\rho+\rho\alpha}$$
$$\omega^{*p} = \frac{x\eta}{(1-\rho)(1-\varepsilon)-(\alpha+\eta)}$$
$$n^{*p} = \frac{\rho[(1-\varepsilon)(1-\rho)-(\alpha+\eta)]}{(1-\rho)x[1+\rho(1-\varepsilon)]}$$

under the public education regime :

$$l^{*g} = \frac{1}{1+\rho(1-\varepsilon)}$$
$$\tau^{*g} = \frac{\rho\alpha}{1+\rho\alpha-\rho\gamma}$$
$$\omega^{*g} = \frac{x\eta}{(1-\rho\gamma)(1-\varepsilon)-\eta}$$
$$n^{*g} = \frac{\rho[(1-\varepsilon)(1-\rho\gamma)-\eta]}{(1-\rho\gamma)x[1+\rho(1-\varepsilon)]}$$

Proof : see appendix (A.1).

According to these results, it is clear that in the private education regime, adult agents allocate an higher fraction of their income on education expenditures (or education quality, following Benabou 1996), choose a lower rate of fertility, allocate an higher fraction of time to work and a lower fraction of time to the education of their offspring. The private education regime dominates the egalitarian public one in terms of efficiency. Under the public education regime, an increase in individual, specific human capital is responsible for an increase in the average level of human capital in the economy, which is an external effect. More formally, the comparison of the two set of equilibrium policy variables can be expressed as follows:

Proposition 2:

The comparison between the public and private education regime is expressed as :

(1) $\tau^{*g} < \tau^{*p} = \nu^{*p}$ *if and only if* $\gamma < 1$.

(2) $n^{*g} > n^{*p}$ *if and only if* $\eta u_1^g < \eta u_1^p + \alpha u_1^p$ *as* $u_1^g < u_1^p$ *and* $\eta > \gamma(\alpha + \eta) - \frac{\alpha}{\rho}$

(3) $l^{*p} > l^{*g}$ *if and only* $\rho\alpha > 0$

(4) $\omega^{*p} > \omega^{*g}$ *if and only* $(\alpha + \eta)u_1^p > \eta u_1^g$ *and* $u_1^p > u_1^g$

Where $u_1^p = \frac{1}{1-\rho} > u_1^g = \frac{1}{1-\rho\gamma}$ stand for Bellman's positive constants and capture the effect of intergenerational transmission of human capital within families .(see appendix (A2)). According to Glomm and Ravikummar (1992), and Cardak (1999), adult agents choose to spend less time rearing their children and less time working under the public education regime. The public regime is responsible for a disincentive as we note that households also choose to devote a lower fraction of their income to education quality: when education is a publicly provided good, adult agents can afford to spend less time working as their budget constraint is relaxed. They also spend less time rearing children, which increases their leisure time (Glomm and Ravikummar 1992) , as education quality depends also on average individual income. More formally, it is useful to analyse the effects of a change in regime on human capital and income of individuals.

2. The Effects of a Change in Education Regime .

The Marginal Return to Human Capital.

In this section, we study the effect of a change in education system. It is assumed that households differ in their children's ability, as learning abilitiy of a student is log-normally distributed over time, with mean $\frac{-s^2}{2}$ and standard deviation s^2: $log\ \epsilon_{it} \sim N(\frac{-s^2}{2}, s^2)$. Individual human capital has mean m_t and variance Δ_t^2: $log\ h_{it} \sim N(m_t, \Delta_t^2)$. In the public regime, government is involved in education provision, as the majority-voting setting determines a tax rate to finance education, at the cost of blunting the adult agents' incentive to exert effort on labour market and to spend time educating children. We determine the human capital transition equation under both settings using the human capital production function and the set of equilibrium variables. The equilibrium transition equations are expressed as:

$$h_{it+1}^g = \kappa\ \epsilon_{it+1} \Omega^g h^\gamma \overline{h^\alpha}$$
$$h_{it+1}^p = \kappa\ \epsilon_{it+1} \Omega^p h^{\alpha+\gamma}$$

where $\Omega^p = \omega^{*p} \frac{\tau^{*p} l^{*p}}{n^{*p}} = \left\{ \frac{\eta x}{(1-\varepsilon)(1-\rho)-(\alpha+\eta)} \right\}^\eta \left\{ \frac{\alpha x}{(1-\varepsilon)(1-\rho)-(\alpha+\eta)} \right\}^\alpha$

therefore, $\Omega^p = \frac{(\eta x)^\eta (\alpha x)^\alpha}{[(1-\varepsilon)(1-\rho)-(\alpha+\eta)]^{\alpha+\eta}}$

$$\Omega^g = \omega^{*g} \frac{\tau^{*g} l^{*g}}{n^{*g}} = \left\{ \frac{\eta x}{(1-\varepsilon)(1-\rho\gamma)-\eta} \right\}^\eta \left\{ \frac{\alpha x(1-\rho\gamma)}{[1+\rho\alpha-\rho\gamma]\ [(1-\varepsilon)(1-\rho\gamma)-\eta]} \right\}^\alpha$$

It is assumed that parental human capital input in the production function has diminishing returns, which ensures that they choose a lower fraction of their income to be devoted to education quality. If the overall return to public and private investment in education is constant, (in this case, we assume that this condition can be formally written as $\alpha + \gamma = 1$), growth is sustained over time. This assumption has several implications within the model: according to the equilibrium conditions, adult agents spend less time working under the public education regime than under the private one. Following this assumption, the coefficient α has to be strictly positive. According to the equilibrium conditions, we observe that this not the case when $\alpha = 0$, $\gamma = 1$, and $\eta = 0$,[2] that is to say, when $\Omega^p = \Omega^g = 1$. The ratio of public to private human capital expenditure equals one if $\alpha = 0$, and $\frac{\overline{h}}{h}$ if $\alpha = 1$: in this case, the difference equation ratio depends on the average level of human capital, as with public education, inequality and heterogeneity are supposed decline over time and the growth rate to rise. In the long run, the parental choices in the private regime do not reduce heterogeneity, (as knowledge and skills are log normally distributed over time), which perpetuates inequality, decreasing the growth rate. Formally,(see Appendix A.2), households allocate more funds to improve education quality if:

$$\left\{\frac{\Omega^g}{\Omega^p}\right\} = \left\{\frac{(1-\varepsilon)(1-\rho)-(\eta+\alpha)}{(1-\varepsilon)(1-\rho\gamma)-\eta}\right\}^{\eta}\left\{\frac{(1-\rho\gamma)\,[(1-\varepsilon)(1-\rho)-(\alpha+\eta)]}{[1+\rho\alpha-\rho\gamma]\,[(1-\varepsilon)(1-\rho\gamma)-\eta]}\right\}^{\alpha} < 1$$

$$\Omega^p > \Omega^g \;\; if \;\; \left\{\frac{(1-\varepsilon)(1-\rho\gamma)-\eta}{(1-\varepsilon)(1-\rho)-(\alpha+\eta)}\right\}^{\alpha+\eta} > \left\{\frac{1-\rho\gamma}{1+\rho\alpha-\rho\gamma}\right\}^{\alpha}$$

Under the private education regime, households still devote a greater fraction of their income to the education finance when $\alpha = 1$ and $\gamma = 0$[3]. Therefore, according to such coefficient values, the function of individual human capital production does not only depend on the level of inherited human capital of the agent. In the public regime, the accumulation of human capital in the following period is subject to the average level of knowledge accumulated in the society,($\overline{h}$) (the mean value of the human capital variable). The majority voting rule and the logarithmic utility function (log-linear preferences) induce all agents to vote for the same tax rate and their contribution to the provision of education is then equal. This borderline case occurs when the knowledge and skills in the period $(t+1)$ are partly determined by innate ability, public funds, and the average level of human capital, which may reduce the intergenerational transmission of inequalities effect. The decreasing return to parental income as an input in children's human capital production function allows agents to devote a greater share of their income to education expenditure under the private regime .

The learning ability effect and the Question of Incentives .

In order to determine a decision rule for the education regime choice, individual and household incomes under the alternative regimes can be compared under the assumption of given and identical initial conditions for all agents (Cardak 1999). We consider now the ratio of public to private education human capital. Dividing Ω^p by Ω^g we find out :

$$\left\{\frac{\Omega^g}{\Omega^p}\right\} = \left\{\frac{(1-\varepsilon)(1-\rho)-(\eta+\alpha)}{(1-\varepsilon)(1-\rho\gamma)-\eta}\right\}^{\eta}\left\{\frac{(1-\rho\gamma)\,[(1-\varepsilon)(1-\rho)-(\alpha+\eta)]}{[1+\rho\alpha-\rho\gamma]\,[(1-\varepsilon)(1-\rho\gamma)-\eta]}\right\}^{\alpha}, \text{ then,} \quad (4)$$

Under the following assumptions:

$$1-\varepsilon>0$$
$$1-\rho\alpha>0$$
$$(1-\varepsilon)(1-\rho)>\alpha+\eta$$
$$(1-\varepsilon)(1-\rho\gamma)>\eta$$
$$1-\rho\gamma>0$$
$$1+\rho\alpha-\rho\gamma>0\ .$$

The determination of a threshold value depends on the existence of an exogenous expression for a steady state level of human capital. The condition for the evolution of human capital to have a steady-state is missing in this framework, because adult agents are forced to allocate a share of their private resources under the assumption of missing credit markets. Then, this condition on the ratio is expressed as:

$$\left\{\frac{h'^g}{h'^p}\right\} = \left\{\frac{\Omega^g}{\Omega^p}\right\}\left[\frac{h^\gamma \overline{h^\alpha}}{h^{\alpha+\gamma}}\right] = \Phi\frac{\overline{h^\alpha}}{h^\alpha} \ < 1 \ .$$

If Ω^g and Ω^p were equal, the corresponding steady state would be determined by the following equation :

$$h^* = \Phi^{\frac{1}{\alpha}}\overline{h} \ = \left\{\frac{(1-\varepsilon)(1-\rho)-(\eta+\alpha)}{(1-\varepsilon)(1-\rho\gamma)-\eta}\right\}^{\frac{\eta}{\alpha}}\left\{\frac{(1-\rho\gamma)\ [(1-\varepsilon)(1-\rho)-(\alpha+\eta)]}{[1+\rho\alpha-\rho\gamma]\ [(1-\varepsilon)(1-\rho\gamma)-\eta]}\right\}\ \overline{h}$$

We note that if $\alpha = \eta$, $\gamma = 0$, then $\Omega^p > 1$ and $\Omega^p > \Omega^g$.

Proof: see Appendix (A.3) .

In our model, the individuals are never indifferent between private and egalitarian public regime : as we assume that $\Omega^g < \Omega^p$. Children's endowment in the public and in the private education system cannot be equal. Following Glomm and Ravikummar (1992), and Cardak (1999), All the parents who have an amount of human capital below the mean $\overline{h}$ would prefer the public education regime, in order their children to receive at least the average level of human capital produced by the society. The number of parents who have an amount of human capital over the mean would then opt for the private education regime. As the distribution of human capital among parents is characterized by a certain degree of heterogeneity, the medium voter will choose, in a majority-voting setting, the public education regime; (the medium voter belongs to the less endowed social group). Then every social group, even those who prefer the private regime have to finance public education. The public choice does not, however, prevent the most fortunate class from devoting a larger amount of resources to the education of their children . Heterogeneity and inequality in the distribution of income in the economy are then able to persist, as the human capital production function and the law of motion of income clearly show. In the public regime, education funds depend more on average income than on individual income: agents whose level of human capital and income are below the mean receive greater education expenditure. The tax implementation in the public regime allows to increase the net income of less endowed agents who are then able to increase their bequests .The offspring receive greater transfers from the adults while the constraints on the accumulation of human capital by getting an education is relaxed . The marginal profit of acquiring knowledge and skills during schooling time grows faster for such individuals. However, according to Aghion and Williamson (1998), such tax policies may reduce incentives to get education as the wage gap between high-skilled jobs and low-skilled jobs get smaller. The aim of these policies is to satisfy a growing demand for public services providing training and schooling . A key issue in the matter is the level of the earnings in the job market and the wage differential. An incentive to raise investment in the education of each child is provided by a relatively high level of wages for higher-skilled jobs. We are now able to set a formal comparison of the alternative regimes in terms of indirect utility and welfare, using the Bellman value functions which incorporates the constant coefficients in the private and in the public regimes. By Merton's principle, the value functions are given by :

$$U(u_0^p, u_1^p) = u_0^p + u_1^p \; log \; h$$
$$U(u_0^g, u_1^g) = u_0^g + u_1^g \; log \; h$$

where $u_1^p = \frac{1}{1-\rho}$ and $u_1^g = \frac{1}{1-\rho\gamma}$

Plugging into the bellman equations, we find that the value function is greater under the private regime if:

$$h \; > e^{\frac{(1-\rho)(1-\rho\gamma)}{\rho(1-\gamma)}(u_0^g - u_0^p)}$$

where u_0^g and u_0^p are exogenous coefficients .

Proof: see Appendix A.3.

The welfare of agents depends on the bellman constant coefficients and on the mean of human capital initial distribution, $m_t = E_t(\log h_t)$. Assigning equal weight to the utilities functions of all agents allows to compare the average level of utility: therefore, the sufficient condition for welfare to be greater under the private education regime is given by:

$$W_t^p = u_0^p + u_1^p \, m_t \, .$$

$$W_t^g = u_0^g + u_1^g \, m_t + u_2^g m_t \, .$$

where $u_2^g = u_1^p - u_1^g$,

and

$$u_0^p = \frac{1}{1-\rho}\left\{\log(1-\delta^p) + \log l^p + \rho(1-\varepsilon)\log n^p + \frac{\rho u_1^p}{1-\rho} E \log \Omega^p + \frac{\rho u_1^p}{1-\rho}\frac{s^2}{2}\right\}$$

$$u_0^g = \frac{1}{1-\rho\gamma}\left\{\log(1-\tau) + \log l^g + \rho(1-\varepsilon)\log n^g + \frac{\rho u_1^g}{1-\rho\gamma} E \log \Omega^g + \frac{\rho\alpha u_1^g \Delta^2}{2(1-\rho\gamma)} + \frac{\rho (u_1^g)^{\,2}}{1-\rho\gamma}\frac{s^2}{2}\right\}$$

We find that a sufficient condition for welfare under the private education regime to be greater than under the public one is $u_0^p - u_0^g > 0$, (see Appendix A.4), that is to say, if the gap between $\left\{\frac{\rho}{1-\rho} E_t \log \Omega^p\right\}$ and $\left\{\frac{\rho}{1-\rho\gamma} E_t \log \Omega^g\right\}$, which are positive functions of education expenditures, is greater than the positive functions of standard deviations of both log normally distributed variables, (human capital and learning ability). The last two terms are a measure of the loss in agent's utility caused by a low level of children's learning abilities. The Bellman constant coefficient u_0^p is greater if the gain in agent's utility, is greater under the public regime. This is an effect of the constraint imposed by the abscence of market to insure against low level of child ability: a rise in intergenerational transmission of human capital follows a rise in low ability risk. Formally, incentives and policies provided by a change in regime, given the risks on learning abilities and distribution of human capital, depend on heterogeneity parameter.The key variables in incentives for agents to stop to rely exclusively on private resources to raise funds to finance education are then $\{\Delta^2\}$, the bellman values and the average level of human capital. The Bellman value functions capture the effect of random distribution of ability and parent's altruist behavior towards their offspring.

3. The Evolution of Human Capital and Income .

Following Glomm and Ravikummar (1992), the inheritance of human capital allows for both endogenous growth and for the transmission of inequalities across generations over time. Persistence of heterogeneity is allowed by the human capital production function. In such similar framework, a steady state can occur when parental income becomes constant over time and equal to h. Considering the difference equation of human capital motion, a steady state with constant growth can still occur when $\alpha + \gamma = 1$. Even if there are diminishing returns to the factor h_{it}, there are overall constant returns to both factors that are supposed to grow over time. If we assume that $\eta + \alpha + \gamma = 1$, then, if $\eta = 0$ the transition equation of human capital becomes[(5)] :

$$h_{it+1} = \kappa \epsilon_{it+1} (\tfrac{\tau l^{*g}}{n^{*g}})^{1-\gamma} \, \overline{h}^{1-\gamma} \, h_{it}^{\gamma} \, .$$

$$\frac{h_{it+1}}{h_{it}} = \kappa \epsilon_{it+1} (\tfrac{\tau l^{*g}}{n^{*g}})^{1-\gamma} \left\{ \frac{\overline{h}}{h_{it}} \right\}^{1-\gamma} \, .$$

We consider now the initial assumption of a log-normally distributed human capital, to analyse the evolution of human capital, income and heterogeneity in the short and in the long run. It is assumed that individual human capital h_{it} and income $y_{it=}h_{it}l_{it}$ remain log-normally distributed over time. If $log\ h_{it} \sim N(m_t, \Delta_t^2)$ the distribution of income at time t will be expressed as: $log\ y_{it} \sim N(m_t + log\ l_t,\ \Delta_t^2)$ where the difference equations of the moments m_t and Δ_t^2 are expressed in the following Proposition :

In the private education regime:

$$m^p_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^p + (\gamma+\alpha)m_t, \text{ where } \theta = \log\kappa$$

$$\Delta^{p\,2}_{t+1} = (\gamma+\alpha)^2\Delta^2_t + s^2$$

In the public education regime:

$$m^g_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^g + (\gamma+\alpha)m_t + \alpha\frac{\Delta^2_t}{2}$$

$$\Delta^{g\,2}_{t+1} = \gamma^2\Delta^2_t + s^2$$

In the long run, the heterogeneity parameter is smaller in the public education regime than in the private one. Public contribution in the provision of education allows a greater homogeneity in the economy. As the growth rate of income is a decreasing function of $\pounds^p$ and $\pounds^g$, the short run gap between the growth rates under the alternative regime is supposed to decrase in the long run, which prevents the economy from converging to a greater equity level. In the long run, the alternative levels of heterogeneity are given by:

under the public regime: $\Delta^{g\,2}(\tau) = \frac{s^2}{1-\gamma^2}$
under the private regime: $\Delta^{p\,2}(\tau) = \frac{s^2}{1-(\alpha+\gamma)^2}$

The difference equations of individual income are given by:

$$log y^p_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^p + (\gamma+\alpha)m_t + log l_{t+1} + \frac{\Delta^2_{p,t+1}}{2}$$

$$log y^g_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^g + (\gamma+\alpha)m_t + log l_{t+1} + \alpha\frac{\Delta^2_{g,t+1}}{2}$$

then, the growth rate of income is given by:

$$log\frac{y^p_{t+1}}{y^p_t} = \theta + E_t log\Omega^p - (1-\gamma-\alpha)m_t + log\ \widetilde{l_t}\ - \pounds^p \frac{\Delta_t^2}{2}$$

$$log\frac{y^g_{t+1}}{y^g_t} = \theta - (1-\alpha)\frac{s^2}{2} + E_t log\Omega^g - (1-\gamma-\alpha)m_t + log\ \widetilde{l_t}\ - \pounds^g \frac{\Delta_t^2}{2}$$

where

$$\pounds^p = [1-(\gamma+\alpha)^2]$$

$$\pounds^g = (1-\alpha\gamma^2)$$

In the long run, the evolution of income under the alternative regime is expressed as:

$$log\ y^p(\tau) = \frac{\theta + E_t log\Omega^p - \pounds^p \frac{\Delta^2}{2}}{(1-\gamma-\alpha)}$$

$$log\ y^g(\tau) = \frac{\theta - (1-\alpha)\frac{s^2}{2} + E_t log\Omega^g - \pounds^g \frac{\Delta^2}{2}}{(1-\gamma-\alpha)}$$

The determination of the growth rate of income in both regimes of education allows to compare the respective loss of efficiency per unit of heterogeneity $\pounds^p$and $\pounds^g$. The evolution of the heterogeneity parameter is described by the expressions of $\left\{log\frac{y^p_{t+1}}{y^p_t}\right\}$ and $\left\{log\frac{y^g_{t+1}}{y^g_t}\right\}$, where $\pounds^p = [1-(\gamma+\alpha)^2]$ and $\pounds^g = (1-\alpha\gamma^2)$. If we assume that $\pounds^p < \pounds^g$, $1-(\gamma+\alpha)^2 < 1-\alpha\gamma^2$, which occurs when the influence of the educational environment is high, the income grows faster in the public education system, and in this case, $log\frac{y^p_{t+1}}{y^p_t} > log\frac{y^g_{t+1}}{y^g_t}$. The growth trajectory is subject to the influence of less endowed young agents, who ihnerit relatively low amount of human capital. Here we agree with Benabou (1996) as the less educated individuals have a stronger influence on a heterogenous community than the well educated ones do. In the long run, however, the evolution is different. We determine the steady states in both education systems, and we find out that the heterogeneity variable for the public regime is lower than the heterogeneity variable for the private regime: the public regime allows faster convergence to a more homogenous society as deviation parameter declines over time as expressed by the following inequality :

$$\Delta_{\infty}^{p,\,2}(\tau) = \frac{s^2}{1-(\alpha+\gamma)^2} > \Delta_{\infty}^{g\,,2}(\tau) = \frac{s^2}{1-\gamma^2}$$

Redistribution policies could reduce poverty by increasing the human capital stock of children raised in poor families who inherit relatively low level of human capital and are affected by the intergenerational transmission of inequalities . As it is assumed that students differ in their own ability to learn, then to increase their human capital stock, (ability is unkown during schooling time), income inequality is perpetuated in the private regime as more education funds are invested on more efficient scholars. The heterogeneity is then about to grow over time . The public education regime is bound to reduce inequality as each student will receive the same amount of education, $\overline{h}$, and young individuals with lower (higher) inherited human capital will benefit from greater (lower) proportional increases in their human capital amount in the public regime. The elasticity of education funds in the human capital production function, α, is then an important parameter determine which regime is more efficient in the short and the long run. The interaction between the two engines of growth, E_{it} and h_{it} and their rate of return have a strong influence on the process of accumulation of human capital: when $\alpha + \gamma = 1^{(6)}$, it is clear that constant return to factors of growth allow inequality to decrease over time, as the less endowed families are supposed to benefit from higher marginal return than wealthier ones . The results concerning the dynamics of wealth, income inequality and distribution of human capital show that the weight of heterogeneity in the evolution of the economy is decreasing when $\alpha + \gamma = 1$, if we compare both regimes (see Appendix A.4). The behavior of the long run income is subject to the influence of the difference $|\Omega^p - \Omega^g|$ as we do notice that $log\ y^p(\tau) > log\ y^g(\tau)$ if the gap between $\Omega^p$and Ω^g grows fast enough to overcome the gap between the respective losses in heterogeneity of both regimes, $|\pounds^p - \pounds^g|$. The loss in efficiency per unit of heterogeneity grows as the human capital level overcomes the average level produced by the economy which continues to be an incentive for the more wealthy agents to get out of the public education market. According to the results obtained in the comparison section, $\alpha \in (0,1)$ and $\gamma = 0$ is a configuration that is still compatible with the assumption $\{\alpha + \gamma = 1\}$. We have stated in the previous section that if $\{\alpha = 1\}$, the necessary conditions for the accumulation of individual human capital process to have equal education expenditures input under both regimes are not valid. Then, we notice that h^* does not converge anymore for $\{\alpha = 0\}$. Then, if $|\Omega^p - \Omega^g| = 1$, as a result, $\alpha = 0$: the assumption of equal level of education quality in both regimes is in contradiction with the existence of a steady state for the human capital production process. If $\alpha = 1$ and $\gamma = 0$, the offspring still receive greater education expenditures in the private education regime, while under the majority-voting decision process, the tax rate $(\tau^* > \delta^{*g})$ set by the medium voter ensures that the less endowed agents would not be harmed by a change in the provision of public goods, and $E^{*p} > E^{*g}$. If we consider the borderline case $\alpha = 1$ and $\gamma = 0$ under the private education regime, the human capital accumulation of the offsprings is not subject to the level of human capital of their parents, but only to the education expenditure. Under this assumption, the human capital of the offspring in the public education regime depends on the average level of human capital, and not only on the bequest left by their parents. Lowering or cancelling the effect of private variables on household's income induce a decrease in heterogenity in the long run. The intergenerational transmission of inequalities that affects the human capital accumulation process of young agents is bounded, as the reference value for the medium voter is the average human capital level. In a case of a private education market, education goods can be provided and purchased without any government envolvment. Government may choose to fund some level of provision common to all agents, and might also prevent agents from getting private insurance (in this setting, the abscence of such market is clearly a liquidity constraint, given the distribution of human capital and endogenous learning abilities of children). In this framework, government policy is set by majority voting rule. Voting equilibrium exists and the point of maximum utility for the median voter (in this case, the level of public expenditure financed by a tax), is chosen. Futhermore, for all voters with above mean income, the preferred expenditure would be $E^* = 0$. (If it is publicly provided, the per capita provision is financed by proportional tax). Redistribution would increase incentives to invest if redistribution allowed people to overcome the effects of credit and insurance market failure. If the bugdet of education grows, it can lower the effects of liquidity constraints that prevent agents from investing in human capital. Parent's utility may depend on bequests, on human capital level of their offspring, on the fractions of time they choose to devote to their education .

4 The Accumulation of Human Capital: The Case of a Poor Household .

The Model .

In this section we a study a simple model of human capital accumulation and optimal time allocation decision in a poor economy . The preferences of the agents do not incorporate an endogenous rate of fertility, as we do not focus on the effect of the number of children on the evolution of parental income, althought they are assumed to be altruistic towards the offspring. There is no government intervention or majority voting process to set a redistributive tax.The individual human capital production depends on the inherited human capital, on schooling time, and on expenditures needed to benefit from education. The social policy depends on decisions of the poor households, and on the choice concerning their children's time allocation. It is assumed that parental income is fixed and is given by their initial human capital endowment. There is no time devoted to the rearing of children, as in this framework, they allocate their one unit of time to work, being unable to devote a fraction of their time to the rearing and the education of their children. Each adult is endowed with a level of human capital $\overline{h}$, and rears a child whose cost of education is λq . In a poor economy, the level of parent's income is supposed to be too low to provide the whole amount of income needed to finance education expenditures: children have to provide a part of these resources by devoting an amount of time $(1-\nu)$ to work. Each child lives two periods and earns a wage $\{w_{\sigma}, w_{\sigma'}\}$ working in the sector σ or in the sector $\sigma^{'}$. The sector $\sigma^{'}$ provides higher retribution but has an higher degree of hazard (Dessy and Palage(2001)). Children enter sector $\sigma^{'}$ if the cost of education becomes too high for parental income. The individual accumulation of a child's human capital depends on time allocated to schooling, on the parent's human capital, and on the quality of education determined by the education expenditure level q. When adult, the human capital for the sectors σ and σ' under the assumption s : $\varepsilon > 0, \quad 0 < \gamma < 1, \quad 0 < \delta < 1, \quad \alpha > 0$, is given by :

$$h_{t+1,\sigma} = \varepsilon q_t^{\gamma} \nu_t^{\delta} \overline{h}^{\alpha} \;\; if\ \lambda = 1 \text{ , where } \varepsilon \text{ is a positive constant .}$$

$$h_{t+1,\sigma'} = \varepsilon \overline{q}_t^{\gamma} \nu_t^{\delta} \overline{h}^{\alpha} \;\; if\ \lambda > 1$$

Household Preferences and optimal decision rules :

We assume that the level of human capital of an individual adult during period $t+1$ is partly determined during period t through schooling time, working time, bequests and the level of parental human capital, which equals the level of parental income. All parents have log-linear preferences which depends upon their level of consumption, and upon their offspring's human capital when reaching the adult age. They have no means, given the market and resource constraints, to substitute child education for child labor. In such a poor economy, there is no technological progress to create a sustainable, increasing wage differential between parental and child labor (Hazan and Berdugo (2001)). As the benefit of child labor decreases, households invest more funds in their offspring education, allowing children to gradually lower their working time to increase their schooling effort. We assume that in this poor economy, there is no such incentive. Following Levy-Garboua(1997), Glomm (1997), the parent's utlity function is expressed as :

$$U_{t+1,\sigma} = log\ C_{t,\sigma} + \beta\ log\ h_{t+1,\sigma}$$

where $\beta \in (0,1)$ is the time discount factor .

The parents make decisions concerning the level of expenditures and schooling time taking account of their offspring future earnings .We assume that parent's wage is given by their human capital endowment, $\overline{h}$, then the budget constraint of a household where a child is working in the sector σ is written as :

$$C_{\sigma} + \ q \leqslant \overline{h} + (1-\nu) w_{\sigma} \ .$$

This allows to write parental's decision and optimal allocation problem :

$$\underset{\{q,\nu\}}{max}\ U_{t+1,\sigma} = log\ C_{t,\sigma} + \beta\ log\ h_{\cdot t+1,\sigma}$$

$$s.t.\quad C = \overline{h} + (1-\nu)w_\sigma - q\ .$$

$$h_\sigma = \ \varepsilon q^\gamma \nu^\delta \overline{h}^\alpha$$

$$y_\sigma = \overline{h}$$

Integrating the equality constraints in this program leads to the dynamic program :

$$V_\sigma(\overline{h}, w_\sigma, \sigma) = \underset{\{q,\nu\}}{max}\ \ log[\overline{h} + (1-\nu)w_\sigma - q]\ + \beta\ log\ h_\sigma \varepsilon q^\gamma \nu^\delta \overline{h}^\alpha\ .$$

The first order conditions for parental optimal choice problem are given by :

$$\frac{-1}{\overline{h}+(1-\nu)w_\sigma - q} + \frac{\beta\gamma}{q} = 0 \qquad \frac{-w_\sigma}{\overline{h}+(1-\nu)w_\sigma - q} + \frac{\beta\delta}{\nu} = 0$$

this leads to the following results for human capital accumulation and equilibrium time allocation:

$$q^* = \theta\gamma[\overline{h} + w_\sigma]$$

$$\nu^* = \frac{\theta\delta}{w_\sigma}[\overline{h} + w_\sigma]$$

where $\theta = \frac{\beta}{1+\beta(\gamma+\delta)}$. Therefore, the optimal accumulation of human capital is given by :

$$h^*_\sigma = \Omega[\overline{h} + w_\sigma]^{\ \gamma+\delta}\overline{h}^{\alpha}$$

where $\Omega = \frac{\varepsilon(\beta\gamma)^{\gamma+\delta}\gamma\delta^{\delta}}{[1+\beta(\gamma+\delta)]^{\gamma+\delta}}\frac{1}{w^{\delta}_\sigma} = \varepsilon\left\{\theta\gamma\right\}^{\gamma}\left\{\frac{\theta\delta}{w_\sigma}\ \right\}^{\delta} = \varepsilon\left\{\theta\right\}^{\gamma+\delta}\left\{\frac{\delta}{w_\sigma}\ \right\}^{\delta}\gamma^{\gamma}$

The set of optimal decisions when a child's income is w'_σ is then :

$$\overline{q}'^{*} = \frac{\theta\gamma}{\lambda}[\overline{h} + w_{\sigma'}]$$

$$\nu'^{*} = \frac{\theta\delta}{w_{\sigma'}}[\overline{h} + w_{\sigma'}]$$

$$h^*_{\sigma'} = \ \varepsilon\left\{\frac{\theta\gamma}{\lambda}\right\}^{\gamma}\left\{\frac{\theta\delta}{w_{\sigma'}}\ \right\}^{\delta}\ [\overline{h} + w_{\sigma'}]^{\ \gamma+\delta}\overline{h}^{\alpha}$$

where $\Omega' = \varepsilon\left\{\frac{\theta\gamma}{\lambda}\right\}^{\gamma}\left\{\frac{\theta\delta}{w_{\sigma'}}\ \right\}^{\delta} = \varepsilon\left\{\theta\right\}^{\gamma+\delta}\left\{\frac{\delta}{w_\sigma\lambda}\ \right\}^{\delta}\gamma^{\gamma}$

In this model, the weight of investment on schooling has a negative effect on liquidity constraint of households: a higher value of λ, by increasing the education cost, increases in turn the fraction of parental resources devoted to education finance. Therefore, parents cannot avoid child labor during schooling years, because such a lack of resources would prevent them from investing on education expenditure, then from raising enough funds to finance education. A rise in the cost of education is supposed to rise the fraction of household's income needed to allow children to remain into an education system. Therefore, children have to supply labour in the sector σ' to increase their earnings, taking more risk while working, and reducing the share of time devoted to school. In this case, children spend more time working, and less time studying, while higher wages allow the household to continue to send them to school. The following simple assumption:

$$\nu'^{*} < \nu^{*} \quad \text{if} \quad w_{\sigma'} > \ w_\sigma\ ,$$

clearly shows that the policy variables are inefficient as an higher level of investment in education induce lower time devoted to study, which has a negative effect on human capital and future earnings. It is assumed that earnings in the sector $\sigma^{'}$ is sufficiently high to compensate for the harmful effect of this type of child labor. The wage premium for working in the sector $\sigma^{'}$rather than in the sector σ is the only incentive provided by the labor market. That wage differential is supposed to be greater than the minimal education expenditure required to send children at school, therefore, to benefit from the higher child's earning in the sector $\sigma^{'}$. According to the previous assumption concerning both wages and optimal time allocation, an increase in the wage of sector σ and σ' will in turn increase the share of time devoted to schooling if the levels of retribution satisfy the following assumptions:[(8)]

$$\frac{\partial q^*}{\partial w_\sigma} = \theta\gamma > 0 \text{ and } \frac{\partial \nu^*}{\partial w_\sigma} = \theta\delta[1 - \frac{\overline{h}}{w_\sigma^2}] > 0 \text{ if } w_\sigma > \sqrt{\overline{h}}\ .$$

$$\frac{\partial q'^*}{\partial w_{\sigma'}} = \theta\gamma > 0 \text{ and } \frac{\partial \nu'^*}{\partial w_{\sigma'}} = \theta\delta[1 - \frac{\overline{h}}{w_{\sigma'}^2}] > 0 \text{ if } w_{\sigma'} > \sqrt{\overline{h}}\ .$$

$$\frac{\partial \log h_\sigma}{\partial w_\sigma} > 0 \quad \text{if} \quad w_\sigma > \frac{\delta}{\gamma}\overline{h}\ .$$

Welfare Determination and household's policy .

According to the parental decision rules, we derive the expression of indirect utilities to set a comparison of respective welfares of households whose child works in sector σ and in sector $\sigma^{'}$:

if child works in sector σ :

$$W = [1 + \beta(\gamma + \delta)]\ log\ (\overline{h} + w_\sigma) + \Phi$$

Where

$$\Phi = \beta\delta\ log\theta\delta + \beta\gamma\ log\theta\gamma - \beta\delta\ logw_\sigma + \beta log\varepsilon + \beta\alpha log\overline{h} + log\ [1 - \theta(\gamma+\delta)]$$

and if child works in sector $\sigma^{'}$:

$$W' = [1 + \beta(\gamma + \delta)] \; log \; (\overline{h} + w_{\sigma'}) + \Phi'$$

Where

$$\Phi' = \; log \; [1 - \theta(\gamma + \delta)] + \beta\gamma log \; \theta\gamma + \beta\delta log \; \theta\delta + \beta\delta log \; w_{\sigma'} - \beta\delta log \; \lambda$$

The welfare comparison is expressed as :

$$W - W' = [1 + \beta(\gamma + \delta)] \; \left\{ log \; \frac{\overline{h}+w_{\sigma}}{\overline{h}+w_{\sigma'}} \right\} + \beta\delta log \; \left\{ \frac{w_{\sigma'} \, \lambda}{w_{\sigma}} \right\} \quad (E.1)$$

This equality leads to the following assumption:

$$W' > W \;\; if \quad \frac{w_{\sigma}}{w_{\sigma'}} > e^{\lambda} \; . \; (\mathrm{A1})$$

The households will benefit from child's work in sector σ' if the wage ratio is lower than the threshold value e^{λ}. More formally, we determine the value of this ratio when both optimal human capital accumulation processes are equal, that is to say, when households' optimal decision policies have the same effect on their offsprings'future earnings, i.e, their human capital when adult. Given w_{σ} and w_{σ}', it is assumed that all parents are indifferent in their investment choice under the two regimes: they act as if their budget constraint was no more subject to the wage differential between their own income $\overline{h}$ and their offsprings'education cost $\{q, \overline{q}\}$. Under this assumption we are able to derive the two following threshold expressions :

$$\text{as} \quad \frac{h^*_{\sigma}}{h^*_{\sigma'}} = \left\{ \frac{w_{\sigma}}{w_{\sigma'}} \right\}^{\delta} \left\{ \frac{1}{\lambda} \right\}^{\gamma} \left\{ \frac{\overline{h}+w_{\sigma}}{\overline{h}+w_{\sigma'}} \right\}^{\gamma+\delta} = 1,$$

$$\text{then} \quad \frac{w_{\sigma}}{w_{\sigma'}} = \lambda^{\frac{\gamma}{\delta}} \left\{ \frac{\overline{h}+w_{\sigma}}{\overline{h}+w_{\sigma'}} \right\}^{-\frac{\gamma+\delta}{\delta}}$$

according with the assumption (A1),

$$\left\{ \frac{\overline{h}+w_{\sigma}}{\overline{h}+w_{\sigma'}} \right\} < e^{\lambda \frac{\delta}{\gamma+\delta}} . \lambda^{\frac{\gamma}{\gamma+\delta}} \ .$$

We note that the sign of the wage gap depends on the parameter λ as the first member of equation (E.1) is negative because of greater earnings provided by working in sector $\sigma^{'}$. As stated previously, an increase in the wage differential between parental and child labor (i.e between $\overline{h}$ and $w_{\sigma'}$), is an incentive for households to substitute child education for child labor, allowing the economy to get out of the poverty trap. More formally, this can occur when $w_{\sigma'} > w_{\sigma} e^{\lambda}$ which, in this economy is clearly inefficient as it is supposed that the parents cannot raise enough education funds without the whole resources provided by child labor in the higher wage sector. In this poor economy, the absence of technological progress and legal restrictions on child labor prevent decision makers from choosing a set of optimal decision rules that allows to avoid children labor while raising enough funds to continue to send them to school. The households' welfare evaluation clearly shows that parental budget constraint is linked to the wage diffenrential between parents and children, and to the gap between adult's income and child's earnings: to reduce the share of time devoted to work, children would have to work in the lower wage sector, and such a decision would prevent their parents from investing more funds in his education. The altruistic agents take into account their offspring's future earnings in their policy decisions while their decisions rules are not socially efficient .

Conclusion .

The distribution of income and the persistence of inequalities accross generations provide incentives to acquire education. Such investment allows to prospect for high-skilled jobs, and to raise enough private funds to allow future generation to benefit from education. This is not the case, however, in the simple model describing a poor economy, where both income of young and old generation are linked and where this gap has a negative effect on child education quality and social condition. The first model analyses the effects of the optimal decision rules of adult agents on the human capital, income distribution and future earnings of their offspring. As the share of income devoted to education is greater under the private education, this private regime is still more efficient, although the agents prefer the public egalitarian regime in terms of welfare. The agents spend less time educating children and less time working under the egalitarian regime, while in a poor economy, children devote a greater amount of time to work when parental income (i.e. liquidity constraint), is too low to provide enough funds given education costs. In the dynastic model, parental human capital matters for the offspring's future earnings, while in the second model, parental decision rules concerning children's time allocation and type of work have strong effects on future generations' income. The parents cannot separate out their own income from their offspring's: both are linked and determine their level of human capital when adult. The gap between parental income and children's wage is a constraint that determines a minimal level of bequest to children in order to benefit from education: such constraint is relaxed when there is a rise in household's income. As parents devote their unit of time to work, children have to enter higher wage sector, which is a disincentive to get education. The prospect of low skilled job which provide enough retribution to relax the liquidity constraint may discourage long term investment such as education.

Appendix .

A.1: the equilibrium .

We derive the first-order condition for optimal labour supply, optimal tax rate, and optimal time allocated to education : We use the Bellman principle of optimization along with the derivation chain rule :

the first-order condition for optimal labour supply:

$$\frac{\rho(1-\epsilon)}{1-l} - \frac{1}{l} = \rho.E\left\{u(h_{t+1})^{1-\nu}.[\frac{\partial \log u(h_{t+1})}{\partial \log h_{t+1}} . \frac{\partial \log h_{t+1}}{\partial \log n}\right\} \frac{1}{Eu(h_{t+1})^{1-\nu}}$$

$$\frac{\rho(1-\epsilon)}{1-l} - \frac{1}{l} = -\frac{l}{l(1-l)} \rho\alpha.E\left\{u(h_{t+1})^{1-\nu}.u_1^p\right\} \frac{1}{Eu(h_{t+1})^{1-\nu}}$$

the first-order condition for optimal time allocation in
the education of the offspring:

$$\frac{1-\varepsilon}{x+\omega} = \frac{\eta(x+\omega)+\alpha\omega}{\omega(x+\omega)} \; E\left\{u(h_{t+1})^{1-\nu}.u_1^p\right\} \frac{1}{Eu(h_{t+1})^{1-\nu}}$$

the first-order condition for optimal tax rate:

$$\frac{\tau^g}{1-\tau^g} = \rho\alpha E\left\{u(h_{t+1})^{1-\nu}.u_1^p\right\} \frac{1}{Eu(h_{t+1})^{1-\nu}}$$

The Bellman value function for the public regime is, by Merton's principle:

$$log\ U(h_t) = u_0^p + u_1^p log\ h_t$$

The Bellman value function for the public regime is, by Merton's principle:

$$log\ U(h_t) = u_0^p + u_1^p log\ h_t + u_2^p m_t$$

where $u_1^p = \frac{1}{1-\rho}$ and $u_2^p = \frac{1}{1-\rho\gamma}$

As $log\ U(h_{t+1}) = u_0^p + u_1^p log\ h_{t+1}$, $\frac{\partial \log u(h_{t+1})}{\partial \log h_{t+1}} = u_1^p$, and $\frac{\partial \log h_{t+1}}{\partial \log n} = \frac{\alpha}{n}$

plugging into the human capital expression:

$$logh_{it+1} = log\kappa\epsilon_{it+1} + \eta log\omega_{it} + \alpha\ log\tau + \alpha\ loghl\ -\ \alpha\ log\ n\ +\gamma\ log\ h_{it}$$

Then we compute the following partial derivatives:

$$\frac{\partial \log h_{t+1}}{\partial \log \omega} = \frac{\eta}{\omega}$$
$$\frac{\partial \log h_{t+1}}{\partial \log \delta} = \frac{\alpha}{\delta}$$
$$\frac{\partial \log h_{t+1}}{\partial \log \tau} = \frac{\alpha}{\tau}$$

As $u_1^{p*} = \frac{1}{1-\rho}$ and $u_1^{g*} = \frac{1}{1-\rho\gamma}$, the Bellman's constants, plugging into the policy variables:

$$l^{*p} = \frac{1+\rho\alpha u_1^p}{1+\rho(1-\varepsilon)]} = \frac{1-\rho+\rho\alpha}{(1-\rho)[1+\rho(1-\varepsilon)]}$$
$$\tau^{*p} = \frac{\rho\alpha u_1^p}{1+\rho\alpha u_1^p} = \frac{\rho\alpha}{1-\rho+\rho\alpha}$$
$$\omega^{*p} = \frac{x\eta u_1^p}{(1-\varepsilon)-(\alpha+\eta)u_1^p} = \frac{x\eta}{(1-\rho)(1-\varepsilon)-(\alpha+\eta)}$$
$$n^{*p} = \frac{\rho[(1-\varepsilon-(\alpha+\eta)u_1^p]}{x[1+\rho(1-\varepsilon)]} = \frac{\rho[(1-\varepsilon)(1-\rho)-(\alpha+\eta)]}{(1-\rho)x[1+\rho(1-\varepsilon)]}$$
$$l^{*g} = \frac{1}{1+\rho(1-\varepsilon)}$$
$$\tau^{*g} = \frac{\rho\alpha u_1^g}{1+\rho\alpha u_1^g} = \frac{\rho\alpha}{1+\rho\alpha-\rho\gamma}$$
$$\omega^{*g} = \frac{x\eta u_1^g}{(1-\varepsilon)-\eta u_1^g} = \frac{x\eta}{(1-\rho\gamma)(1-\varepsilon)-\eta}$$
$$n^{*g} = \frac{\rho[(1-\varepsilon-\eta u_1^g]}{x[1+\rho(1-\varepsilon)]} = \frac{\rho[(1-\varepsilon)(1-\rho\gamma)-\eta]}{(1-\rho\gamma)x[1+\rho(1-\varepsilon)]}$$

A.2 *Constant and Decreasing Returns.*

If $\alpha = \eta$ and $\gamma = 0$, then:

$(1-\varepsilon)(1-\rho) - (\alpha+\eta) < (1-\varepsilon) - \eta$

we find that $\rho(\varepsilon - 1) < \alpha$ as $\varepsilon < 1$.

and $\frac{1}{1+\rho} < 1$ is given by the second member of equation

defining h. Then, $\Omega^p > \Omega^g$.

We notice that if $\alpha \neq \eta$ these assumptions, the corresponding values of the parameters will be :

$(1-\varepsilon)(1-\rho) - (\eta+\alpha) = (1-\varepsilon)(1-\rho\gamma) - \eta$

$(1-\rho\gamma)[(1-\varepsilon)(1-\rho) - (\alpha+\eta)] = (1+\rho\alpha-\rho\gamma)[(1-\varepsilon)(1-\rho\gamma) - \eta]$

then,

if $\alpha = 0,\ \gamma = 1$,

$(1-\rho\gamma) = (1+\rho\alpha-\rho\gamma)$

We notice that:

$lim_{\alpha\to 0}\ h^* \ = \infty$.

$log\ h_{it+1} = log\ \kappa\epsilon_{it+1\,+}\ \eta\ log\frac{x\eta}{(1-\rho)(1-\varepsilon)-(\alpha+\eta)}\ + log\ h$.

A steady state can occur when $\alpha + \gamma = 1$, it is still the case when $\alpha = 0$ and $\gamma = 1$.
The parental expenditure on children education is then normalized to one in the human capital production function.

We check the bordeline case :

if $\gamma + \alpha = 1$, then $lim_\infty \, log \, y^u(\tau) = \infty$.

If $\alpha = \eta$ and $\gamma = 0$, then:

$(1-\varepsilon)(1-\rho) - (\alpha+\eta) < (1-\varepsilon) - \eta$
we find that $\rho(\varepsilon - 1) < \alpha$ as $\varepsilon < 1$.
and $\frac{1}{1+\rho} < 1$

A3 *The Steady State.*

The threshold value of the human capital is given by :
$h^* = \Phi^{\frac{1}{\alpha}} \overline{h}$
$\left\{\frac{h'^g}{h'^p}\right\} = \left\{\frac{\Omega^g}{\Omega^p}\right\} \left[\frac{h^\gamma \overline{h^\alpha}}{h^{\alpha+\gamma}}\right] = \Phi \frac{\overline{h^\alpha}}{h^\alpha} \; = 1$.

If we assume instead that : $\left\{\frac{h'^g}{h'^p}\right\} = \left\{\frac{\Omega^g}{\Omega^p}\right\} \left[\frac{h^\gamma \overline{h^\alpha}}{h^{\alpha+\gamma}}\right] = \Phi \frac{\overline{h^\alpha}}{h^\alpha} \; > 1$,
then $\Phi \overline{h^\alpha} > h^\alpha$,
and $\Phi^{\frac{1}{\alpha}} \overline{h} > h$, finally, $h < \Phi^{\frac{1}{\alpha}} \overline{h} = h^*(\varepsilon, \ \rho, \ \alpha, \ \eta, \ \gamma, \ \rho; \overline{h}\ )$.
If $\left\{\frac{h'^g}{h'^p}\right\} < 1$, then $h > \Phi^{\frac{1}{\alpha}} \overline{h} = h^*(\varepsilon, \ \rho, \ \alpha, \ \eta, \ \gamma, \ \rho; \overline{h}\ )$.
Then, if parental income is below the threshold, their offspring's income will be higher in the public education regime.

A.4 *The Difference Equation of Mean and Variance Under Constant Returns.*

We assume that $(\alpha + \gamma) = 1$, then we derive the difference equations for mean and variance:

$m^p_{t+1} = \theta - \frac{s^2}{2} + E_t log \Omega^p + m_t$

$\Delta^2_{p,t+1} = \Delta^2_{p,t} + s^2$

$m^g_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^g + m_t + \alpha\frac{\Delta^2_t}{2}$

$\Delta^2_{g,t+1} = \gamma^2\Delta^2_{g,t} + s^2$

$logy^p_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^p + m_t + logl_{t+1} + \frac{\Delta^2_{p,t+1}}{2}$

$logy^g_{t+1} = \theta - \frac{s^2}{2} + E_t log\Omega^g + m_t + logl_{t+1} + \alpha\frac{\Delta^2_{g,t+1}}{2}$

$log\frac{y^p_{t+1}}{y^p_t} = \theta + E_t log\Omega^p + log\ \widetilde{l_t}$

$log\frac{y^g_{t+1}}{y^g_t} = \theta - (1-\alpha)\frac{s^2}{2} + E_t log\Omega^g + log\ \widetilde{l_t}\ - (1-\alpha\gamma^2)\frac{\Delta^2_t}{2}$

In the long run, we find out the following heterogeneity coefficients:

$\Delta^{g,2}(\tau) = \frac{s^2}{1-\gamma^2}$

$\Delta^{p,2}(\tau) = \infty$

$\Delta^{p,2}(\tau) > \Delta^{g,2}(\tau) = \frac{s^2}{1-\gamma^2}$

The degree of heterogeneity is greater under the private education regime in the long run, under the assumption: $\alpha + \gamma = 1$.

A5 *The Interpretation of Value Functions.*

$U(u^g_0, u^g_1) > U(u^p_0, u^p_1)$

we obtain:

$$logh > \frac{(1-\rho)(1-\rho\gamma)}{\rho(1-\gamma)}(u_0^g - u_0^p)$$

therefore,

$$h > e^{\frac{(1-\rho)(1-\rho\gamma)}{\rho(1-\gamma)}(u_0^g - u_0^p)}$$

A.6 *Welfare Evaluation.*

$$W_t^p = u_0^p + u_1^p \int log\ h_{it}di = u_0^p + u_1^p\ E \log\ h_{it}di = u_0^p + u_1^p\ m_t\ .$$

$$W_t^g = u_0^g + u_1^g \int log\ h_{it}di + u_2^g m_t = u_0^g + u_1^g\ E \log\ h_{it}di = u_0^g + u_1^g\ m_t + u_2^g m_t\ .$$

Then, we compute the difference:

$$u_0^p - u_0^g > u_1^g\ m_t + u_2^g m_t - u_1^p\ m_t\ .$$

$$u_0^p - u_0^g > \frac{\rho(1-\gamma)}{(1-\rho)(1-\rho\gamma)} m_t - \frac{\rho(1-\gamma)}{(1-\rho)(1-\rho\gamma)} m_t = 0$$

$$u_0^p - u_0^g > 0\ .$$

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Notes

(1) We do not take into account the wages of adult agents as their level of labour supply and human capital is a measure of households'income.

(2) This condition depends on the overall rate of return as under the assumption of constant returns, $\alpha+\gamma+\eta=1$, if $\alpha=0, \eta=0$ are the steady state existence conditions, then it is clear that $\gamma=1$.

(3) If $\alpha+\gamma+\eta=1,\ \alpha=1,\ \gamma=0,\ \eta=0$, then,

$\frac{1-\varepsilon}{(1-\varepsilon)(1-\rho)-1} > 1$ as $\frac{1}{1+\rho} < 1$ and $1+\rho > 1$.

When $\eta \in (0,1)$, however, the condition is:

$\rho(\varepsilon-1) < 1$ which is always true as $\varepsilon < 1$.

If $\alpha=0$, then the second term

$\frac{1-\rho\gamma}{1+\rho\alpha-\rho\gamma} = 1$, then it is correct to write: $\frac{1-\rho\gamma}{1+\rho\alpha-\rho\gamma} \leqslant 1$.
under both assumptions $\alpha=(0,1)$.

If $\gamma=0$, then $\forall\alpha \in (0,1)$, $\frac{1-\rho\gamma}{1+\rho\alpha-\rho\gamma} < 0$.

(4) If the level of education expenditure is equal under both regimes, then the ratio $\frac{\Omega^p}{\Omega^g} = 1$ if $\alpha=\eta=0$.
It is not a sufficient condition, however, for the existence of a steady state.

(5) The difference equation of human capital depends on the ratio of average level of human capital to first period human capital It is still the case under the assumption $\eta = 0$, but not if $\gamma = 1$: then, the difference equation depends on log-normal learning ability variable.

(6) The assumption of overall constant return to education inputs does not change when $\eta = 0$.

(7) This assumption is in contradiction with condition $\left\{\frac{\Omega^p}{\Omega^g} = 1\right\}$, although it allows h^* to converge.

(8) The threshold value is the same in both child labor regime, as the equilibrium decision rules are not affected by a change in regime.