

# Financial Development, Financing Choice and Economic Growth\*

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August 31, 2001

## Abstract

In an overlapping generations economy households (lenders) fund risky investment projects of firms (borrowers) by drawing up loan contracts on the basis of asymmetric information. An optimal contract entails either the issue of only debt or the issue of both debt and equity according to whether a household faces a single or a double enforcement problem as a result of its own decision about whether or not to undertake costly information acquisition. The equilibrium choice of contract depends on the state of the economy which, in turn, depends on the contracting regime. Based on this analysis, the paper provides a theory of the joint determination of real and financial development with the ability to explain both the endogenous emergence of stock markets and the complementarity between debt finance and equity finance.

## 1 Introduction

Over the past decade, a substantial volume of research has been devoted towards verifying and understanding the existence of linkages between the real and financial development of economies. One such linkage is evidenced by the strong positive (and possibly two-way causal) correlation between long-run economic growth and the degree of financial intermediation (e.g., De Gregorio and Guidotti 1992; King and Levine 1993a,b; Levine and Renelt 1992;

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\*The authors are grateful for the financial support of the ESRC (Grant No.R000222871). The usual disclaimer applies.

Roubini and Sala-i-Martin 1991). Another is demonstrated by the similar correlation between long-run growth and stock market activity (e.g., Atje and Jovanovic 1993; Demirgüç-Kunt and Levine 1996a,b; Levine and Zervos 1995, 1996, 1998). Broadly speaking, both of these relationships may be explained by appealing to the opportunities made available for channelling a larger fraction of savings into investment and for undertaking more productive types of investment as a result of costly improvements in financial arrangements which wealthier economies are better able to afford. In the case of financial intermediation, to which most of the theoretical literature is devoted, these opportunities may arise from a greater pooling of risks, a higher quality of information, a lower cost of monitoring and a lower cost of transactions (e.g., Bencivenga and Smith 1991; Blackburn and Hung 1998; Bose and Cothren 1996, 1997; Boyd and Smith 1992; Cooley and Smith 1998; De La Fuente and Marin 1994; Greenwood and Jovanovic 1990; King and Levine 1993b; Sussman 1993; Sussman and Zeira 1993; Zilibotti 1994). In the case of stock markets, where the body of theory is a little thinner, they may reflect a wider diversification of portfolios and a re-direction of resources towards longer run, less liquid but higher yielding projects (e.g., Bencivenga *et al.* 1995; Devereux and Smith 1994; Levine 1991; Obstfeld 1994; Saint-Paul 1992).<sup>1</sup>

Each of the above relationships may be viewed as being particularly relevant for a certain class of economies - for example, developing economies (where commercial and central banks are the dominant financial institutions), or more advanced economies (where the market capitalisation of firms is much higher). What is notable is that, almost without exception, each relationship has been studied in isolation with no connection to the other. Yet it has been widely recognised for some time that financial development is a multi-faceted process that takes place through various distinct stages - from the emergence and expansion of debt-oriented finance to the materialisation of stock markets and the increasing use of equity as an additional instrument by which firms are able to raise funds (e.g., Goldsmith 1969; Gurley and Shaw 1955, 1960). Modelling this process has so far eluded the attention of most researchers and there remains little by way of a complete account of events that lead an economy to undergo transition from a financial system based wholly or predominantly on the issue of debt to one involving a much greater (though not exclusive) reliance on the issue of equity.

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<sup>1</sup>Of course, there are exceptions to the conventional wisdom that financial development is necessarily conducive to real development. In some of the above models, for example, a reduction in uncertainty due to a greater sharing of risks may reduce precautionary savings by far enough to reduce growth as well (e.g., Bencivenga and Smith 1991; Devereux and Smith 1994; Obstfeld 1994).

The only exceptions to the above that we know of are the contributions by Boyd and Smith (1996, 1998). These authors develop a dynamic general equilibrium model in which producers of capital choose between two different types of technology that are financed in two different ways. The first type of technology is one that yields a relatively low expected return, is publicly observable and is financed by means of equity at no expense. The second type of technology is one that yields a relatively high expected return, is not directly observable by lenders and is financed by means of debt subject to a standard costly state verification problem (e.g., Diamond 1984; Townsend 1979).<sup>2</sup> Assuming plausible parameter values, it is shown that there is a critical level of per capita income below which only a debt market exists. As capital accumulation takes place, however, the cost of state verification increases due to a fall in the relative price of capital. Eventually (i.e., once the threshold level of income is reached), a stock market emerges as firms begin to make more use of the observable technology and less use of the unobservable technology, implying an increase in the amount of equity finance relative to debt finance.

The present paper shares the same basic objective as that of Boyd and Smith (1996, 1998) - namely, to model the co-evolution of the real and financial sectors of an economy in a way that enables one to explain why the emergence of stock markets occurs relatively late in this process and why debt markets continue to remain active thereafter. Our approach to this is based on an analysis that is quite different, however, from the analysis presented by those authors. We imagine an economy in which households (lenders) fund risky investment projects of firms (borrowers) by drawing up loan contracts involving some optimally-determined combination of debt and equity. Capital market imperfections arise due to asymmetric information: each household is unable to observe (and therefore control) directly both the type of project selected by a firm and the level of effort that a firm devotes to running its project. The first of these difficulties can be resolved if a household is prepared to spend a fixed amount of resources on choosing a project, itself, in which case the household is confronted by a single enforcement problem in terms of the firm's input of effort. If, on the other hand, a household is not willing to undertake such action, then it is faced with a double enforcement problem over a firm's selection of both its effort and project. We

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<sup>2</sup>In the type of framework considered by the authors, the assumption of two technologies (the innovation of their analysis) is crucial. If there was only a single (unobservable) technology, then a stock market would never emerge since the verification costs associated with the use of debt would always be lower than the verification costs associated with the use of equity because debt claims, unlike equity claims, make contractual payments contingent on a firm's performance only in the event of bankruptcy.

show that the optimal contract that solves the single enforcement problem is a pure debt contract, whereas the optimal contract that solves the double enforcement problem is a mixed debt-equity contract. Significantly, the actual (preferred) choice of contract both influences and is influenced by the state of the economy such that only debt financing takes place at relatively low levels of development along a relatively low capital accumulation path, while both debt and equity financing occur at relatively high levels of development along a relatively high capital accumulation path. Accordingly, the economy displays multiple development regimes associated with different growth trajectories and different financial systems. Transition between these regimes is characterised by the endogenous emergence of a stock market which has a positive feedback effect on growth. Transition is not inevitable, however, and there exists the possibility of multiple long-run outcomes which depend fundamentally on initial conditions.

The implication of our model that both debt and equity markets are active at same time during the later stages of development may appear to be somewhat at odds with the popular belief that debt and equity represent substitute sources of corporate finance. Yet there is strong evidence to suggest that this belief is misguided and that debt and equity act more as complements to each other (e.g., Demirgüç-Kunt and Levine 1996a,b; Demirgüç-Kunt and Maksimovic 1996). In Boyd and Smith (1996, 1998) this complementarity arises because of an increase in the cost of pure debt financing to such an extent that it eventually becomes infeasible for firms to continue issuing debt without also issuing equity. In our analysis complementarity is realised as a natural outcome of lenders' optimal decisions in the face of multiple enforcement problems. A positive growth effect from the emergence of a stock market in response to these problems is explained by the cost savings to lenders from leaving project investment decisions entirely up to borrowers. This accords with the view that one of the major impediments to real economic development is the loss of resources associated with informational frictions in financial markets (e.g., McKinnon 1973; Shaw 1973).

The paper is organised as follows. In Section 2 we present a description of the economic environment. In Section 3 we define and solve the optimal loan contracting problem under different scenarios. In Section 4 we study the optimal choice of contract in dynamic general equilibrium and examine the implications for capital accumulation and growth. In Section 5 we conclude with a discussion of our results.

## 2 The Economy

Time is discrete and indexed by  $t = 0, 1, \dots, \infty$ . There is a countable infinite number of two-period-lived agents belonging to overlapping generations of non-altruistic families. Each generation is divided at birth into two groups of market participants - households (or workers, or lenders) and firms (or entrepreneurs, or borrowers). To fix ideas, we normalise the total population to 2, assume equal sized groups of mass 1, and unite newly-born lenders with newly-born borrowers in randomly-matched pairs.<sup>3</sup> Each household is a financier of a risky capital project when young, a supplier of a fixed amount of labour when young and a consumer of final output when old. Each firm is an operator of a risky project when young, an employer of labour when old and a producer and consumer of final output when old. All agents are risk neutral and all markets are competitive. We proceed with our formal description of the economy with reference to the circumstances facing agents of generation  $t$ .

Each firm begins life with zero resources, except for a unit endowment of time. Each firm has an opportunity, however, to undertake a risky investment project from which capital is produced. To exploit such an opportunity, a firm must acquire external finance from a household (of the same generation). There is a continuum of projects, indexed by  $x_t \in \mathfrak{R}_+$ , each of which gives access to a stochastic technology for converting output and effort at time  $t$  into capital at time  $t+1$ . To be precise, we assume that  $l_t$  units of loans and  $h_t \in (0, 1)$  units of entrepreneurial time may be combined to produce  $\kappa_{t+1}$  units of capital according to

$$\begin{aligned} \kappa_{t+1} &= A(x_t)l_t^\alpha h_t^\beta, \quad \alpha, \beta \in (0, 1), \alpha + \beta \leq 1, \\ A(x_t) &= \begin{cases} x_t & \text{with probability } p(x_t), \\ 0 & \text{with probability } 1 - p(x_t), \end{cases} \end{aligned} \tag{1}$$

where  $p(\cdot) \in (0, 1)$ . Differences in the riskiness of projects are captured by

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<sup>3</sup>The assumption of a one-to-one matching between lenders and borrowers is not uncommon in the literature (e.g., Bencivenga and Smith 1993; Bose and Cothren 1996, 1997) and is made in the present context largely to save on notation. As will become apparent, if a lender were to be approached by more than one borrower (each of whom is identical *ex ante*), then the lender would either divide her loanable funds equally (and on the same terms) between borrowers, or lend only to a single borrower, depending on the nature of the loan contract. Given that there are equal numbers of lenders and borrowers, the equilibrium outcome in each case would be equivalent to a one-to-one matching. Alternatively, the assumption might be justified by appealing to the existence of search costs which prohibit the break-up of any initial lender-borrower pairing. The model is also consistent with the case in which lenders delegate the task of designing and implementing contracts to a manager of a mutual fund.

the assumption that  $p'(\cdot) < 0$  so that higher yielding projects have a lower probability of being successful. A convenient specification of this innovation technology is  $p(\cdot) = e^{-\rho x_t}$  ( $\rho > 0$ ) which implies an expected level of capital equal to

$$E(\kappa_{t+1}) = e^{-\rho x_t} x_t l_t^\alpha h_t^\beta. \quad (2)$$

Thus, for given  $l_t$  and  $h_t$ , there is a unique choice of project,  $x_t = \frac{1}{\rho}$ , that maximises expected capital production.

In addition to the above, a young entrepreneur has access to a linear home production technology that enables her to convert her own labour into output.<sup>4</sup> Productivity in home production is both stochastic and contingent on whether a project is being operated at the same time: with probability  $q$ , an entrepreneur who runs a project obtains  $\phi(1 - h_t)$  units of home-produced output, while an entrepreneur who does not run a project receives  $\phi_0$  units of home-produced output; with probability  $1 - q$ , home-produced output is zero whatever the entrepreneur's circumstances. We assume that  $\phi > \phi_0$  due to positive externalities (spillovers of knowledge) from capital production to home production. This assumption is meant to capture the plausible idea that the operation of a project entails the acquisition of certain transferable skills and expertise that can be used to raise efficiency in other, more basic, productive activities.<sup>5</sup> In what follows we define  $\Phi = \phi - \phi_0$ .

The ultimate activity of entrepreneurs is the manufacture of final output in the second period of their lives using a common, non-stochastic technology. The inputs to manufacturing are labour (hired from young households of the next generation) and capital (acquired from risky investments undertaken previously by firms of the current generation). A mature entrepreneur employing  $n_{t+1}$  units of labour and  $k_{t+1}$  units of capital is able to produce  $y_{t+1}$  units of output according to

$$y_{t+1} = \Theta n_{t+1}^\theta k_{t+1}^{1-\theta} K_{t+1}^\theta, \quad \Theta > 0, \theta \in (0, 1), \quad (3)$$

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<sup>4</sup>It makes no difference to our analysis as to whether one assumes that entrepreneurs consume this output in the first or second period of their lives. We opt for the latter (envisaging either a storage technology or a one period lag in home production) merely for the sake of comparability with the consumption profile of households.

<sup>5</sup>This idea has featured in other analyses which emphasise the potential benefits from the transference of skills acquired in one task to other occupations (e.g., Jovanovic and Nyarko 1996). Empirically, it has been observed that firms are often able to increase the productivity of labour in some of their branches by drawing on various knowledge and expertise gained in others (e.g., Blomstrom *et al.* 1994). The idea may also be captured by assuming that the operation of a project raises the probability of successful home production.

where  $K_{t+1}$  denotes the aggregate stock of capital.<sup>6</sup> Labour is hired at the competitively-determined wage rate  $w_{t+1}$ , while capital is rented at the competitively-determined interest rate  $r_{t+1}$ . If an entrepreneur produced  $\kappa_{t+1}$  units of capital when young, then she is a net borrower of capital if  $k_{t+1} - \kappa_{t+1} > 0$  and a net lender of capital if  $k_{t+1} - \kappa_{t+1} < 0$ . Her profit is therefore  $\pi_{t+1} = \Theta n_{t+1}^\theta k_{t+1}^{1-\theta} K_{t+1}^\theta - w_{t+1} n_{t+1} - r_{t+1} (k_{t+1} - \kappa_{t+1})$  which, for given values of  $w_{t+1}$ ,  $r_{t+1}$ ,  $K_{t+1}$  and  $\kappa_{t+1}$ , is maximised by choosing  $n_{t+1}$  and  $k_{t+1}$  so as to satisfy  $\theta \Theta n_{t+1}^{\theta-1} k_{t+1}^{1-\theta} K_{t+1}^\theta = w_{t+1}$  and  $(1-\theta) \Theta n_{t+1}^\theta k_{t+1}^{-\theta} K_{t+1}^\theta = r_{t+1}$ . In equilibrium, where  $n_{t+1} = n$  (the fixed supply of labour) and  $k_{t+1} = K_{t+1}$ , these conditions become

$$w_{t+1} = \theta \Theta n^{\theta-1} k_{t+1}, \quad (4)$$

$$r_{t+1} = r = (1-\theta) \Theta n^\theta. \quad (5)$$

Correspondingly,  $\pi_{t+1} = r \kappa_{t+1}$  so that the choice of project which maximises expected capital production (i.e.,  $x_t = \frac{1}{\rho}$ ) is the same as that which maximises expected profits,  $E(\pi_{t+1}) = r e^{-\rho x_t} x_t l_t^\alpha h_t^\beta$ .<sup>7</sup>

Each young household is endowed with one unit of labour, part or all of which is supplied inelastically to old entrepreneurs (producing output) in return for the wage  $w_t$ . Each young household then lends all of her labour income to a young entrepreneur (producing capital) in return for a payment next period. Imperfections in the loan market arise due to asymmetric information between borrowers and lenders. Specifically, we assume that a lender is unable to observe (and therefore control) directly both a borrower's choice of project,  $x_t$ , and a borrower's allocation of time to that project,  $h_t$ . The first of these difficulties can be resolved if the lender is prepared to devote a fixed amount of her own time endowment,  $1 - \eta \in (0, 1)$ , to acquiring information and choosing a project for herself. Under such circumstances, the lender has only  $n = \eta$  units of labour with which to earn income so that the size of her loan is  $l_t = \eta w_t$ . Alternatively, if the lender does not intervene in project choice, then she loses none of her labour time so that  $n = 1$  and her loanable fund is  $l_t = w_t$ . We also assume that, while the random yield from capital production is publicly observable, the stochastic output of a borrower's home production is private information and cannot, therefore, be used to condition loan repayments.<sup>8</sup>

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<sup>6</sup>Thus we allow for an aggregate externality in the production of goods, as in many types of endogenous growth model (e.g., Romer (1986)).

<sup>7</sup>Note that the actual production of capital and the actual value of profits will both be zero if a project either fails or commands no entrepreneurial time.

<sup>8</sup>This assumption would be necessary even if a borrower was to consume her home production of output in the first period of life since observation of this output would

The precise functioning of the credit market is as follows. At the beginning of each period, a newly-born lender is approached by a newly-born borrower with a request for a loan to finance a risky investment project. The lender offers a contract to the borrower, acceptance of which implies a binding agreement that commits the former to a transfer of funds from her current wage earnings and the latter to a repayment of these funds from her future profit. As indicated above, only this profit (which is publicly observable and which reflects the realised yield of the project) can be used to set the terms of repayment, there being no other income of the borrower to which the lender can lay any contingent claim. Consequently, a firm that fails in its project but continues to produce final output is unable to make any repayment since the profit of such a firm is driven to zero by competition.

In general, a loan contract between a firm and a household allows for both debt and equity finance. The terms of such a contract state that, conditional on the success of its project, a firm must repay a certain amount,  $d_{t+1}$ , out of its profit (its debt payment) plus a share,  $s_{t+1} \in (0, 1)$ , of any remaining surplus (its equity payment). Given this, together with the above description of events, we may write the expected lifetime levels of consumption, or utility, of a firm and a household engaged in a loan agreement as, respectively,

$$E(u_t) = (1 - s_{t+1})e^{-\rho x_t}(rx_t l_t^\alpha h_t^\beta - d_{t+1}) + q\phi(1 - h_t), \quad (6)$$

$$E(v_t) = e^{-\rho x_t}[s_{t+1}(rx_t l_t^\alpha h_t^\beta - d_{t+1}) + d_{t+1}]. \quad (7)$$

The decision problem for the firm is to maximise (6), given the financial contract offered by the household, while the decision problem for the household is to maximise (7), subject to the behaviour of the firm. It is worth noting that, because of our assumption that the outcomes of projects are publicly observable, lenders do not face the same type of costly state verification problem as that studied in the early contracting literature under the opposite assumption (e.g., Diamond 1984; Gale and Hellwig 1985; Townsend 1979). In fact, our model implies that lenders could, in principle, execute loan repayments through a system of contingent claim contracts based on observed project yields. In an Appendix we characterise such contracts and show that the outcomes to which they give rise can be implemented exactly by the types of financial arrangement that we are more interested in (i.e., debt and equity contracts) and that resemble more closely the sorts of arrangement observed in actual economies.

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enable a lender to infer the borrower's effort on a project (which would contradict our other assumption).

### 3 Optimal Loan Contracts

An equilibrium loan contract is a pair of debt and equity claims,  $(d_{t+1}, s_{t+1})$ , that is feasible to implement and agreeable to both a household and a firm. At one extreme, a pure debt contract emerges if the household finds it optimal to offer  $(d_{t+1}, 0)$  and the firm is willing to accept this, in which case the loan repayment is independent of the firm's profits. At the other extreme, a pure equity contract transpires if the household's preferred offer is  $(0, s_{t+1})$  which the firm is willing to accept, implying a loan repayment that is wholly conditional on profits. Between these extremes lies the situation in which debt and equity are used optimally together as different means of finance.

Before proceeding to study problems of asymmetric information, it is instructive to consider a frictionless environment where each household is able to costlessly observe and control both the type of project operated by a firm and the level of effort that the firm devotes to this operation. This serves as a useful benchmark case in which each household achieves its first-best solution by maximising its expected utility in (7) subject only to a firm's participation (or individual rationality) constraint which requires that the firm's expected payoff in (6) is no less than its expected payoff from home production in the absence of project investment - that is,  $E(u_t) \geq q\phi_0$ . The solution is characterised by  $x_t = \frac{1}{\rho}$  and  $h_t = (r\beta x_t l_t^\alpha q\phi e^{\rho x_t})^{\frac{1}{1-\beta}} = (r\beta l_t^\alpha q\phi \rho e^{\frac{1}{1-\beta}})^{\frac{1}{1-\beta}}$ , where  $l_t = w_t$ ,  $w_t = \theta\Theta k_t$  (from (4)) and  $r = (1 - \theta)\Theta$  (from (5)). Thus the household selects that project which maximises expected capital production for any given level of effort which, in turn, is selected optimally conditional on this project and the size of the loan. The method of financing is irrelevant in this case.

In fact a household cannot impose its preferred choices on a firm without cost. Neither the type of project nor the level of effort selected by the firm, itself, is directly observable by the household. As such, the household has no immediate control over the firm's actions and problems of enforcement arise, to which an optimal loan contract must provide a solution. As indicated earlier, a household is endowed with the option of eliminating part of its uncertainty by investing  $1 - \eta$  units of its own labour time in acquiring information and securing the project of its choice. If the household exercises this option, then it is confronted by a single enforcement problem with respect to the firm's input of effort; if not, then it is faced with a double enforcement problem over the firm's selection of both effort and project. These different scenarios have different implications not only for the amount of loanable funds available, but also for the structure of the optimal financial arrangement. We consider each of them in turn.

The decision problem for a household that takes charge of project selection is defined as

$$\begin{aligned}
\mathbf{P1} : \quad & \max_{x_t, d_{t+1}, s_{t+1}} e^{-\rho x_t} [s_{t+1}(\widehat{r}x_t \widehat{l}_t^\alpha h_{f,t}^\beta - d_{t+1}) + d_{t+1}], \\
\text{s.t. } & h_{f,t} = \arg \max_{h_t} (1 - s_{t+1})e^{-\rho x_t} (\widehat{r}x_t \widehat{l}_t^\alpha h_t^\beta - d_{t+1}) + q\phi(1 - h_t), \\
& (1 - s_{t+1})e^{-\rho x_t} (\widehat{r}x_t \widehat{l}_t^\alpha h_{f,t}^\beta - d_{t+1}) + q\phi(1 - h_{f,t}) \geq q\phi_0, \\
& d_{t+1} \geq 0, \quad 0 \leq s_{t+1} \leq 1,
\end{aligned}$$

where  $\widehat{l}_t = \eta \widehat{w}_t$ ,  $\widehat{w}_t = \theta \Theta \eta^{1-\theta} k_t$  and  $\widehat{r} = (1 - \theta) \Theta \eta^\theta$ . The first constraint specifies the firm's optimal level of effort,  $h_{f,t}$ , given the behaviour of the household, while the second constraint is the firm's individual rationality condition. The solution to problem **P1**, established in an Appendix, is given by the quadruplet  $\{\widehat{x}_t, \widehat{h}_t, \widehat{d}_{t+1}, \widehat{s}_{t+1}\}$  such that

$$\widehat{x}_t = \frac{1}{\rho}, \quad (8)$$

$$\widehat{h}_t = (\widehat{r} \beta \widehat{l}_t^\alpha q \phi \rho e^{\frac{1}{1-\beta}}), \quad (9)$$

$$\widehat{d}_{t+1} = (1 - \beta) \widehat{r} \beta \widehat{l}_t^\alpha (q \phi)^\beta \rho e^{\beta \frac{1}{1-\beta}} + e q \Phi, \quad (10)$$

$$\widehat{s}_{t+1} = 0. \quad (11)$$

In this case, therefore, the optimal loan contract is a pure debt contract. The reason is straightforward. Observe that the firm's optimising response to the actions of the household is to set its level of effort according to  $h_{f,t} = \widehat{r} \beta \widehat{l}_t^\alpha q \phi e^{\frac{1}{1-\beta}}$ . Given this, then the household's best strategy is to implement directly its first-best choice of project (over which it has complete control) and to induce a level of effort as close as possible to its first-best level by setting  $s_{t+1}$  at its minimum permissible value (i.e., zero). The household's offer of debt as a single means of finance is an optimal solution in the presence of a single enforcement problem. The firm's participation constraint is binding in this case (i.e.,  $E(u_t) = q\phi_0$ ) so that the household extracts all of the surplus from the arrangement.<sup>9</sup>

By contrast, the decision problem for a household that does not intervene in the choice of project is defined as

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<sup>9</sup>Note that the expression for debt in (10) can be written as  $\widehat{d}_{t+1} = \widehat{R}(\widehat{l}_t) \widehat{l}_t$ , where  $\widehat{R}(\cdot)$  is the implicit interest rate on debt which is decreasing in the size of the loan (i.e.,  $\widehat{R}'(\cdot) < 0$ ).

$$\begin{aligned}
\mathbf{P2} : \quad & \max_{d_{t+1}, s_{t+1}} e^{-\rho x_{f,t}} [s_{t+1}(\tilde{r}x_{f,t}\tilde{l}_t^\alpha h_{f,t}^\beta - d_{t+1}) + d_{t+1}], \\
\text{s.t. } & h_{f,t} = \arg \max_{h_t} (1 - s_{t+1})e^{-\rho x_t}(\tilde{r}x_t\tilde{l}_t^\alpha h_t^\beta - d_{t+1}) + q\phi(1 - h_t), \\
& x_{f,t} = \arg \max_{x_t} (1 - s_{t+1})e^{-\rho x_t}(\tilde{r}x_t\tilde{l}_t^\alpha h_t^\beta - d_{t+1}) + q\phi(1 - h_t), \\
& (1 - s_{t+1})e^{-\rho x_{f,t}}(\tilde{r}x_{f,t}\tilde{l}_t^\alpha h_{f,t}^\beta - d_{t+1}) + \phi(1 - h_{f,t}) \geq q\phi_0, \\
& d_{t+1} \geq 0, \quad 0 \leq s_{t+1} \leq 1,
\end{aligned}$$

where  $\tilde{l}_t = \tilde{w}_t$ ,  $\tilde{w}_t = \theta\Theta k_t$  and  $\tilde{r} = (1 - \theta)\Theta$ . The set of constraints now includes the firm's reaction functions for determining both its input of effort,  $h_{f,t}$ , and its selection of project,  $x_{f,t}$ . The solution to problem **P2**, also derived in an Appendix, is given by the quadruplet  $\{\tilde{x}_t, \tilde{h}_t, \tilde{d}_{t+1}, \tilde{s}_{t+1}\}$  such that

$$\tilde{x}_t = \frac{1 + \beta}{\rho}, \quad (12)$$

$$\tilde{h}_t = \lceil \tilde{r}\beta^2(1 + \beta)^2\tilde{l}_t^\alpha q\phi\rho e^{1+\beta\frac{1}{1-\beta}}, \quad (13)$$

$$\tilde{d}_{t+1} = \lceil \tilde{r}\beta^{1+\beta}(1 + \beta)^{2\beta}\tilde{l}_t^\alpha (q\phi)^\beta \rho e^{\beta(1+\beta)\frac{1}{1-\beta}}, \quad (14)$$

$$\tilde{s}_{t+1} = 1 - \beta(1 + \beta). \quad (15)$$

Thus, under the assumption that  $\beta(1 + \beta) < 1$ , the optimal loan contract is now a mixed debt-equity contract. The intuition in this case runs as follows. The firm's two reaction functions are  $h_{f,t} = \lceil (1 - s_{t+1})\tilde{r}\beta x_t\tilde{l}_t^\alpha q\phi e^{\rho x_t\frac{1}{1-\beta}}$  and  $x_{f,t} = \frac{1}{\rho} + \frac{d_{t+1}}{\tilde{r}\tilde{l}_t^\alpha h_t^\beta}$ . Suppose that the household was to offer a pure debt contract so that  $d_{t+1} > 0$  while  $s_{t+1} = 0$ . The firm's optimal response would be to select a type of project and a level of effort that are both different from the household's first-best choices. Conversely, suppose that the household was to offer a pure equity contract so that  $d_{t+1} = 0$  while  $s_{t+1} > 0$ . Then the firm would deliver the household its first-best choice of project but not its first-best choice of effort. Neither of these contracts is able to yield the complete set of first-best outcomes for the household and each of them turns out to be dominated by a mixed contract. Relative to those first-best outcomes, this contract induces a riskier choice of project and a lower input of effort. The firm's participation constraint is non-binding in this case (i.e.,  $E(u_t) > q\phi_0$ ) so that the firm is able to retain a share of its profits in excess of its reservation level. Analogous to the above, the household's offer of both debt and equity as two means of finance is an optimal solution in the presence of two enforcement problems.<sup>10</sup>

<sup>10</sup>Also as before, it is possible to write the expression for debt in (14) as  $d_{t+1} = \tilde{R}(\tilde{l}_t)\tilde{l}_t$ ,

## 4 Equilibrium Finance and Capital Accumulation

The foregoing analysis identifies the optimal loan contract for a given set of circumstances. If a lender decides to take control of project choice, then she will offer a contract based solely on debt finance. If a lender decides not to take control of project choice, then she will offer a contract based on both debt and equity finance. Our immediate next task is to determine which of these arrangements is preferred by a lender, from which we may then proceed to characterise the complete dynamic general equilibrium of the economy.

A lender's expected utility under each contractual arrangement can be computed straightforwardly by substituting the relevant set of expressions obtained above - either (8)-(11) or (12)-(15) - into (7). Thus, in the case of a debt-only contract,

$$\begin{aligned}\widehat{E}(v_t) &= \prod (1 - \theta) \Theta (\theta \Theta)^\alpha \beta^\beta \eta^{\theta(\alpha+1)} (q\phi)^\beta \rho e^{\frac{1}{1-\beta}} (1 - \beta) k_t^{\frac{\alpha}{1-\beta}} + q\Phi \\ &\equiv \widehat{V} k_t^{\frac{\alpha}{1-\beta}} + q\Phi.\end{aligned}\tag{16}$$

And in the case of a debt-plus-equity contract,

$$\begin{aligned}\widetilde{E}(v_t) &= \prod (1 - \theta) \Theta (\theta \Theta)^\alpha \beta^{2\beta} (1 + \beta)^{1+\beta} (q\phi)^\beta \rho e^{1+\beta \frac{1}{1-\beta}} (1 - \beta) k_t^{\frac{\alpha}{1-\beta}} \\ &\equiv \widetilde{V} k_t^{\frac{\alpha}{1-\beta}}.\end{aligned}\tag{17}$$

Under both arrangements, therefore, the lender's expected utility depends on the level of capital. It is this feature that accounts for the endogeneity of the contracting regime along the economy's path of development. A comparison of (16) and (17) reveals that the lender's preferred choice of contract depends on whether  $\widetilde{V} k_t^{\frac{\alpha}{1-\beta}} \geq \widehat{V} k_t^{\frac{\alpha}{1-\beta}} + q\Phi$ . From this we can determine a critical level of capital,  $k^c = (\frac{q\Phi \widetilde{V}}{\widehat{V}})^{\frac{1-\beta}{\alpha}}$ , below which the preferred choice is a pure debt contract and above which the preferred choice is a mixed debt-equity contract. For our analysis to be non-trivial, we require  $k^c > 0$  which entails the parameter restriction  $\widetilde{V} > \widehat{V}$ , or  $\frac{\beta^\beta (1+\beta)^{1+\beta}}{e^\beta} > \eta^{\theta(\alpha+1)}$ . This states that the cost to the lender of implementing a pure debt contract (i.e., the cost of enforcing her own choice of project) cannot be too small. If this cost is negligible, then the lender would always be better off under such a contract since she would always be closer to her first-best solution and would always be

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where  $\widetilde{R}(\cdot)$  - the implicit interest rate on debt - is decreasing in the size of the loan except if there are constant returns in capital production, in which case it is constant (i.e.,  $\widetilde{R}'(\cdot) \leq 0$  according to whether  $\alpha + \beta \leq 1$ ).

able to extract the entire surplus. Consequently, no stock market would ever emerge.<sup>11</sup> As it is, the above restriction is necessarily satisfied by virtue of a similar condition that we impose later - namely,  $\frac{\beta^\beta(1+\beta)^{1+\beta}}{e^\beta} > \eta^{\theta(\alpha+\beta)}$ . Given this, then our analysis yields the result that the equilibrium loan contract is  $(\widehat{d}_{t+1}, \widehat{s}_{t+1})$  if  $k_t < k^c$  but  $(\widetilde{d}_{t+1}, \widetilde{s}_{t+1})$  if  $k_t \geq k^c$ . Accordingly, only a debt market exists at relatively low levels of development, while both debt and equity markets are active at relatively high levels of development. Whether or not the latter outcome actually transpires depends on whether or not the economy reaches the critical stage of evolution corresponding to  $k^c$ , a matter to which we now turn.

Our model implies that the process of capital accumulation not only determines, but is also determined by, the equilibrium contract outcome. This follows from the fact that each of the inputs to capital production - the type of project, level of effort and size of loan - is chosen differently across the two contracting regimes. Given these choices, it is possible to compute the expected amount of capital produced by each firm under each regime by referring back to (2): that is,  $E(\kappa_{t+1}) = e^{-\rho\widehat{x}_t}\widehat{x}_t\widehat{l}_t^\alpha\widehat{h}_t^\beta$  or  $E(\kappa_{t+1}) = e^{-\rho\widetilde{x}_t}\widetilde{x}_t\widetilde{l}_t^\alpha\widetilde{h}_t^\beta$ . Since there is a countable infinite number (a unit mass) of firms, we may then appeal to the law of large numbers to deduce the following transition paths for the actual stock of capital:

$$k_{t+1} = \begin{cases} \left[ ((1-\theta)\Theta)^\beta (\theta\Theta)^\alpha \beta^\beta \eta^{\theta(\alpha+\beta)} q^\beta \phi^\beta \rho e^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}} \right] \equiv \widehat{F} k_t^{\frac{\alpha}{1-\beta}} & \text{if } k_t < k^c, \\ \left[ ((1-\theta)\Theta)^\beta (\theta\Theta)^\alpha \beta^{2\beta} (1+\beta)^{1+\beta} q^\beta \phi^\beta \rho e^{1+\beta\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}} \right] \equiv \widetilde{F} k_t^{\frac{\alpha}{1-\beta}} & \text{if } k_t \geq k^c. \end{cases} \quad (18)$$

Under the assumption (alluded to above) that  $\frac{\beta^\beta(1+\beta)^{1+\beta}}{e^\beta} > \eta^{\theta(\alpha+\beta)}$ ,  $\widetilde{F} > \widehat{F}$  so that the transition path  $\widetilde{F} k_t^{\frac{\alpha}{1-\beta}}$  always lies above the transition path  $\widehat{F} k_t^{\frac{\alpha}{1-\beta}}$ . In this way, we ensure that the existence of a stock market, should it ever emerge, is conducive to real economic development, as suggested by the empirical evidence.<sup>12</sup> For illustrative purposes, we confine our subsequent analysis to the case in which  $\alpha + \beta < 1$ . Under such circumstances, each of the transition paths is strictly concave and each of them implies convergence

<sup>11</sup>The same result is possible in the models of Boyd and Smith (1996, 1998), where a stock market fails to materialise under certain parameter values.

<sup>12</sup>For example, among the references cited earlier, Levine and Zervos (1998) report significant positive correlations between various indicators of stock market activity (such as the value of stock trading relative to the size of the loan market, or relative to the size of the economy) and various measures of economic performance (such as current and future rates of capital accumulation, or productivity growth).

to a non-trivial steady state equilibrium, as defined by  $\widehat{k} = \widehat{F}^{\frac{1-\beta}{1-\alpha-\beta}}$  and  $\widetilde{k} = \widetilde{F}^{\frac{1-\beta}{1-\alpha-\beta}}$  (where  $\widehat{k} < \widetilde{k}$ ). These properties are depicted in Figure 1 to which the reader is referred during the following discussion.<sup>13</sup>

Given that  $k^c < \widetilde{k}$  (otherwise the analysis is trivial), one may distinguish between two types of development regime for the economy: the first - a low development regime - is characterised by relatively low levels of economic activity and a financial system in which only the debt market is open; the second - a high development regime - is characterised by relatively high levels of economic activity and a financial system in which both debt and equity markets are open. The overall evolution of the economy depends essentially on the relationship between  $k^c$  and  $\widehat{k}$ , together with  $k_0$  (the initial stock of capital). Suppose that  $k_0 < k^c < \widehat{k}$ . Under such circumstances, the economy evolves initially along the low growth path,  $\widehat{F}k_t^{\frac{\alpha}{1-\beta}}$ , with debt being the only mode of finance. On reaching  $k^c$ , the economy jumps to the high growth path,  $\widetilde{F}k_t^{\frac{\alpha}{1-\beta}}$ , as a stock market materialises to cater for the use of equity as an additional means of finance. Thereafter, the economy proceeds along this trajectory and converges to the high steady state equilibrium,  $\widetilde{k}$ . This chain of events describes a process of transition from the low development regime to the high development regime with the endogenous emergence of a stock market. Such an outcome is not inevitable, however. To be sure, suppose that  $k_0 < \widehat{k} < k^c$ . In this case the economy is destined for the low steady state equilibrium,  $\widehat{k}$ , being locked forever on the low growth path with only a debt market ever active. To the extent that a stock market would emerge and that the high steady state equilibrium would be attained if  $k_0 > k^c$ , the model now describes a scenario in which limiting outcomes depend fundamentally on initial conditions.

## 5 Discussion and Conclusions

Why do stock markets emerge relatively late in the development process of economies? Why do stock markets emerge at all when there are already other established markets for loans? Why do these other markets continue to operate after stock markets have been formed? And why do economies experience positive growth effects from stock market formation? These are fundamental questions that are posed by a large empirical literature and that are in need of answers from theory. As far as we know, this paper is one of only a few contributions that seek to provide some answers within the

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<sup>13</sup>If  $\alpha + \beta = 1$ , then the transition paths are linear and it is possible for our analysis to generate a switch in contracting regime coupled with long-run endogenous growth.

context of fully-specified dynamic general equilibrium models.

The linchpin of our analysis is the existence of asymmetric information between borrowers and lenders who must resolve problems of enforcement through the appropriate design of financial contracts. What is significant about these problems (and the solutions to them) is that they change endogenously with endogenous changes in both the structure of information and the level of economic activity as lenders make optimal decisions about whether or not to spend resources on strengthening their control over borrowers. At low levels of development, it pays lenders to do this and to offer a single means of finance - debt - in response to a single enforcement problem. At high levels of development, it pays lenders to abstain from such action and to offer two means of finance - debt and equity - in response to two enforcement problems. Transition from the low to the high development regime is therefore characterised by the endogenous emergence of a stock market which has a positive feedback effect on capital accumulation. Transition is not inevitable, however, and there exists the possibility of multiple long-run outcomes that depend on the initial circumstances of the economy.

A notable feature of our model as it presently stands is that, if financial transition takes place, it does so abruptly as soon as the critical level of capital,  $k^c$ , is reached. This feature has been instilled deliberately in order to focus and simplify the analysis. In principle, the model could be extended to allow for a smoother process of transition and a more gradual adoption of equity finance. For example, suppose that entrepreneurs are heterogeneous in their reservation levels of utility,  $q\phi_0$ . Under such circumstances, not all entrepreneurs would begin to issue equity at the same time: only those for whom  $\tilde{V}k_t^{\frac{\alpha}{1-\beta}} \geq \hat{V}k_t^{\frac{\alpha}{1-\beta}} + q\Phi$  would do so. *Ceteris paribus*, as capital accumulation proceeds, this condition would become satisfied for a growing number of firms, implying a gradual increase in stock market trading. The analysis of this case is more complicated because of changes in the market wage due to changes in labour supply associated with changes in the distribution of firms engaged in different types of contract. Nevertheless, one may surmise that there would be certain parameter configurations of the model under which all firms end up issuing equity, and certain other parameter configurations under which only a subset (including the empty set) of firms become part equity financed.

Our analysis is not only successful in stylising the widely-observed positive correlation between real economic development and stock market activity (e.g., Atje and Jovanovic 1993; Demirgüç-Kunt and Levine 1996a,b; Levine and Zervos 1995, 1996, 1998), but is also consistent with the long-standing view that one of the major obstacles to growth is the loss of resources due

to informational frictions in the capital market (e.g., McKinnon 1973; Shaw 1973). This view is also shared by Boyd and Smith (1996, 1998) whose analysis, like ours, predicts that these resource costs decline as economies develop and stock markets become more established. In addition, the potential of our model to generate multiple steady state equilibria means that it is possible to talk of underdevelopment traps in which an economy's real and financial sectors display little co-evolution. An implication of this case is that an exogenous shock which raises the capital stock above its critical level could have a profound effect on the fortunes of an economy as the financial system is transformed by the rapid materialisation of a stock market. This may be allied to the observation that the deregulation of international capital flows has been associated with surges in stock market activity among many different countries (e.g., Levine and Zervos 1995).

A popular belief is that debt and equity represent substitute sources of corporate finance. Recent evidence suggests the opposite that debt and equity act more as complements to each other (e.g., Demirgüç-Kunt and Levine 1996a,b; Demirgüç-Kunt and Maksimovic 1996). Indeed, an increase in the size of the stock market is typically associated with an increase in the volume of debt in many economies. Such evidence has been regarded as a puzzle in demand of an explanation. In Boyd and Smith (1996, 1998) the puzzle is resolved on account of an increase in the cost of pure debt finance as an economy develops. Eventually, this cost becomes too high for firms to feasibly continue their operations by relying on debt finance alone and equity emerges as a complementary means of raising funds. In our analysis the cost of pure debt financing also increases with the level of development. At some point, lenders find it optimal to forego incurring this cost and to complement the use of debt with the use of equity as a double-edged strategy for dealing with a double (rather than single) enforcement problem.

## Appendix A: The Optimal Debt-Equity Contract

In what follows we suppress the indices on variables to save on notation. In addition, we exploit the fact that the dimensions of each problem can be reduced slightly by ignoring the case in which  $s = 1$  since this would imply that  $h = 0$ , meaning that no project would be undertaken.

### A.1 Solution to Problem P1

The firm's reaction function is

$$h = [(1 - s)(q\phi)^{-1}\beta e^{-\rho x_t} \widehat{r}x\widehat{l}^\alpha]^{1-\beta}. \quad (\text{A1})$$

Given (A1), it is possible to simplify the problem by converting it to the form

$$\begin{aligned} \max_{x,h,d} \quad & e^{-\rho x} \widehat{r}x\widehat{l}^\alpha h^\beta - q\phi\beta^{-1}h + q\phi(\beta\widehat{r}\widehat{l}^\alpha)^{-1}x^{-1}h^{1-\beta}d, \\ \text{s.t.} \quad & q\phi\beta^{-1}(1 - \beta)h - q\phi(\beta\widehat{r}\widehat{l}^\alpha)^{-1}x^{-1}h^{1-\beta}d + q\Phi \geq 0, \\ & d \geq 0, \\ & 1 - q\phi(\beta\widehat{r}\widehat{l}^\alpha)^{-1}e^{\rho x}x^{-1}h^{1-\beta} \geq 0, \end{aligned}$$

where  $s = 1 - q\phi(\beta\widehat{r}\widehat{l}^\alpha)^{-1}e^{\rho x}x^{-1}h^{1-\beta}$ . After various manipulations, the optimality conditions may be written as

$$0 = e^{-\rho x} \widehat{r}x\widehat{l}^\alpha h^\beta (1 - \rho x) - e^{-\rho x} (1 - s)(1 - \lambda_1) + \lambda_3 (1 - s)(1 - \rho x), \quad (\text{A2})$$

$$\begin{aligned} 0 = e^{-\rho x} \widehat{r}x\widehat{l}^\alpha h^\beta \beta - [1 - \lambda_1(1 - \beta)]q\phi\beta^{-1}h \\ + e^{-\rho x} (1 - \beta)(1 - s)(1 - \lambda_1)d - \lambda_3(1 - \beta)(1 - s), \end{aligned} \quad (\text{A3})$$

$$0 = e^{-\rho x} (1 - s)(1 - \lambda_1) + \lambda_2, \quad (\text{A4})$$

$$0 = \lambda_1[q\phi\beta^{-1}(1 - \beta)h - e^{-\rho x}(1 - s)d + q\Phi], \quad \lambda_1 \geq 0, \quad [\cdot] \geq 0, \quad (\text{A5})$$

$$0 = \lambda_2 d, \quad \lambda_2 \geq 0, \quad d \geq 0, \quad (\text{A6})$$

$$0 = \lambda_3 s, \quad \lambda_3 \geq 0, \quad s \geq 0, \quad (\text{A7})$$

where  $\lambda_i$  ( $i = 1, 2, 3$ ) are the Lagrange multipliers. It is evident from (A4) that, since  $\lambda_2 \geq 0$ ,  $\lambda_1 \neq 0$  so that the firm's participation constraint always binds in compliance with (A5).

**Case 1**  $d = 0$ ,  $s > 0$ . From (A5), it is required that  $q\phi\beta^{-1}(1 - \beta)h + q\Phi = 0$  which is not possible.

**Case 2**  $d > 0, s > 0$ . By virtue of (A6) and (A7),  $\lambda_2 = \lambda_3 = 0$ . Thus  $\lambda_1 = 1$  from (A4) and  $x = \rho^{-1}$  from (A2). Combining (A3) with (A1) implies  $s = 0$  which is a contradiction.

**Case 3**  $d > 0, s = 0$ . Since  $\lambda_2 = 0$  from (A6), then  $\lambda_1 = 1$  from (A4). Hence  $x = \rho^{-1}$  in accordance with (A3). The solution for  $h$  is obtained from (A1), while the solution for  $d$  is computed from  $q\phi\beta^{-1}(1-\beta)h - e^{-\rho x}d + q\Phi = 0$  by virtue of (A5). These results are summarised in (8)-(11) of the main text.

## A.2 Solution to Problem P2

The firm's reaction functions are

$$h = [(1-s)(q\phi)^{-1}\beta e^{-\rho x}\tilde{r}x\tilde{l}^\alpha]^{1-\beta}, \quad (\text{A8})$$

$$x = \rho^{-1} + (\tilde{r}\tilde{l}^\alpha)^{-1}h^{-\beta}d. \quad (\text{A9})$$

As before, (A8) and (A9) may be used to simplify the problem by transforming it to read

$$\begin{aligned} \max_{x,h} \quad & e^{-\rho x}\tilde{r}x\tilde{l}^\alpha h^\beta - \rho^{-1}q\phi\beta^{-1}x^{-1}h, \\ \text{s.t.} \quad & (\rho^{-1}\beta^{-1}x^{-1} - 1)q\phi h + q\Phi \geq 0, \\ & \rho^{-1}(\rho x - 1)\tilde{r}\tilde{l}^\alpha h^\beta \geq 0, \\ & 1 - q\phi(\beta\tilde{r}\tilde{l}^\alpha)^{-1}e^{\rho x}x^{-1}h^{1-\beta} \geq 0, \end{aligned}$$

where  $d = \rho^{-1}(\rho x - 1)\tilde{r}\tilde{l}^\alpha h^\beta$  and  $s = 1 - q\phi(\beta\tilde{r}\tilde{l}^\alpha)^{-1}e^{\rho x}x^{-1}h^{1-\beta}$ . Appropriate manipulation yields the following expressions for the optimality conditions:

$$\begin{aligned} 0 = \quad & e^{-\rho x}\tilde{r}x\tilde{l}^\alpha h^\beta(1 - \rho x) + \rho^{-1}\tilde{r}\tilde{l}^\alpha h^\beta e^{-\rho x}(1-s)(1 - \mu_1) \\ & + \mu_2\tilde{r}x\tilde{l}^\alpha h^\beta + \mu_3(1-s)(1 - \rho x), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} 0 = \quad & e^{-\rho x}\tilde{r}x\tilde{l}^\alpha h^\beta\beta - \rho^{-1}\tilde{r}\tilde{l}^\alpha h^\beta e^{-\rho x}(1-s)(1 - \mu_1) - \mu_1q\phi h \\ & - \mu_2\beta\rho^{-1}(1 - \rho x)\tilde{r}\tilde{l}^\alpha h^\beta - \mu_3(1 - \beta)(1 - s), \end{aligned} \quad (\text{A11})$$

$$0 = \mu_1[(\rho^{-1}\beta^{-1}x^{-1} - 1)q\phi h + q\Phi] = 0, \quad \mu_1 \geq 0, \quad [\cdot] \geq 0, \quad (\text{A12})$$

$$0 = \mu_2\rho^{-1}(\rho x - 1)\tilde{r}\tilde{l}^\alpha h^\beta, \quad \mu_2 \geq 0, \quad \rho^{-1}(\rho x - 1)\tilde{r}\tilde{l}^\alpha h^\beta \geq 0, \quad (\text{A13})$$

$$0 = \mu_3s, \quad \mu_3 \geq 0, \quad s \geq 0, \quad (\text{A14})$$

where  $\mu_i$  ( $i = 1, 2, 3$ ) are the Lagrange multipliers.

**Case 1**  $d = 0, s > 0$ . From (A13) (recalling that  $d = \rho^{-1}(\rho x - 1)\tilde{r}l^\alpha h^\beta$ ) and (A14),  $\rho x = 1$  and  $\mu_3 = 0$ .

i) Suppose that  $\mu_1 = 0$ . Then (A10) cannot be satisfied.

ii) Suppose that  $\mu_1 > 0$ . Then (A12) requires that  $\beta^{-1}(1 - \beta)q\phi h + q\Phi = 0$  which is not possible.

**Case 2**  $d > 0, s = 0$ . By virtue of (A13),  $\mu_2 = 0$  and  $\rho x > 1$ . Combining (A10) with (A11) reveals that  $1 - \rho x + \beta \geq 0$  must hold.

i) Suppose that  $\mu_1 > 0$ . Then (A12) requires that  $(\rho^{-1}\beta^{-1}x^{-1} - 1)q\phi h + q\Phi = 0$  so that  $\rho x \beta > 1$  must hold. This is consistent with  $1 - \rho x + \beta > 0$  only if  $\beta(1 + \beta) > 1$  which is ruled out by assumption.

ii) Suppose that  $\mu_1 = 0$ . If  $\mu_3 \neq 0$ , then (A10) and (A11) imply the quadratic equation  $(\rho x)^2 - 2(\rho x) + \beta = 0$ , to which the only feasible solution is  $\rho x = 1 + (1 - \beta)^{\frac{1}{2}}$  which must satisfy  $1 - \rho x + \beta > 0$ . This is possible only if  $\beta(1 + \beta) > 1$  which is ruled out by assumption. If  $\mu_3 = 0$ , then (A10) and (A11) cannot be satisfied simultaneously.

**Case 3**  $d > 0, s > 0$ . From (A13) and (A14),  $\mu_2 = \mu_3 = 0$  and  $\rho x > 1$ . Combining (A10) with (A11) reveals that  $1 - \rho x + \beta \geq 0$  must hold.

i) Suppose that  $\mu_1 > 0$ . Then (A12) requires that  $(\rho^{-1}\beta^{-1}x^{-1} - 1)q\phi h + q\Phi = 0$  so that  $\rho x \beta > 1$  must hold. This is consistent with  $1 - \rho x + \beta > 0$  only if  $\beta(1 + \beta) > 1$  which is ruled out by assumption.

ii) Suppose that  $\mu_1 = 0$ . Then (A10) and (A11) imply  $x = \rho^{-1}(1 + \beta)$  and  $s = 1 - \beta(1 + \beta)$ . The solution for  $h$  is obtained from (A8), while the solution for  $d$  is computed from (A9). These results are summarised in (12)-(15) of the main text.

## Appendix B: A Generalised Loan Contract

We show that, for each scenario considered, the optimal financial contract expressed in terms of debt and equity implements exactly the same outcomes as those that would transpire under a generalised contingent claims contract based on lenders' *ex post* observations of events. The latter arrangement is characterised by a repayment schedule,  $R(\cdot)$ , which specifies the amount that a borrower must repay to a lender conditional on a set of observed outcomes which includes, at a minimum, the realised yield of a project,  $\kappa = xl^\alpha h^\beta$ . The expected payoffs to a borrower and a lender are then given, respectively, by

$$E(u) = e^{-\rho x}[rxl^\alpha h^\beta - R(\cdot)] + q\phi(1 - h), \quad (\text{B1})$$

$$E(v) = e^{-\rho x}R(\cdot). \quad (\text{B2})$$

In the case of a frictionless environment, where the lender is able to costlessly observe and control both  $x$  and  $h$ , the problem for the lender is straightforward, being to maximise (B2) subject only to the borrower's participation constraint,  $E(u) \geq q\phi_0$ . The solution to this problem is given by  $x = \frac{1}{\rho}$  and  $h = (\rho\beta x l^\alpha q\phi \rho e^{\rho x \frac{1}{1-\beta}})^{\frac{1}{1-\beta}}$ , with  $E(u) = q\phi_0$  and  $R(\cdot) = r x l^\alpha h^\beta + e^{\rho x} q\phi(1-h) - e^{\rho x} q\phi_0$ . This is the first-best solution reported in the main text.

For the case in which the lender chooses  $x$ , but the borrower chooses  $h$ , the problem is similarly straightforward since the lender is able to infer  $h$  from observations of  $\kappa$  and  $x$ . This means that the lender can credibly threaten to take all the proceeds from a project if the borrower does not choose the preferred level of effort: that is,  $R(\cdot) = \kappa$  if  $\kappa \neq \widehat{x} \widehat{l}^\alpha \widehat{h}^\beta$ . Given this, then the lender is able to enforce the (near) first-best outcomes,  $\widehat{x} = \frac{1}{\rho}$  and  $\widehat{h} = (\widehat{r} \beta \widehat{x} l^\alpha q\phi e^{\rho \widehat{x} \frac{1}{1-\beta}})^{\frac{1}{1-\beta}}$ , which correspond to the solutions of the problem of maximising (B2) subject to  $E(u) = q\phi_0$ , and which are identical to the outcomes obtained in (8) and (9) under a pure debt contract. In addition,  $e^{-\rho \widehat{x}} R(\cdot) = \widehat{E}(v)$  in (16).

For the case in which the borrower controls both  $x$  and  $h$  (neither of which are directly observable by the lender), the problem is a little less straightforward but still solvable. Under such circumstances, the borrower maximises (B1) by setting

$$\widetilde{l}^\alpha h^\beta [\widetilde{r} - R_\kappa(\cdot)] - \rho [\widetilde{r} \widetilde{l}^\alpha h^\beta - R(\cdot)] = 0, \quad (\text{B3})$$

$$e^{-\rho x} \beta x \widetilde{l}^\alpha h^{\beta-1} [\widetilde{r} - R_\kappa(\cdot)] - q\phi = 0. \quad (\text{B4})$$

The lender is then faced with the problem of maximising (B2) subject to  $E(u) \geq q\phi_0$ , where  $R(\cdot) = \widetilde{r} x \widetilde{l}^\alpha h^\beta - \frac{q\phi e^{\rho x}}{\rho \beta x}$  from (B3) and (B4). It may be verified that, given our assumption that  $\beta(1+\beta) < 1$ , the participation constraint is never binding in this case and that the lender's optimality conditions are

$$e^{-\rho x} \widetilde{r} \widetilde{l}^\alpha h^\beta (1 - \rho x) + \frac{q\phi h}{\rho \beta x^2} = 0 \quad (\text{B5})$$

$$e^{-\rho x} \beta \widetilde{r} \widetilde{l}^\alpha h^{\beta-1} - \frac{q\phi}{\rho \beta x} = 0. \quad (\text{B6})$$

These conditions imply  $\widetilde{x} = \frac{1+\beta}{\rho}$  and  $\widetilde{h} = (\widetilde{r} \beta^2 \rho \widetilde{x}^2 \widetilde{l}^\alpha q\phi e^{\rho \widetilde{x} \frac{1}{1-\beta}})^{\frac{1}{1-\beta}}$  which are equivalent to the outcomes obtained in (12) and (13) under a mixed debt-equity contract. In addition,  $e^{-\rho \widetilde{x}} R(\cdot) = \widetilde{E}(u)$  in (17).

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