When Romer meets Lucas: on Human Capital, Imperfect Competition and Growth[†]

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Abstract

This paper studies the economic determinants of the inter-sectoral allocation of skills and the long-run consequences of imperfect competition on growth within an R&D-based growth model with human capital accumulation. I find that steady-state growth is driven only by incentives to accumulate skills and is independent of scale effects. In the model imperfect competition has a positive growth effect, while influencing the allocation of human capital to the different economic activities. Contrary to general wisdom, the share of resources invested in R&D turns out not to be monotonically increasing in the equilibrium output growth rate and the market power.

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Introduction

Recent theoretical and empirical advancements in economic growth literature suggest that technological progress and human capital accumulation are primary determinants of economic growth. The majority of growth models, however, focus on only one of these engines at a time.[1] Exceptions are represented by Stokey (1988) and Young (1993), Grossman and Helpman (1991, Ch.5.2), and Eicher (1996), Redding (1996) and Restuccia (1997). Even though all these works take explicitly into account the interaction between endogenous technological change and human capital formation, they still remain limited in many respects. In the first two (Stokey, 1988 and Young, 1993), for example, skill accumulation happens through learning-by-doing and on-the-job-training in the production activity, rather than a separate education sector. In Grossman and Helpman (1991, Ch.5.2), a separate education sector does exist but, strangely enough, it does not require any skilled worker to operate. Eicher (1996) develops a rich model in which both human capital and technological innovation are endogenous. However, this paper is solely concerned with steady-state predictions on the relationship between relative supply of skilled labour and relative wage. Restuccia (1997), on the other hand, builds a dynamic general equilibrium model with schooling and technology adoption. But the primary concern of the paper is to study how schooling and technology adoption may be amplifying the effects of productivity differences on income disparity. Finally, Redding (1996) emphasises the potential interaction between investment in education and investment in research and shows under which conditions such an interaction may give rise to coordination problems and under-development traps.

Given all this, it is quite surprising that little attention has been paid until now to the analysis of both the long-run determinants of the equilibrium shares of human capital devoted to each economic sector employing this factor input and the impact of imperfect competition on steady-state growth within the context of an integrated model of purposive R&D activity and human capital accumulation.

[1] "Ak-based growth models" (e.g. Lucas, 1988; Jones and Manuelli, 1990 and Rebelo, 1991) imply that sustainable growth is the outcome of reproducible inputs (such as physical and human capital), whereas "Research and Development (R&D)-based models" (e.g. Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Jones, 1995a and Young, 1998) all maintain that technological progress, rather than any accumulable input, is the main engine of growth.

The aim of this chapter is to fill this gap in the literature and in order to do this it combines in the simplest possible way the basic Lucas (1988) model of skills accumulation, on the one hand, with the Grossman and Helpman (1991, Ch. 3) and Romer's (1990) models of endogenous technical change, on the other.

In more detail, we consider a model economy that is made up of a representative household and firms. For simplicity purposes the representative household consists of only one agent who is involved in four types of activities: consumption goods production, intermediate goods manufacturing, human capital investment and R&D effort. Population is stationary in the economy and consumption goods are produced within a perfectly competitive environment in which prices are taken as given and each input is compensated according to its own marginal product. The intermediate-goods sector consists of monopolistic producers of differentiated products entering the production function of the homogeneous final good as an input. The representative household invests portions of its fixed-time endowment to acquire formal education. Finally, purposive R&D activity is the source of technological progress in the model. Technical progress happens, indeed, through inventing new varieties of horizontally differentiated capital goods within a separate and competitive R&D sector. When a new blueprint is discovered in the R&D sector, an intermediate-goods producer acquires the perpetual patent over it. This allows the intermediate firm to manufacture the new variety and practice monopoly pricing. A peculiarity of the model is that all the sectors composing this economy do employ human capital. This is done because, as already mentioned, it is one of our objectives here to study in detail the economic forces underlying the inter-sectoral allocation of such an input in this model economy.

Our main findings are threefold. First of all, as in the basic Lucas (1988) model, growth is driven by human capital accumulation and depends on the parameters describing preferences and the skill acquisition technology. At the same time, and unlike Lucas (1988), the presence of imperfect competition conditions in the intermediate sector both has growth effects and influences the allocation of the available human capital stock to the different sectors employing this input. Secondly, as it is common in recent endogenous growth theory[2], our model does not display any *scale effect*, since growth does not depend on the total available human capital stock. Finally, we find that the relationship between the equilibrium growth rate and the share of resources invested

[2] See Eicher and Turnovsky (1999) for a detailed discussion on *non-scale models of economic growth*.

in R&D, on the one hand, and that between this last and the mark-up rate, on the other, are absolutely non-monotonic.

Our analysis here is related to other works, both in its scope and its methodological approach. Arnold (1998) also develops an endogenous growth model that integrates purposive R&D activity with human capital accumulation. But his work is mainly motivated by the attempt of rejecting, on theoretical grounds, two main predictions of standard growth models based on R&D (namely that the equilibrium growth rate is very much sensitive to policy changes and to the level of resources used in research). Blackburn *et alii* (2000) extend Arnold's model (1998) in the direction of a fuller micro-foundation of the R&D process and obtain the same results with no further new insights. Then, both these two papers do not deal at all with the determinants of the inter-sectoral allocation of skilled workers and with the long-run influences of imperfect competition on growth. Indeed, in Blackburn *et ali* (2000) intermediate firms do not employ directly human capital, since they use forgone consumption to produce. This is the main reason why we consider that framework as truly inadequate to answer the questions we would like to answer in this paper.

To the best of our knowledge, and within a similar framework, this is the first attempt in this direction. In this respect, we should probably mention a recent paper by Jones and Williams (2000) aimed at analysing whether a decentralised economy undertakes too little or too much R&D in the presence of some distortions to the research activity.[3] However, in this paper there is no human capital accumulation and capital goods and research are produced devoting units of foregone consumption. In addition, in the paper there is no evaluation of the possible long-run links between (im)perfect competion and growth. Finally, another work which comes closer to our work is Bucci (2001). The main difference is that in the present analysis the shares of human capital devoted to each economic activity are endogenous.

The rest of the chapter is organised as follows. Section 1 introduces the basic model and Section 2 presents the solution of it. Section 3 examines the steady-state properties of the model and Section 4 computes the equilibrium output growth rate. In Section 5, we solve for the inter-

[3] These distortions are represented respectively by the *surplus appropriability problem*, the presence of *knowledge spillovers* and the *creative destruction* and *congestion externalities*.

sectoral allocation of human capital, present some comparative statics results about the main economic determinants of the shares of the reproducible input (skilled work) devoted to each economic activity and briefly discuss the most important findings. Section 6 presents the results concerning the steady-state predictions of the model about the relationship between imperfect competition and growth and Section 7 concludes.

1. The Basic Model

Consider an economy with three different productive sectors. There exists an undifferentiated consumers good, which is produced using skilled labour and capital goods (intermediate inputs). These are available, at time t, in n_t different varieties. In order to produce such inputs, intermediate firms employ only human capital. Technical progress takes place as a continuous expansion, through purposive Research and Development (R&D) activity, of the set of available horizontally differentiated intermediates. R&D is skill intensive as well. Unlike the traditional *R&D-based growth models*, I assume that the supply of human capital may grow over time. In this connection, following the pathbreaking papers by Uzawa (1965) and Lucas (1988), I postulate the existence of a representative household that chooses plans for consumption (c), asset holdings (a, to be defined later) and human capital (h). For the sake of simplicity, I also assume that the representative household of this economy has unit measure. In the model there is no physical capital and unskilled labour. Human capital is a homogeneous input and can be employed to produce the final output, intermediates, new human capital and to invent new varieties of capital goods (research).

1.1 The Consumers Good Sector

The homogeneous, undifferentiated consumers good is produced within a *competitive industry*. Such an industry is populated by a large number of identical firms and employs the following constant returns to scale aggregate production function[4]:

(1) $$Y_t = AH_{Yt}^{1-a}\int_0^{n_t}(x_{jt})^a\,dj\,, \qquad A>0\,, \qquad a\in(0,1).$$

Therefore, output at time t (Y_t) is obtained combining skilled work (H_{Yt}) and n different varieties of intermediate inputs, each of which is employed in the quantity x_j. A and a are technological parameters. The former (total factor productivity) is strictly positive, whereas the latter is between 0 and 1. As the industry is competitive, in equilibrium each variety of intermediates receives its own marginal product (in terms of the numeraire, the only final good):

(2) $$p_{jt} = AaH_{Yt}^{1-a}(x_{jt})^{a-1}, \qquad \forall j\in(0,n_t)\,.$$

In (2), p_{jt} is the inverse demand function faced, at time *t*, by the generic *j-th* intermediate producer. From (2), the direct demand function for the *j-th* type of intermediates reads as:

(3) $$x_{jt} = \left(\frac{AaH_{Yt}^{1-a}}{p_{jt}}\right)^{\frac{1}{1-a}}, \qquad \forall j\in(0,n_t).$$

As it is common in the *innovation-based* growth literature, the elasticity of substitution between two generic intermediates coincides with the price-demand elasticity faced by each

[4] A similar specification for the aggregate final output technology is employed by Blackburn *et alii* (2000).

capital goods producer and is equal to $1/(1-a)$.

1.2 The Intermediate Goods Sector

The capital goods industry is *monopolistically competitive* and each intermediate input is produced using the same technology[5]:

$$(4) \qquad x_{jt} = B \cdot h_{jt} \quad , \qquad \forall j \in (0, n_t), \qquad B \geq 1$$

This production function is characterised by constant returns to scale in the only input employed (human capital) and, according to it, one unit of skills is able to produce (at each time) the same constant quantity of whatever variety. B measures the productivity of human capital employed in this sector. Following Romer (1990) and Grossman and Helpman (1991, Chap.3), we continue to assume that each intermediate good embodies a design created in the R&D sector and that there exists a patent law which prohibits any firm from manufacturing an intermediate good without the consent of the patent holder of the design.

The generic *j-th* firm maximises (with respect to x_{jt}) its own instantaneous profit, under the demand constraint (given by (2)). From the first order conditions, it is immediate to obtain:

$$(5) \qquad w_{jt} = ABa^2 H_{Yt}^{1-a} (x_{jt})^{a-1},$$

where w_{jt} is the wage rate paid (at time *t*) to one unit of human capital employed in this sector. In a symmetric equilibrium (where $x_{jt} = x_t$, $\forall j \in (0, n_t)$), each local monopolist faces the same wage rate ($w_{jt} = w_t$, $\forall j \in (0, n_t)$). The hypothesis of symmetry is dictated by the way through which each variety of intermediates enters the final output technology.

[5] The reason why in equation (4) below I set the restriction on the parameter B will be clear in a moment.

Plugging (3) into (5) yields:

$$(6) \qquad p_{jt} = \frac{1}{aB} \cdot w_{jt} = \frac{1}{aB} \cdot w_t = p_t \quad , \qquad \forall j \in (0, n_t).$$

Hence, when all the capital goods firms are identical, they produce the same quantity, face the same wage rate accruing to intermediate skilled workers and fix the same price for their own output. The price is equal to a constant *mark-up* $(1/aB)$ over the marginal cost (w_t). Notice that, in order for the mark-up ratio to be strictly greater than one, a should be strictly less than 1/B. Since $0 < a < 1$, the condition on B ($B \geq 1$) follows immediately. In addition, we define by $H_{jt} \equiv \int_0^{n_t} h_{jt} dj$ the total amount of human capital employed in the intermediate sector at time t obtaining, from equation (4) and under the hypothesis of symmetry among capital goods producers:

$$(7) \qquad x_{jt} = \frac{B \cdot H_{jt}}{n_t} = x_t, \qquad \forall j \in (0, n_t).$$

Finally, the profit function of a generic *j-th* intermediate firm is given by:

$$(8) \qquad p_{jt} = \left(p_t - \frac{1}{B} w_t \right) \cdot x_t = Aa \cdot (1-a) \cdot H_{Yt}^{1-a} \cdot \left(\frac{B \cdot H_{jt}}{n_t} \right)^a = p_t, \qquad \forall j \in (0, n_t).$$

Equation (8) says that, just as *x* and *p*, so too the instantaneous profit is equal for each variety of intermediates in a symmetric equilibrium.

1.3 The Research Sector

Producing the generic *j-th* variety of capital goods entails the purchase of a specific blueprint (the *j-th* one) from the competitive research sector, characterised by the following aggregate technology:

$$(9) \qquad \dot{n}_t = C \cdot H_{nt}, \qquad C>0,$$

where n_t denotes the number of capital goods varieties existing at time *t*, H_n is the total amount of human capital employed in the sector and C is the productivity of the research skilled workers. The production function of new ideas displays two peculiar features that are worth pointing out. First of all, it is a deterministic linear function of H_n. Secondly, it does not depend on n_t. This is an alternative to the canonical assumption one may find in the literature in the sense that, unlike the Romer's (1990) and Grossman and Helpman's (1991, Chap.3) models, we explicitly assume that no positive *spillover* effect is attached to the available stock of disembodied knowledge (approximated by the existing number of designs, n_t) in discovering a new product variety. This simple formulation of the R&D technology has been chosen in line with the aims of the present chapter. Another, and more important, reason is that in the present model economic growth is exclusively driven by human capital accumulation. As a consequence, unlike the Romer (1990) and Grossman and Helpman's (1991, Chap. 3) models, we do not need to introduce any kind of pecuniary externality in the R&D sector in order to make growth sustainable in the long run.[6] At the same time we think that this formulation of the R&D technology allows to avoid some recent criticisms about the nature of knowledge spillovers, the way these last are modelled in the new growth theory and the way this branch of economic theory deals with the problem of technology

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[6] Indeed, in Romer (1990) and Grossman and Helpman (1991, Chp. 3) the only growth engine is represented by the research sector and the presence of externalities in it serves the only scope of allowing constant and positive equilibrium growth rates.

diffusion[7].

As the research sector is competitive, new firms will enter it till when all profit opportunities will be completely exhausted. The zero profit condition amounts, in this case, to set:

(10) $$\frac{1}{C} w_{nt} = V_{nt}$$

(11) $$V_{nt} = \int_{t}^{\infty} \exp\left[-\int_{t}^{t} r(s) ds \right] p_{jt}\, dt \,, \qquad t > t \,.$$

Symbols used in (10) and (11) have the following meaning: w_n is the wage rate accruing to one unit of human capital devoted to research; the term $\exp\left[-\int_{t}^{t} r(s) ds \right]$ is a present value factor which converts a unit of profit at time t into an equivalent unit of profit at time *t; r* is the real rate of return on the consumers' asset holdings; p_j is the profit accruing to the *j-th* intermediate producer (once the *j-th* infinitely-lived patent has been acquired) and V_n is the market value of one unit of research output (the generic *j-th* idea allowing to produce the *j-th* variety of capital goods). Notice that V_n is equal to the discounted present value of the profit flow a local monopolist can potentially earn from *t* to infinity and coincides with the market value of the *j-th* intermediate firm (since there is a one to one relationship between number of patents and number of capital goods producers).

[7] See Fagerberg (1994), Fagerberg and Verspagen (1998) and Verspagen and Schoenmakers (2000) for theoretical as well as empirical considerations about the spatial dimension of knowledge spillovers in Europe and the relationship among productivity growth, R&D spillovers and trade. See also Breschi and Lissoni (2001) for a recent critical survey of the literature. According to L.C. Keely (2001): *"...the information in the patent is used in further innovation (see, for example, Aghion and Howitt (1992), Grossman and Helpman (1991), and Romer (1986, 1990)). Although in principle a patent's information spills over to other firms, there is a large empirical literature that suggests such spillovers are in practice neither so immediate nor widespread".*

1.4 Consumers

We consider a closed economy in which there exists only one representative infinitely-lived household that holds assets in the form of ownership claims on firms and chooses plans for consumption (*c*), asset holdings (*a*) and human capital (*h*). For the sake of simplicity, I assume that the only household populating this economy has unit measure and there is no population growth. This hypothesis implies that, at each *t*, the household's own stock of human capital (*h*) equals the economy aggregate stock of human capital (*H*). Following Lucas (1988), we also assume that the household is endowed with one unit of time and optimally allocates a fraction *u* of its time endowment to productive activities (research, capital goods and consumer goods production) and the remaining fraction (*1-u*) to non-productive activities (education or skills accumulation). As it will be clearer later on, given the household's choice of the optimal *u* (that we denote by *u**), the labour market clearing conditions will determine the decentralised allocation of the productive human capital between manufacturing of intermediate and consumers goods and invention of new ideas (research).

With an instantaneous felicity function $u(c_t) = \log(c_t)$, the decision problem of the household can be written as:

$$(12) \qquad \underset{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-rt} \log(c_t) dt \ , \qquad r > 0$$

s.t.:

$$(13) \qquad \dot{a}_t = r_t a_t + w_t u_t h_t - c_t$$

$$(14) \qquad \dot{h}_t = d(1 - u_t) h_t \ , \qquad d > 0$$

$$a_0 \ , \ h_0 \text{ given.}$$

The control variables of this problem are c_t and u_t, whereas a_t and h_t are the state variables. Equation (12) is the intertemporal utility function; equation (13) is the budget constraint and equation (14) represents the human capital supply function.[8] The symbols used have the following meaning: r is the subjective discount rate; c denotes consumption of the homogeneous final good; w is the wage rate accruing to one unit of skilled labour[9] and d is a parameter reflecting the productivity of the education technology.

With l_{1t} and l_{2t} denoting respectively the shadow price of the household's asset holdings and human capital stock, the first order conditions are:

(15) $$\frac{e^{-rt}}{c_t} = l_{1t}$$

(16) $$l_{1t} = l_{2t}\frac{d}{w_t}$$

(17) $$l_{1t} r_t = -\dot{l}_{1t}$$

(18) $$l_{1t} w_t u_t + l_{2t}\, d(1-u_t) = -\dot{l}_{2t}$$

Conditions (15) through (18) must satisfy the constraints (13) and (14), together with the transversality conditions:

$$\lim_{t\to\infty} l_{1t} a_t = 0$$

$$\lim_{t\to\infty} l_{2t} h_t = 0$$

2. General Equilibrium

In this paragraph we solve for the general equilibrium of the model. In order to build such an equilibrium, I use the symmetry hypothesis $(x_{jt} = B \cdot H_{jt} / n_t = x_t,\ \forall j \in (0, n_t))$ and, for notation

[8] Notice that I assume no depreciation for human capital. This hypothesis is completely harmless in the present context and serves the scope of simplifying the analysis.

[9] In equilibrium there exists only one wage rate accruing to skilled workers since human capital is homogeneous.

simplicity, drop the index t on the variables depending on time. Next, for given $u*$ (the optimal fraction of skills devoted by consumers to production activities), the optimal allocation of human capital among research, capital and consumers goods production is found solving simultaneously the following labour market clearing conditions:

(19) $$H_Y + H_j + H_n = u^* H, \qquad \forall t$$

(20a) $$w_j = w_n$$

(20b) $$w_j = w_Y$$

Since human capital is perfectly homogeneous in the model, we impose that: 1) it is paid the same wage rate across all the productive sectors where this input is employed (equations (20a) and (20b)); 2) the sum of the human capital stocks allocated to each market is equal to the total stock of productive human capital available at time t (equation (19)).

Finally, as the total value of the household's assets must equal the total value of firms, the following condition must be checked in a symmetric equilibrium:

(21) $$a = nV_n$$

where V_n is given by (11) and satisfies the following asset pricing equation:

(21a) $$\dot{V}_n = rV_n - p_j$$

with:

(21b) $$p_j = \frac{aH_Y w_Y}{n}, \qquad \text{and}$$

(21c) $$w_Y = \frac{A(1-a)n}{H_Y^a} \cdot \left(\frac{BH_j}{n} \right)^a .$$

Recall that one new idea allows a new intermediate firm to produce one new variety of capital goods. In other words, there exists a one-to-one relationship between number of ideas, number of capital goods producers and number of intermediate input varieties. This explains why, in equation (21), the total value of the household's assets (*a*) is equal to the number of profit-making intermediate firms (*n*) times the market value (V_n) of each of them (equal to the market value of the corresponding idea). Finally, equation (21a) simply suggests that the interest on the value of the *j-th* generic intermediate firm (rV_n) should be equal, in equilibrium, to the sum of two terms:

- the instantaneous monopoly profit (p_j) coming from the production of the *j-th* capital good;
- the capital gain or loss matured on V_n during the time interval *dt* ($\dot{V}_n$).

We can now move to the steady-state equilibrium.

3. Steady State Equilibrium

We first start with a formal definition of balanced growth path equilibrium:

Definition (Balanced Growth Path or Steady-State Equilibrium):

*A balanced growth path (or steady-state) equilibrium is an equilibrium where the growth rate of all the variables depending on time is constant, human (H) and knowledge (*n*) capital are complements ($R \equiv H_t / n_t$ is constant) and H_Y, H_j, H_n all grow at the same constant rate as H.*

Defining with $g_z = \dot{z}/z$ the growth rate of variable z, when g_H is constant, u is constant as well (see equation (14)).[10] This means that, along a balanced growth path, the household will optimally decide to devote a constant fraction of its fixed time endowment to working (u*) and education (1-u*) activities.

Solving explicitly the consumers' problem, it is possible to show that the following results do hold in the long-run equilibrium (see the Appendix for details):

[10] As already said in paragraph 1.4, given the assumptions on the size of the representative household and the population growth rate, $h \equiv H$ (which implies that we can use g_H instead of g_h).

(22) $r = d(2-a) - r(1-a)$

(23) $g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r$

(24) $g_c = g_a = (2-a)(d - r)$

(25) $\dfrac{H_j}{n} = \dfrac{a\,d}{C(1-a)}$

(26) $\dfrac{H_Y}{n} = \dfrac{d}{aC}$

(27) $u^* = \dfrac{r}{d}.$

According to result (22), the real interest rate (*r*) is constant. Equation (23) states that along a balanced growth path, the number of new ideas (*n*), the household's total human capital stock (*H*) and the human capital stocks devoted respectively to the consumers goods production (H_Y), to the intermediate sector (H_j) and to research (H_n) all grow at the same constant rate, given by the difference between the human capital accumulation technology productivity parameter (d) and the subjective discount rate (r). Equation (24) gives the equilibrium growth rate of consumption and household's asset holdings. Equations (25) and (26), instead, give respectively the equilibrium values of the constant H_j / n and H_Y / n ratios, whereas equation (27) represents the optimal constant fraction of the household's time endowment that it will decide to allocate to working activities (*u**). For the growth rate of the variables in equations (23) and (24) to be positive and bounded , d should be strictly greater than r and bounded. The condition $d > r$ also assures that $0 < u^* < 1$.

4. Endogenous Growth

To compute the output growth rate of this economy in a symmetric, balanced growth equilibrium, first rewrite equation (1) as follows:

$$Y_t = AH_{Yt}^{1-a} n_t \left(\frac{B \cdot H_{jt}}{n_t} \right)^a = \Psi H_{Yt}^{1-a} n_t \,, \qquad \Psi \equiv A \left(\frac{B \cdot H_{jt}}{n_t} \right)^a .$$

Then, taking logs of both sides of this expression, totally differentiating with respect to time and recalling that in the steady-state equilibrium $g_{H_Y} = g_n = g_H = d - r$ (see equation (23) above), I obtain:

$$\frac{\dot{Y}_t}{Y_t} \equiv g_Y = g_c = g_a = (2-a)(d-r) = (2-a)g_H \,.$$

Thus, unlike the Lucas' (1988) and Blackburn *et alii's* (2000) models, output growth depends not only on human capital accumulation (g_H) but also on the technological parameter a that, for given *B*, can be easily interpreted as a measure of the monopoly power enjoyed by each intermediate local monopolist.[11] In Section 6, I'll come back to the long-run relationship between market power and growth as implied by the present model.

Since I am particularly interested in analysing those factors potentially able to influence the *inter-sectoral competition* for the acquisition of human capital in the present context, we have first to determine an expression for the equilibrium human to technological capital ratio ($R \equiv H/n$). At this aim, we use equation (19), with $u^* = r/d$, $H_j/n = a\,d/(1-a)C$ and $H_Y/n = d/aC$, and obtain:

(28) $$\frac{H_n}{n} = R\frac{r}{d} - \frac{d}{aC} - \frac{a\,d}{(1-a)C} \Rightarrow g_n = C\frac{H_n}{n} = \frac{r}{d}CR - \frac{d}{a} - \frac{a\,d}{(1-a)} \,.$$

Equating the last expression above to equation (23) yields:

(29) $$R \equiv \frac{H_t}{n_t} = \frac{d\left[d - ar(1-a)\right]}{ar(1-a)C} \,.$$

[11] Recall that the mark-up charged over the marginal cost by each capital goods producer is indeed equal to $1/aB$.

In equation (29), the human to technological capital ratio (R) has been expressed as a function of the human capital accumulation process productivity parameter (d), the technological capital accumulation process productivity parameter (C), the subjective discount rate (r), and a .

In the next section, I compute the equilibrium shares of human capital devoted to research (s_n), capital goods production (s_j), final good manufacturing (s_Y) and human capital accumulation (s_H).

5. Human Capital, R&D and Growth

Given R, computed in the last paragraph, the shares of human capital devoted to each sector employing this input in the decentralised long-run equilibrium are the following:

$$(30) \qquad s_j \equiv \frac{H_j}{H} = \frac{H_j}{n} \cdot \frac{n}{H} = \frac{H_j}{nR} = \frac{a^2 r}{d - ar(1-a)}$$

$$(31) \qquad s_Y \equiv \frac{H_Y}{H} = \frac{H_Y}{n} \cdot \frac{n}{H} = \frac{H_Y}{nR} = \frac{r(1-a)}{d - ar(1-a)}$$

$$(32) \qquad s_n \equiv \frac{H_n}{H} = \frac{H_n}{n} \cdot \frac{n}{H} = \frac{H_n}{nR} = \frac{ar(d-r)(1-a)}{d\left[d - ar(1-a)\right]}$$

$$(33) \qquad s_H \equiv \frac{H_H}{H} = 1 - u^* = \frac{d-r}{d}\ .$$

5.1. Some Comparative Statics Results

From equation (30) it is possible to state the following comparative statics results (throughout this analysis I'll continue to assume $g_H > 0$, which implies $d > r > 0$):

$$(30a)\ \frac{\partial s_j}{\partial a} > 0; \qquad (30b)\ \frac{\partial s_j}{\partial d} < 0; \qquad (30c)\ \frac{\partial s_j}{\partial r} > 0.$$

Equations (30a) through (30c) say that the equilibrium share of human capital devoted to the capital goods sector depends negatively on the human capital accumulation productivity parameter (d) and positively on a and the subjective discount rate (r). I am particularly interested in studying the impact that the monopoly position enjoyed by each local intermediate producer may have on the main variables of the model in the long-run equilibrium. At this aim, first notice that, for given *B*, $1/a$ does represent, as already mentioned, a *proxy* for the mark-up charged over the marginal cost by the intermediate producers. Indeed, the higher a, the higher the elasticity of substitution between two generic intermediate inputs (equal to $1/(1-a)$). This means that they become more and more alike when a grows and, accordingly, the price elasticity of the derived demand curve faced by a local monopolist (equal, again, to $1/(1-a)$) tends to be infinitely large when a tends to one. In a word, the toughness of competition in the intermediate sector is strictly (and positively) depending on the level of a. Conversely, the inverse of a ($1/a$), can be viewed as a proxy for how uncompetitive the sector is.

Intuitively, what equation (30a) tells us is that when a increases, the degree of competition within the capital goods market increases and, then, the aggregate intermediate output and the human capital demand coming from this sector do increase as well (s_j goes up). Therefore, a re-allocation of the available human capital among all the sectors employing this input does happen. In other words, when the monopoly power enjoyed by intermediate local monopolists rises, then a decentralised market equilibrium will allocate less and less resources to capital goods production (which, indeed, is not the true engine of growth within this economy).

As for the equilibrium share of human capital devoted to the consumer good sector, we conclude that:

$$(31a)\ \frac{\partial s_Y}{\partial a} < 0; \qquad (31b)\ \frac{\partial s_Y}{\partial d} < 0; \qquad (31c)\ \frac{\partial s_Y}{\partial r} > 0.$$

Hence, unlike what happens for s_j, now an increase in the mark-up rate does increase the decentralised equilibrium share of human capital devoted to the production of the final good. Again, the economic intuition behind this result is quite simple: an increase in the mark-up rate (and in this way in the price) of all the intermediate inputs, *ceteris paribus*, makes it more profitable for the final good producers to substitute human capital for capital goods. As a consequence, the demand for this factor input (H_Y) increases and, for given total human capital stock, s_Y increases as well. The effects of r and d on s_Y are exactly the same as those found on s_j.

Coming now to the comparative statics results for the equilibrium share of human capital devoted to R&D activity (s_n), I find that:

(32a) $\frac{\partial s_n}{\partial a} > 0$, when $0 < a < 1/2$ and

$\frac{\partial s_n}{\partial a} < 0$, when $1/2 < a < 1$.

This means that the impact that the intermediate sector monopoly power exerts upon s_n is not unambiguous and crucially depends on the absolute size of the monopoly power itself: when the level of competition among local intermediate monopolists is low (a is low), a further increase in market power contributes to reduce the amount of resources employed by the R&D sector, whereas, when the level of competition is high (a is high), it is possible to increase s_n through an increase in the mark-up rate. The relationship between market power and the share of human capital to the R&D sector is illustrated below:[12]

[12] In the graph I have set $d = 0.10$ and $r = 0.08$. As long as one assumes $d > r$, the behaviour of $s_n(1/a)$ does not change at all.

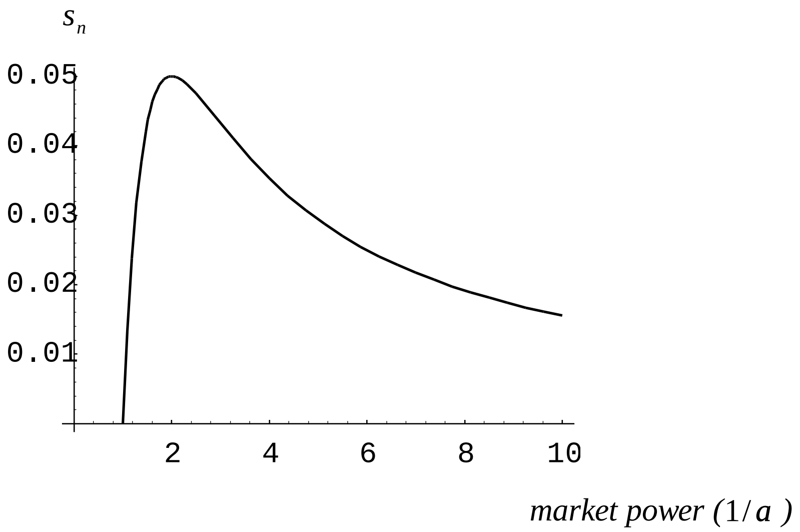

Figure 1

The relationship between (a proxy of) the level of competition in the intermediate sector ($1/a$) and the equilibrium share of human capital devoted to research (s_n).

As it is evident from the figure above, there exists a critical level of (the proxy for) the monopoly power enjoyed by capital goods producers that maximizes s_n: this level is equal to $1/a = 2$. Therefore, unlike Jones and Williams (2000)[13], our analysis underlines the fact that in the presence of human capital accumulation and when all the sectors employ skilled workers it is no more obvious that in the decentralized equilibrium steady-state the R&D share is always increasing in the mark-up.[14] On the contrary, we may conclude that the relationship between monopoly power and s_n is highly non-monotonic in the present context. This striking result is explained as follows: when the level of competition in the capital goods sector is high, a higher mark-up increases, *ceteris paribus*, the flow of profits accruing to intermediate producers, which in turn increases the market value of one unit of research output, raising R&D investment. On the contrary, when the level of competition among intermediate firms is low, a further increase in the mark-up rate leads final output producers to use more and more human capital. This *substitution*

[13] Where there is no human capital accumulation and the inter-sectoral competition for the same resource (foregone consumption) is restricted to the intermediate and research sectors.

[14] As in the present model, in Jones and Willimas (2000) the mark-up is determined by the elasticity of substitution between intermediate capital goods, too.

effect of capital goods with skilled workers in the downstream sector is so strong to induce a shift of resources away from the research and the intermediate sectors.[15]

Concerning the effect that d and r do have on s_n, again it is possible to see that it is not unambiguous and crucially depends on the absolute value of the human capital accumulation productivity parameter (d). Indeed, one can easily show that:

(32b) $\frac{\partial s_n}{\partial d} > 0$ and $\frac{\partial s_n}{\partial r} < 0$ when $r < d < r\left(1+\sqrt{1-a(1-a)}\right)$

(32c) $\frac{\partial s_n}{\partial d} < 0$ and $\frac{\partial s_n}{\partial r} > 0$ when $d > r\left(1+\sqrt{1-a(1-a)}\right)$

Finally, the comparative statics results for s_H and R are as follows:

(33a) $\frac{\partial s_H}{\partial a} = 0$; $\frac{\partial s_H}{\partial d} > 0$; $\frac{\partial s_H}{\partial r} < 0$

(29a) $\frac{\partial R}{\partial a} > 0$ when $1/2 < a < 1$ and $\frac{\partial R}{\partial a} < 0$ when $0 < a < 1/2$;

(29b) $\frac{\partial R}{\partial r} < 0$; $\frac{\partial R}{\partial d} > 0$; $\frac{\partial R}{\partial C} < 0$.

Below I report a table that summarizes all the comparative statics results:

[15] Notice, indeed, that in this case ($0 < a < 1/2$), $\partial s_j / \partial a > 0$, $\partial s_n / \partial a > 0$ and $\partial s_Y / \partial a < 0$. In a moment, I'll also show that $\partial s_H / \partial a = 0$ for each value of a .

	d	r	a	g_Y	g_H	$R \equiv \frac{H}{n}$	s_n	s_j	s_Y	s_H
$r < d < r\left(1+\sqrt{1-a(1-a)}\right)$	$\uparrow$			+	+	+	+	-	-	+
$d > r\left(1+\sqrt{1-a(1-a)}\right)$	$\uparrow$			+	+	+	-	-	-	+
$r < d < r\left(1+\sqrt{1-a(1-a)}\right)$		$\uparrow$		-	-	-	-	+	+	-
$d > r\left(1+\sqrt{1-a(1-a)}\right)$		$\uparrow$		-	-	-	+	+	+	-
$0 < a < 1/2$			$\uparrow$	-	0	-	+	+	-	0
$1/2 < a < 1$			$\uparrow$	-	0	+	-	+	-	0

Table 1: *Comparative Statics Results Summary*

In the first and second row I see what happens to g_Y, g_H, R, s_n, s_j, s_Y and s_H when d increases and falls respectively in the two following intervals: 1) $r < d < r\left(1+\sqrt{1-a(1-a)}\right)$ (first row); 2) $d > r\left(1+\sqrt{1-a(1-a)}\right)$ (second row). In rows number 3 and 4 I do the same with r. In the third row I analyse the sign of the impact of an increase in r on g_Y, g_H, R, s_n, s_j, s_Y and s_H when $r < d < r\left(1+\sqrt{1-a(1-a)}\right)$, whereas in the fourth row I analyse the sign of the impact of the same increase in r on the above-mentioned variables when $d > r\left(1+\sqrt{1-a(1-a)}\right)$. Finally, in the last two rows I analyse what happens in the long run to the main variables of the model when a increases (the level of competition in the intermediate sector becomes tougher and tougher) and again I keep distinguished into the analysis two intervals: $0 < a < 1/2$ and $1/2 < a < 1$.

5.2. Discussion

First of all it is important to notice that g_Y, g_H, s_n, s_j, s_Y and s_H do not depend on the productivity with which human capital is employed in the intermediate sector (the parameter B) and the research one (C). These variables are also independent on the total factor productivity (A). Instead, the ratio of human to technological capital (R) does depend on the parameter C (the higher the research human capital productivity, the higher the amount of resources invested in this sector, the higher the number of available capital goods, the lower R).

Looking at Table 1 (first row), one can also see that when $r < d < r\left(1+\sqrt{1-a(1-a)}\right)$, then g_Y, g_H, R, s_n and s_H are positively correlated to each other. On the contrary, s_n and s_H turn out to be negatively correlated with s_j and s_Y, meaning that within the above mentioned interval when the productivity of human capital in the formation sector increases, then the economy allocates more resources to education and research and less resources to intemediates and final output production. This, in turn, has the effect to boost economic growth (which depends on human capital accumulation) and to make human capital relatively abundant with respect to technological capital (R increases[16]). On the other hand, when d is sufficiently high ($d > r(1+\sqrt{1-a(1-a)})$), s_n, s_j and s_Y are positively correlated with each other and negatively correlated with g_Y, g_H, s_H and *R*. Hence, the hypothesis one can infer is that when human capital is particularly productive in the education sector, further increases in d push the investment in formation up and reduce the investment (in terms of human capital) in the other three sectors competing for the same input. All this implies a generalised increase in both the steady-state growth rate of the economy and the ratio *R*.

The effect that an increase in r has on the main variables of the model is perfectly consistent with what we have just said. Indeed, in the interval $r < d < r(1+\sqrt{1-a(1-a)})$, g_Y, g_H, R, s_n and s_H are positively correlated with each other and negatively correlated with s_j and s_Y.

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[16] Evidently *R* can also be written as: $R = \dfrac{a\,d}{s_j(1-a)C}$. An increase in d determines an increase in the numerator, a reduction in the denominator (through the effect on s_j) and an increase in *R*.

Instead, in the interval $d > r(1+\sqrt{1-a(1-a)})$, g_Y, g_H, R and s_H are positively correlated with each other and negatively correlated with s_n, s_j and s_Y.

Overall, the result comes out that, contrary to Jones and Williams (2000) where the steady state share of R&D is monotonically increasing in the steady state output growth rate, the relationship between s_n and g_Y is not monotonic in a context where human capital is allowed to grow over time through optimizing behaviour of rational agents and human and technological capital are complements. The result stated by Jones and Williams remains true either when the productivity parameter of human capital accumulation is sufficiently low or when the level of competition in the intermediate sector is sufficiently high.

Finally, as for the impact of a (the *proxy* for the monopoly power in the model) on the other variables, we notice in general that an increase in a increases s_j and reduces s_Y without any ambiguity. At the same time, this parameter does not play any role both on g_H and s_H. Indeed, as in the recent Blackburn *et alii*'s (2000) paper, the eventual market power enjoyed in the monopolistic sector does not play any role on the consumers' decision about how much time to invest in education and training (such a decision being solely driven by the parameters describing preferences and human capital technology and absolutely independent on the R&D activity). However, variations in a do influence the allocation of human capital between the capital goods, final output and research sectors ($1/a$ is positively correlated with s_Y; negatively correlated with s_j and its relationship with s_n is ambiguous a priori, as already explained). More importantly, and unlike the Blackburn *et alii*'s (2000) paper, the present analysis shows that introducing in the simplest possible way a model of endogenous technological change (*à la* Romer, 1990 and Grossman and Helpman, 1991, Chap.3) within a basic Lucas (1988) model allows us to predict an unambiguously positive relationship between imperfect competition and growth. The next section is devoted to a deeper study of such a link.

6. Imperfect competition, scale effects and growth

As I have shown in Section 4, the output growth rate of this economy is:

$$\frac{\dot{Y}_t}{Y_t} \equiv g_Y = (2-a)g_H = (2-a)(d-r) .$$

Hence economic growth depends only on the technological parameter a and the accumulation rate of human capital (g_H). In this last respect, the model supports the main conclusion of that branch of the endogenous growth literature pioneered by Uzawa (1965) and Lucas (1988).[17] As a consequence, our analyses does not display any scale effect, since g_Y depends neither on the absolute dimension of the economy (its total human capital stock), nor on the population growth rate.[18] Another feature of this model is that when a tends to one it allows us to obtain the same equilibrium growth rate as in the seminal Lucas' (1988) work, whereas when a tends to zero we get a growth rate which is the double of the Lucas' (1988) one. Consider, first, the case where $a \to 1$. Under this circumstance, all the existing intermediate inputs tend to be perfect substitutes with each other and prices equal marginal costs in each sector. In addition, the technologies in use in our economy remain unchanged, with the exception of the one employed in the consumers good sector, which now becomes:

$$Y_t = A\int_0^{n_t} x_{jt} dj .$$

[17] Benhabib and Spiegel (1994), Islam (1995) and Pritchett (1996) all suggest that, unlike Lucas (1988), international differences in per-capita growth rates depend exclusively on differences in the respective human capital stocks each country is endowed with. However Jones (1995a,b) points out that the *scale effect hypothesis* should be rejected.

[18] The prediction of no scale effects is indeed shared by many other models (e.g. Kortum, 1997; Aghion and Howitt, 1998a, Chap.12; Dinopoulos and Thompson, 1998; Peretto, 1998; Peretto and Smulders, 1998; Segerstrom, 1998; Young, 1998; Howitt, 1999; Blackburn et alii, 2000; Bucci, 2001, among others). See Jones, 1999 and Eicher and Turnovsky (1999) for recent surveys.

Therefore, human capital enters only indirectly (through x_j) the final good manufacturing and the household devotes the entire fraction *u* of its own fixed time endowment to the production of intermediates ($s_j \to r/d$) and continues to devote the fraction *(1-u)* to human capital accumulation ($s_H \to (d-r)/d$). A share of human capital equal to zero is devoted in equilibrium both to the final output sector and to research (s_Y and s_n both tend to zero). In the long-run, when *x* is constant and equal for each intermediates producer, the growth rate of the economy is given by the growth rate of the total human capital stock devoted to the capital goods sector (H_j), which, in turn, is equal to the growth rate of *H* along a balanced growth path.

Consider now the case where $a \to 0$. In this case, the technology in use in the downstream sector becomes:

$$Y_t = AH_{Yt}n_t.$$

It is evident that now the demand function faced by each intermediates producer is negatively sloped and they consider the price for their own output no longer as given. Since the market power enjoyed by these producers tend to be infinitely large when $a \to 0$, final good producers will prefer to substitute human capital for intermediates. This is reflected in the fact that now $s_j \to 0$ (no human capital is devoted in equilibrium to the intermediate sector). On the other hand, the household devotes the entire fraction *u* of its own fixed time endowment to the production of the consumers good ($s_Y \to r/d$) and continues to devote the fraction *(1-u)* to human capital accumulation ($s_H \to (d-r)/d$). Just as s_j, so too a share of human capital equal to zero is devoted in equilibrium to research (s_n tends to zero as well). Even though $s_n \to 0$ when $a \to 0$, the number of existing varieties of capital goods (*n*) may grow over time, since $\dot{n}/n = C \cdot s_n (H/n) = d - r$ is independent of a in the long run. Given all this, the growth rate of the economy is now given by the growth rate of the total human capital stock devoted to the final good sector (H_Y), plus the growth rate of *n* (both equal to the growth rate of *H* along a balanced growth path). This explains why in this economy the growth rate of output is twice as big as the output growth rate of the basic Lucas (1988) model when we allow a going to zero within our context.

It is probably worth noting at this point that all the results of this model (including its prediction about the absence of any scale effects) are obtained using explicitly the hypothesis that human and technological capital are complements (the value of the ratio $R = H_t / n_t$ remains invariant along the balanced growth path). Such a hypothesis may be justified on both theoretical and empirical grounds. From the theoretical point of view, Redding (1996) clearly shows that the complementarity relationship between skilled workers and technology does represent a crucial element in explaining the existence of poverty traps in many less developed countries, due to the joint presence of low levels of skills and R&D investment in these areas. He also shows that, under particular conditions, the complementarity hypothesis between human capital and R&D is also responsible for the existence of multiple steady states. On the empirical side, instead, many contributions claim the relevance of the skill-technology connections even at the sectoral level (Goldin and Katz, 1998), whereas de la Fuente and da Rocha (1996) also find evidence of strong complementarities between human capital stock and investment in R&D for the OECD countries.[19]

Coming now to the relationship between market power and growth within the present framework, we see that such a relationship is definitely unambiguously positive. In the figure below, I report the behaviour of g_Y as a function of $1/a$:[20]

[19] Other works where the skill-biased technical progress hypothesis is analysed (both theoretically and empirically) are, among others, Bartel and Lichtenberg (1987), Berndt, Morrison and Rosenblum (1992), Bell (1996), Machin, Ryan and Van Reenen (1996), Doms, Dunne and Troske (1997), Bartel and Sicherman (1998), Machin and Van Reenen (1998).

[20] In the figure I continue to assume $d = 0.10$ and $r = 0.08$.

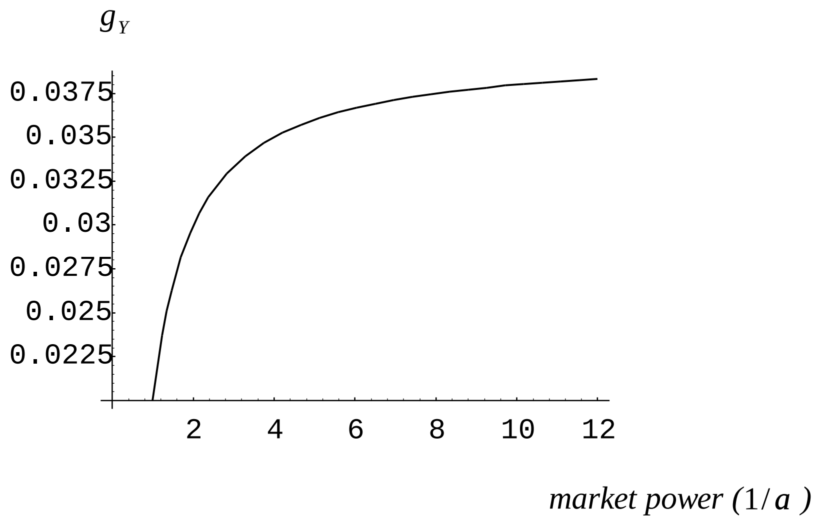

Figure 2

The relationship between (a proxy of) the level of competition in the intermediate sector ($1/a$) *and the equilibrium output growth rate* (g_Y).

Intutively, looking at the production function of the homogeneous final good, one realises that in the steady-state equilibrium (with x constant), an increase in the level of output may be determined either by the growth of H_Y or the growth of n (or the growth of both). Since the growth of n is itself (for the complementarity hypothesis between human and technological capital) induced by the growth of H (independent of the mark-up rate), the only way for the market power to influence output growth is through varying the level of H_Y. In turn, H_Y can be decomposed in two parts:

$$H_Y \equiv s_Y \cdot H \,, \quad s_Y = H_Y / H \,.$$

While the impact of a variation in $1/a$ on human capital accumulation is null, an increase in the market power variable exerts an unambiguously positive effect on s_Y. In other words, it is through allocating an higher share of human capital towards the final output sector that monopoly power positively affects growth in the model.

The positive relationship between competition and growth is now empirically widely accepted. Recent works (Blundell *et al.*, 1995 and Nickell, 1996) suggest indeed a positive correlation between product market competition, on the one hand, and firm/industry level productivity growth, on the other. This definitely corroborates the idea that product market competition is unambiguously good for growth. On the theoretical side, Aghion *et al.* (1997a,b) and Aghion and Howitt (1996; 1998b) do reconcile this evidence with the *Schumpeterian growth paradigm* taking into account three possible explanations (respectively based on agency considerations, the tacit nature of knowledge, and the decomposition of R&D activities into research and development). Still, their result relies on a model in which the engine of growth is represented by the continuous improvement of the quality level of already existing goods. What the present paper has shown is that results might change within a *horizontal differentiation model* of endogenous growth, in which the engine of growth is represented by human capital accumulation and the choice of utility-maximising agents to accumulate human capital complements the one of profit-seeking firms to invent new varieties of intermediate goods.

7. Concluding Remarks

In this paper I have analysed the steady-state predictions of an endogenous growth model with both purposive R&D activity and human capital accumulation. In the economy, human and technological capital are complements to each other and there exists no pecuniary externality in their accumulation process. Finally, I have assumed that human capital enters as an input in all the activities performed in this economy in order to analyse the economic forces underlying the inter-sectoral allocation of skilled workers. Using a theoretical framework where technological progress shows up in the form of the creation of new horizontally differentiated capital gods, the long-run relationship between imperfect competition and growth has also been deeply studied.

The results of the model can be summerised as follows. First of all, the steady-state output growth rate depends solely on the parameters describing preferences and the human capital accumulation technology and is completely independent of R&D activity. As a consequence, the model does not display any scale effect. This property is nowadays shared by many other endogenous growth models. Secondly, I find that the share of human capital devoted to research is not monotonically increasing either in the steady-state growth rate or the market power enjoyed by intermediate producers. Indeed, the result found by Jones and Williams (2000) – according to which the share of resources invested in R&D increases without any ambiguity with the aggregate output growth rate - remains true in the present context when the productivity parameter of human capital accumulation is not sufficiently high and the intermediate sector is competitive enough. Finally, as for the impact of monopoly power on the other main variables of the model, I find that the presence of imperfect competition conditions among the capital goods producers has positive growth effects and may dramatically influence the allocation of the reproducible factor input (human capital) to the economic sectors employing it. We think this is as an important as an alternative result in comparison with other papers that, unlike the approach taken here, consider the technological progress as basically stemming from a continuous vertical differentiation process.

APPENDIX

In this Appendix, I derive the set of results (22) through (27) in the main text.

As already mentioned in the main text, from equation (14), when g_h is constant, u_t turns out to be constant as well. This means that in equilibrium the household devotes a constant fraction of its own time endowment to working (*u*) and to education (*1-u*) activities. Consequently, the optimal *u* (*u**) will be constant and endogenously determined through the solution to the household decision problem. Consider now this problem (equations (12) through (14) in the main text), whose first order conditions (equations (15)-(18)) are reported here below for convenience, together with the consumer's constraints and the transversality conditions:

(15) $\frac{e^{-rt}}{c_t} = l_{1t}$

(16) $l_{1t} = l_{2t}\frac{d}{w_t}$

(17) $l_{1t}r_t = -\dot{l}_{1t}$

(18) $l_{1t}w_tu_t + l_{2t}d(1-u_t) = -\dot{l}_{2t}$

(13) $\dot{a}_t = r_ta_t + w_tu_th_t - c_t$

(14) $\dot{h}_t = d(1-u_t)h_t, \qquad d > 0$

$\lim_{t\to\infty} l_{1t}a_t = 0$

$\lim_{t\to\infty} l_{2t}h_t = 0$

From now on I will omit in this appendix the index *t* near the time dependant variables. Combining equations (16) and (18) we get:

(1) $\frac{\dot{l}_2}{l_2} = -d$

whereas, from (17):

(2) $\frac{\dot{l}_1}{l_1} = -r$

In a symmetric, steady-state long-run equilibrium H_Y, H_j, H_n and n all grow at the same constant rate as H (denoted by g_H). This definition of steady-state implies that x (the output produced in the symmetric equilibrium by each local monopolist) is constant over time and (from equations (5) and (21c) in the main text) the wage rate accruing to one unit of skilled labour (w_t) grows at a rate equal to $(1-a)g_H$. Then, using equation (16) in this appendix, we get:

(3) $\frac{\dot{l}_1}{l_1} = \frac{\dot{l}_2}{l_2} - (1-a)g_H \qquad \Rightarrow$

(3') $r = d + (1-a)g_H$.

This means that in equilibrium (when g_H is constant), the real interest rate (r) is constant as well. From equation (8) in the main text, it follows that the profit rate accruing to capital goods producers does grow at the rate $(1-a)g_H$, as well. In turn, this allows us to re-write equation (11) in the main text as:

$$\text{(11a)} \quad V_{nt} = \int_t^{\infty} Aa(1-a)H_{Yt}^{1-a}\left(\frac{BH_{jt}}{n_t}\right)^a e^{-r(t-t)}dt = Aa(1-a)\left(\frac{BH_{jt}}{n_t}\right)^a \frac{H_{Yt}^{1-a}}{[r - g_H(1-a)]} =$$

$$= Aa(1-a)\left(\frac{BH_{jt}}{n_t}\right)^a \frac{H_{Yt}^{1-a}}{d}.$$

According to the equation above, the market value (the discounted flow of future profits) of a generic *j-th* intermediate firm (equal to the market value of the corresponding *j-th* idea) grows in the long run equilibrium at the rate $(1-a)g_H$. Using equations (10) in the main text and (11a) above, it is possible to conclude that:

$$\text{(11b)} \quad w_{nt} = ACa(1-a)\left(\frac{BH_{jt}}{n_t}\right)^a \frac{H_{Yt}^{1-a}}{d}.$$

Employing equations (20a) and (5) in the main text and equation (11b) above, we get:

(4) $\frac{H_j}{n} = \frac{a\,d}{C(1-a)}$,

whereas using equations (20b), (5) and (21c) in the main text, the result comes out that:

(5) $\frac{H_Y}{n} = \frac{1-a}{a^2} \cdot \frac{H_j}{n} = \frac{d}{aC}$.

Combining equations (15) and (17) of this appendix, I find the usual Euler equation, giving the optimal household's consumption path:

(6) $\frac{\dot{c}}{c} \equiv g_c = r - r = d - r + (1-a)g_H$.

Dividing both sides of equation (13) by *a*, we get:

(7) $\frac{c}{a} = r + wu\frac{h}{a} - g_a$.

We already know that in the steady-state equilibrium *r*, *u* and g_a are constant. Therefore, for the ratio *c/a* to be constant it should be the case that wh/a is constant. Indeed, *h* grows at the rate g_H, *w* grows at the rate $(1-a)g_H$ and $g_a = (2-a)g_H$ since $a = nV_n$ (from equation (21) in the main text), $g_n = g_H$ (for the complementarity hypothesis between human and knowledge capital) and $g_{V_n} = (1-a)g_H$ (see equation (11a) in this appendix). Hence, we can conclude that in equilibrium the growth rate of wh/a is equal to zero and the ratio *c/a* is constant. In other words, consumption (*c*) and asset holdings (*a*) do grow at the same constant rate along a balanced growth path. This implies that:

(8) $g_c = g_a = r - r = d - r + (1-a)g_H$.

Finally, to find out the optimal *u**, one first equates equation (6) in this appendix with the value of g_a and obtains:

(9) $g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r$.

Then, plugging equation (9) into (14):

(10) $\frac{\dot{h}}{h} \equiv g_H = d(1-u) = d - r \Rightarrow u^* = \frac{r}{d}$.

For g_H to be strictly positive, d should be strictly greater than r, which in turn implies $0 < u^* < 1$.

When $g_H = d - r > 0$, the real interest rate and the growth rate of consumption and asset holdings become respectively:

(3'') $r = d(2-a) - r(1-a) > 0$;

(8') $g_c = g_a = (2-a)(d-r)>0$.

Also notice that, when $r = d(2-a) - r(1-a)$ and $g_H = d - r$, then the two transversality conditions are trivially checked since:

$\lim\limits_{t\to+\infty} l_{1t} \cdot a_t = \lim\limits_{t\to+\infty} l_{10} \cdot a_0 \cdot e^{-rt} = 0$, and

$\lim\limits_{t\to+\infty} l_{2t} \cdot h_t = \lim\limits_{t\to+\infty} l_{20} \cdot h_0 \cdot e^{-rt} = 0$.

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