KALDORIAN IDEAS IN THE FRAMEWORK OF "OLD" AND "NEW" THEORIES OF CYCLICAL GROWTH : AN ASSESSMENT [*]

(Provisional draft)

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The purpose of this paper is to offer an assessment of the importance of the role played by kaldorian ideas in the framework of both "old" and "new" theories of cyclical growth. Kaldor's contribution to the theory of business cycles (BC) is far from being negligible. A large part of it is mainly critical and concerned the main various versions of the BC models of the thirties, forties and fifties. Hawtrey's economic theory provided the first target of Kaldor's reflections: Kaldor showed how the consequences of Hawtrey's distinction between short-term and long-term investments were underestimated and had to be faced systematically (Kaldor, 1938). Kaldor also devoted three articles (Kaldor, 1939, 1940a, 1942) to the critique of Hayek's theory of BC, pointing out some of the shortcomings of the Austrian theory of capital. In the beginning of the forties, Kaldor modified Pigou's macroeconomic model in **Employment and equilibrium**, showing how these changes could affect its ability to prove the existence of a persistent tendency of the economic towards a long-period equilibrium (Kaldor, 1941). After the war, he finally criticised Hick's model of the trade cycle and especially its treatment of expectations (Kaldor, 1951). Kaldor's contribution to BC theory was also constructive however. In 1940, he built his famous non linear model (Kaldor, 1940b) coming back to it thirty years later (Kaldor, 1971). In 1954, he faced the problem of the relation of economic growth and cyclical fluctuations (Kaldor, 1954). He touched again the topic in connection with his use of Okun's law in his last book (Kaldor, 1985). By the way, Kaldor never neglected the empirical aspects of business cycles (see, for instance, Kaldor, 1955).

Kaldor also devoted a substantial part of his reflections to the theory of economic growth, from his first model (Kaldor, 1957) to his last article (Kaldor, 1986). The volumes 2 and 9 of his **Collected Papers** (Kaldor, 1960, 1980 and 1989) show the evolution of his thought on the subject: in the thirties, he seemed to have strong doubts on the natural tendency of the economic system towards a steady state growth path and insisted on the fundamental cyclical instability of capitalism; in the fifties, he switched to a more optimistic view of market economies evolution, arguing that with the help of an adequate economic policy, industrialised economies could tend to a balanced growth trajectory; on the contrary, in his later growth models, "Kaldor laws" implied cumulative changes which could hardly converge to any form of long-term dynamic equilibrium.

The importance of Kaldor's contribution to the theories of growth and BC does not imply however that the author convincingly connected both analytical fields. We already noted that,

in the early phase of his professional life, Kaldor considered that capitalist economies were naturally unstable. Moreover, even economic policy could reduce the extent of fluctuations but could never eliminate them. Referring to the opportunity of such an adequate policy, Kaldor wrote in 1938:

> "Even if it succeeded, I do not think that we could hope from it the complete absence of cyclical tendencies; it would only ensure that fluctuations were confined to much narrower limits" (Kaldor, 1938/1960, p.118).

This conviction and the fact that, in the thirties and the forties, economists did not pay much attention to growth theory explain why we cannot find any clear consideration on cyclical growth in this period of Kaldor's life. During the fifties and the sixties, things could have been different. The problem of the relation of economic growth and BC became a major issue for economic theorists and Kaldor (1954) explicitly faced it. However, Kaldor argued that

> "it is not necessary to assume economic growth or dynamic change in order to account for the existence of fluctuations" (Kaldor, 1954/1960, p.221).

This fundamental assumption lead him to note that

> "it is thus seen that all the "dynamic" models that were recently presented to the world –such as Kalecki's amended models, Marrama's, Hick's or Goodwin's (…) are all variants of the same thing and, essentially, all consist of the superimposition of a linear trend introduced from the outside on an other wise trendless model without altering, in any way, its basic character. Some of the authors mentioned deny this. Thus Professor Hicks thinks that "it is on the trend rate of growth that the whole cyclical mechanism depends" and that in the absence of the growth in autonomous investment the cycle could not get off the floor– or, at any rate, not for an inordinately long time. Similar ideas were expressed also by Mr Goodwin. This, in my view, is mistaken, and must be due to an insufficient appreciation by the authors of the properties of their own models" (Kaldor, 1954/1960, pp.224-225).

From this standpoint, BC could be viewed "as a special case of the "dynamic equilibrium" where the equilibrium rate of growth happens to be zero" and economic growth could be analysed in itself or in combination with BC (p.226).

This type of solution is not convincing at all, as we know today, and Pasinetti pointed it out as soon as 1960. Referring to the various models of BC based respectively on the multiplier/accelerator principle, and to the steady state growth models, he noted:

> "The situation is that, on the one hand, the macro-economic models which provide a cyclical interpretation of the economic activity cannot give any explanation of economic growth and, on the other hand, these theories which define, or rely on the conditions for a dynamic equilibrium to be reached and maintained cannot give an explanation of business cycles" (Pasinetti, 1960, p.69).

This is one of the various reasons why, in the seventies and the eighties, Kaldor changed again his mind. He introduced in his analysis dynamic increasing returns (along to the Marshall-Young line of thought (Kaldor, 1977/1989, pp.424-426 and 1985, pp.63-72)), structural change (on this issue, see Targetti, 1992, section 15.5 "a structuralist political economist", pp.353-354) and cumulative causation (Kaldor, 1985, chapter III). This new conception changed substantially Kaldor's views. As Targetti stressed it, analysing Kaldor's life late years view:

> "the very nature of the dynamic process entails the passage of time: this is why changes occur in the parameters and in the structure of the functions that represent it. Consequently, any attempt to assert once-and-for-all general laws, or a general theory is vacuous. Kaldor's method, above all when it deals with the theory of growth, is the building of an economic theory with its roots in history. In consists in the identification of processes and phenomena which develop over the long period, and with such regularity that they stand out from a myriad of contingent data to constitute stylised facts, thus acquiring economic and social, and therefore political significance" (Targetti, 1992, p.351).

In this framework, it is clear that, to use a robinsonian terminology, history excludes equilibrium. The presence of dynamic increasing returns is sufficient in itself according to Kaldor to exclude any theory of steady-state growth:

"The existence of increasing returns, even if confined to particular sectors of an economy, such as manufacturing, are bound to cause very large differences to the reaction pattern of the economy. There is no inherent tendency to anything that could be called an equilibrium, or an equilibrium path" (Kaldor, 1985, p.68).

These ideas which correspond to the last part of Kaldor's life fit with the conception of "economics without equilibrium". However, they do not give any clear indication on the possible ways of elaborating a theory of cyclical growth. They confirm therefore the existence of what Targetti has called "a discrepancy between his theories on the cycle and on growth" (Targetti, 1992, p.349). Commenting, by the way, one of his discussions with Kaldor in the eighties, referring to these two theories, Targetti noted:

"When I asked him how the two theoretical components could be fitted together, he simply replied that they should be treated as different theories dealing with different historical periods" (Targetti, 1992, p.349).

It could be therefore surprising to focus here on the role played by kaldorian ideas in the framework of cyclical growth theory. However, the richness of a theory is also related to its potentialities, namely, to its ability to cope with new problems which did not exist or were not seen when it was built. And we think that Kaldor's approach belongs to this type of theory: all the ingredients are present in it for building a model of cyclical growth.

This is why we will first remind what these ingredients are, referring to Kaldor's own writings. We will then combine them in a very simple model which captures what a more complete kaldorian theory of cyclical growth could consist in. Secondly, we will investigate how these kaldorian ideas could also be incorporated in a "new" growth type model and analyse if they might affect the usual analytical results which have already been obtained.

Cyclical growth and kaldorian ideas

We will not focus here on the demand side but on the supply side of a process of cyclical growth. Our only justification is simplification but we will come back to the problem of demand in the end of this section. Moreover, this choice fits with the fact that in Kaldor's

approach, demand is fundamentally exogenous, even in the framework of growth theory (see Targetti, 1992, p.350).

We know that, in Kaldor's model of the trade cycle, the basics of dynamics was provided by the interaction between a saving function and an investment function. Focusing here on the supply side of the kaldorian approach, we will first welcome the second of these functions. Therefore, investment is supposed to vary positively with the level of activity. The foundation of this functional relation rests on "the assumption that the demand for capital goods will be greater the greater the level of production" (Kaldor, 1940/1989, p.149). In accordance with Kaldor's model, however, the investment function is not linear, and also depends negatively on the level of the total amount of equipment; the conditions of capital accumulation indeed vary according to the volume of existing capital: this is the reason why Kaldor distinguished different **levels** of capital related to various **levels** of the investment S-shaped curve (see Kaldor, 1940/1989, appendix).

Secondly, we know that one of the main critiques Kaldor addressed to the usual neo-classical production function was its assumption of substitutability. Kaldor's criticism of the neo-classical principle of factor substitution, referring to the **creative** rather than to the **allocative** function of economic activity, is well known. In the theories of allocation,

> "the principle of substitution (as Marshall called it) or the "law of variable proportions" or of "limited substitutability" is elevated to the central principle on the basis of which both the price system and the production system are explained; and it is implied that the world is one where elasticities of substitution are all important. This approach ignores the essential complementarity between different factors of production (such as capital and labour) or different types of activities (such as that between primary, secondary and tertiary sectors of economy [we might add today "as between sector-specific intermediate inputs" – AR and RA]) which is far more important for the understanding of the laws of change and development of the economy than the substitution aspect" (Kaldor, 1975/1989, p.400).

This analytical choice is favour of the principle of good complementarity explains why, in accordance with Palley's kaldorian model (Palley, 1997) we will favour an AK type of endogenous growth model. In this context, it might be noted that our approach also has a "classical" flavour since it

"assumes implicitly that (simple) labour is a producible resource, whose cost of production is constant, that is, equal to the real wage rate: at the given wage labour is in elastic supply, and the supply is fully adjusted to the needs of capital accumulation. Second, natural resources such as land are implicitly kept in a state of abundance by a sufficiently plentiful exogenous technical change of a resource-saving nature" (Kurz, 1998, p.55).

We will also consider that the learning mechanisms investigated by Kaldor in his various versions of his theory of growth play a major role in his conception of economic dynamics. Kaldor's interest in these mechanisms began in 1957 when the author introduced a technical progress function in his "model of economic growth":

"The second main respect in which the present model departs from its predecessors is that it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labour and those induced by technical invention or innovation –i.e. the introduction of new knowledge (…) Hence the speed with which a society can "absorb" capital (…) depends on its technical dynamism, its ability to invent and introduce new techniques of production (…) The converse of this proposition is also true: the rate at which a society can absorb and exploit new techniques is limited by its ability to accumulate capital.

It follows that any sharp or clear-cut distinction between the movement **along** a "production function" with a given state of knowledge, and a **shift** in the "production function" caused by a change in the state of knowledge, is arbitrary and artificial" (Kaldor, 1957/1960, pp.264-265).

The same ideas are also present in the Kaldor-Mirlees 1962 article (Kaldor and Mirlees, 1962) where it is assumed that in every time interval (year) new machinery is introduced that is more productive than the machinery it replaces. It is interesting to note that, in the same year, the **Review of Economic Studies** organised a symposium in which Arrow investigated the "economic implications of learning by doing" (Arrow, 1962), namely the extent to which new ideas are exploited is measured by the amount of gross investment made in the past; and, commenting on Arrow's paper, Kaldor stressed the similarity between Arrow's arguments and the ones he developed in Kaldor, 1958 and 1961.

Finally, in the last years of his life, Kaldor re-shaped the same ideas, emphasising how important they were and referring to Young's article on increasing returns. It is this version of the learning mechanism we will retain here. This version implies two important consequences.

The first one consists in the introduction of increasing returns and, therefore, implies the abandon of linearity inherent to the assumption of constant returns (on this point, see again Targetti, 1992, p.140).

The second consequence refers to the relation between accumulation and technical progress. Following Kaldor, we will assume that learning is closely related to firms' investment activities: the absence of capital accumulation there is neither learning nor knowledge accumulation or technical change. This view is obviously strongly kaldorian and derives from Smith's and Allyn Young's famous argument:

> "Finally, there are the inventions and innovations induced by experience to which Adam Smith paid the main emphasis –what we know call "learning by doing" or "dynamic economics of scale". The advance in scientific knowledge in physics or in the science of engineering in the laboratory cannot by itself secure the innumerable design improvements that result from the repeated application of particular engineering principles. The optimum design for the steam engine or for the diesel engine or the sewing machine has only be achieved after many years or decades of experience: that for the nuclear power plant is still far away. The gain in design through experience is even more important in the making of plant and equipment; hence the **annual** gain of productivity due to "embodied technical progress" will tend to be all the greater the larger the number of plants constructed per year" (Kaldor, 1972/1989, p.381).

The origin of technological change is not, therefore, related to an exogenous probability law governing the arrival rates of innovation (Aghion and Howitt, 1998, p.55). Technological change is here to a large extent a by-product of production and capital accumulation processes. Combining these kaldorian ideas, it is possible to build a very simple model of cyclical growth.

A simple Kaldorian model : endogenous cyclical growth with exogenous trend

The aim of this section is therefore to combine Kaldor's views on endogenous business cycles, learning and technical progress, in a very stylised model of cyclical growth. This framework refers to the so called "old" growth theory in which the representation of the economy at the aggregate level is not explicitly related to some form of optimal agents behaviour.

Accordingly, let us denote by, $Y(t)$ the global output, $K(t)$ the stock of capital and by $N(t)$ the labour supply of the economy. Moreover, we assume that $N(t)$ grows at the constant natural rate n. Production is carried on according to the following technology:

$$Y(t) = A(t)f\left(\frac{K(t)}{N(t)}\right)N(t) \tag{0.1}$$

where $A(t)$ refers to the level of knowledge in the economy at time t and f is a continuous, positive and increasing function in $k(t) = \dfrac{K(t)}{N(t)}$.

Knowledge evolves according to the following leaning mechanism:

$$\dot{A}(t) = \left(\frac{I(t)}{K(t)}\right) - \gamma A(t) \tag{0.2}$$

where, γ is a small positive parameter and the positive continuous function satisfies $\phi_k > 0$, $\phi_k < 0$. This function refers to Kaldor's insights according to which technical progress is to a large extend a by product of production and capital accumulation, and the parameter γ simply indicates that knowledge may also be subject to depreciation.

The law of motion of capital is given by the traditional relation:

$$\frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)} - \delta \tag{0.3}$$

where $0 < \delta < 1$ is the rate of depreciation of capital and $I(t)$ refers to the level of investment. Finally, we suppose that investment is determined according to Kaldor's 1940 business cycle model. That is, investment is function of the level of activity $Y(t)$ and of the capital stock $K(t)$. Thus, we have :

$$I(t) = I\big(I(t), K(t)\big) \tag{0.4}$$

with[1] $I_Y > 0, I_K < 0$.

Relations (0.1) to (0.4) completely describe our simplified economy. Substituting (0.1) and (0.4) into (0.3) and (0.2), and disregarding until now for simplicity the time factor t, we obtain the following dynamical system in (k, A) :

$$
\begin{aligned}
\dot{k} &= \tilde{I}(k, A) - (\delta + n)k \\
\dot{A} &= \tilde{}(k, A) - \gamma A
\end{aligned}
\tag{0.5}
$$

We shall not investigate here the dynamic properties of this non linear system. Let us simply notice that due to the properties of the learning function and of the shape of the investment function, it be can be shown using e.g. Hopf bifurcation theory that the model may admit limit cycles. Consequently, the model exhibits endogenous cyclical growth paths around a steady state without learning where the economy grows at the natural rate.

Therefore, this example shows that Kaldor's ideas on learning by doing and on business cycles can be formulated in a unified framework of cyclical growth. However, the main deficiency of this very stylised setting is that the long term growth trend of the economy, i.e. the natural rate of growth, still remains exogenous and independent of the cyclical dynamic paths followed by the economy.

Moreover, as noted earlier, this model only focuses on the supply side of cyclical growth seen from a Kaldorian standpoint. The demand side has also been investigated in the literature, and, recently, Palley (1997) has developed a "generalised Keynes-Kaldor model of economic growth" compatible, in many respects, with the model we first presented.

The main analogies between Palley's model and ours concern the output supply and the dynamic technical progress functions, since in both approaches there is a direct relation between capital accumulation and technical progress. Some slight difference appears in the characterisation of the investment function; ours is analogous to Kaldor's function of the 1940 trade cycle model, while Palley's one does not only include the level of equipment capital but also the influence of aggregate demand growth and entrepreneurial net profitability.

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[1] It is also convenient in for technical reasons to assume that I is homogenous of degree one in Y and K

The main differences appear on the demand side of the model. Palley introduces a consumption function in which social groups appear to consume their entire wages while profit-earners save a part of their net profits [2].

The dynamics of the model is mainly related to the strength of capital deepening and to the growth states or steady-states neighbourhood. Cyclical growth is a major possible issue since multiple local equilibria as well as the supply side of a kaldorian model of cyclical growth generate complex dynamics even if the equations of both models are pretty simple.

To some extent, Palley and ours models belong to the field of endogenous growth theory since they both use an AK type function and consider technical progress as an endogenous consequence of capital accumulation. They do not exhibit however all the usual features of standard endogenous growth theory. It could be interesting therefore to go a bit further and to investigate now the extent to which the introduction of Kaldorian ingredients might affect the analytical results which are generally obtained in the standard literature.

Kaldorian ideas in an endogenous growth framework

In this section we consider a model which gives a significant example of incorporation of Kaldorian ideas on growth and cycles within the framework of endogenous growth theory. We will favour Romer's (1994) definition of endogenous growth, according to which "economic growth is an endogenous outcome of an economic system, not the outset of forces that impel from outside." (Romer 1994 p.3). In this perspective, the essential inside facto is learning by doing. By contrast, population growth does not play any role either exogenous nor endogenous in the process we describe[3]. Time is assumed to be continuous and all variables are expressed in real terms. The economy produces a single good in quantity $Q(t)$ by means of an amount of capital $K(t)$. Knowledge $A(t)$ evolves in an endogenous way, according to a global learning mechanism specified below, but appears to be an externality for the individual firm. This usual setting is modified by the introduction of adjustment costs, induced by changes in the stock of capital which grows in step with the rate of investment. Simple models of investment (e.g. user cost models) have at least two major shortcomings. First, they do not identify the mechanism through which financial variables and expectations affect investment.

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[2] By the way, this treatment of demand is close to the one privileged in Arena and Raybaut, 1995.
[3] The introduction of the demographic factor in our framework is possible, but it would also imply to consider the problems of income distribution and of consumption patterns among social groups.

Second, each invested unit does lead to an instantaneous and automatic growth of the productive stock of capital $K(t)$. The introduction of adjustment costs gives at least a remotely realistic picture of actual investment decisions. Such costs come in two forms. On the one hand, internal adjustment costs refer to the costs of installing new capital goods and training new workers to operate the new machines. On the other hand, external adjustment costs refer to financial aspects of investment, based on the q theory of investment.

From this perspective, we consider that real and financial costs of investment are encapsulated in an adjustment cost function. Let $\varphi\left(\dfrac{I(t)}{K(t)}\right)K(t)$ be this cost function, where $\varphi(.)$ is positive, continuous, increasing and convex in $k(t) = \dfrac{I(t)}{K(t)}$, with $K(t) > 0$. Moreover, let us suppose, in accordance with the literature (e.g. Hubbard, Kashyap and Whited 1995), that $\varphi(.)$ is quadratic and specified as follows :

$$\varphi(k(t)) = \frac{(k(t) - \delta)^2}{a}$$

We obtain :

$$\varphi'(k(t)) = \frac{2(k(t) - \delta)}{a}$$

$$\varphi''(k(t)) = \frac{2}{a}$$

Consequently, the net value of a representative firm is given by :

$$\pi_{Net}(K(t), I(t)) = \pi(K(t)) - \varphi\left(\frac{I(t)}{K(t)}\right)K(t) \tag{0.6}$$

where $\pi(K(t))$ is the gross level of profits. In this "AK" framework, with exogenous wages and prices, gross profits are simply given by $\pi(K(t)) = \eta A K(t)$, with $\eta > 0$ (Arena and Raybaut 1998).

Investment decisions of the representative firm $I^*(t)$ are the outcome of the maximisation of its net present value. Thus, we have the following intertemporal optimisation problem :

$$\underset{I(t)}{Max} \int_{t=0}^{\infty} e^{-rt}\left[\pi_{Net}(K(t), I(t))\right]dt \tag{0.7}$$

$$\dot{K}(t) = I(t) - \delta K(t) \tag{0.8}$$

$$K(t) > 0, I(t) \quad 0 \tag{0.9}$$

where, r denotes the discount rate and $0 \quad \delta \quad 1$ refers to the rate of depreciation of capital.

It can be shown, given the concavity of $\quad_N(K(t), I(t))$, that a unique optimal investment path $I^*(t)$ exists to solve our problem.

Let us now investigate the dynamic behaviour of the economy. To do so, it is convenient to denote investment per capital as follows : $k(t) = \dfrac{I(t)}{K(t)}$. For simplicity, we disregard the time factor (t) until now. We obtain the following first order conditions :

FOC.1
$$\lambda = \quad '(k) \tag{0.10}$$

FOC.2
$$\dot{\lambda} = (r + \delta)\lambda + \quad (k) - k \quad '(k) - \eta A \tag{0.11}$$

The derivative with respect to time of (0.10) combined with (0.11), gives :

$$\dot{k} = \frac{'(k)(r + \delta - k) + \quad (k) - \eta A}{''(k)} \tag{0.12}$$

Relation (0.12) describes the dynamics of $k(t)$, that is investment per capital.

Note that since $Q = AK$, the rate of growth of the economy is given by $g = \dfrac{\dot{A}}{A} + \dfrac{\dot{K}}{K}$. That is, the net growth rate g is equal to $\dfrac{\dot{A}}{A} + k - \delta$.

Thus, in the stationary state $\dot{A} = 0$ and $A = \overline{A} = cte$, therefore $g = k - \delta$.

We first investigate the dynamical properties of an economy without learning. In this case, $A = \overline{A} = cte$, and the dynamics of the growth rate is fully described by the one dimension differential equation in k(t) (0.12).

Substituting the respective values of $\quad , \quad , \quad$ into (0.12), we finally obtain :

$$\dot{k} = \frac{(k - \delta)(2r + \delta - k) - a\eta\overline{A}}{2}$$ (0.13)

It is obvious that equation (0.13) has two positive stationary values for k,

$k_1^* = r + \delta - \sqrt{r^2 - a\eta\overline{A}}$ and $k_2^* = r + \delta + \sqrt{r^2 - a\eta\overline{A}}$, where the first is below and the second above the golden rule.

The lower equilibrium k_1^* is locally unstable, while the higher k_2^* is locally stable. Moreover, it can be shown that for all initial conditions $k_1^0 < k(0)$, the economy converges monotonously to the higher growth rate k_2^*. Thus, without learning growth is steady, stable and non-cyclical. The phase diagram is depicted in fig.1 infra.
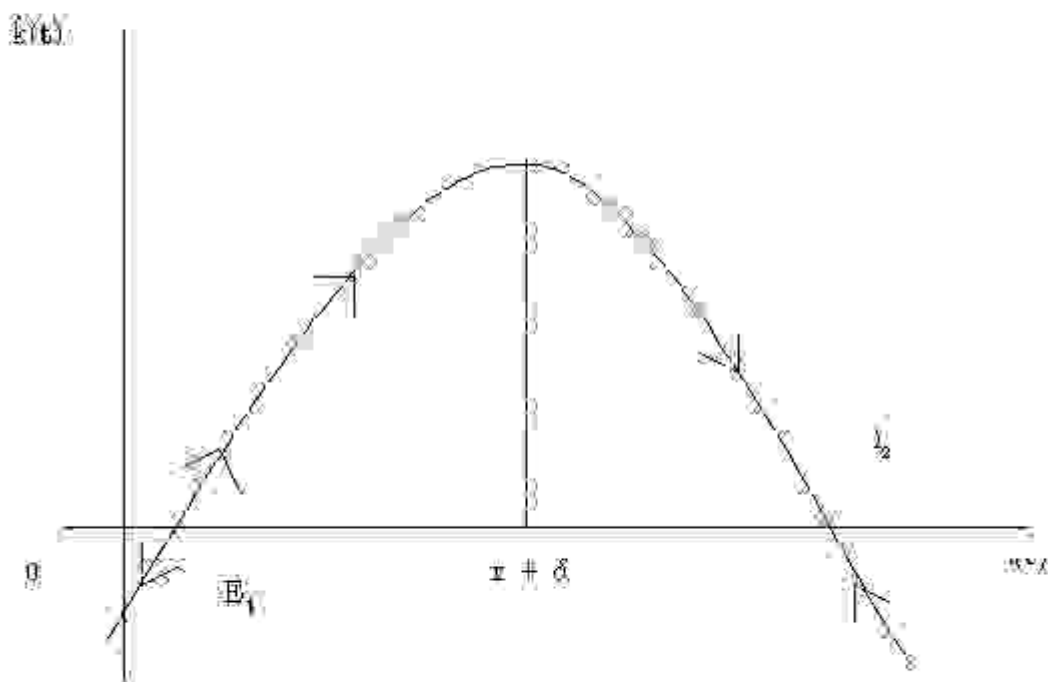
We now introduce a learning mechanism into this framework. A recurring difficulty in the representation of technological change, raised in particular by Young (1993), is that an investment in new technologies does not realise all its productive potential from the moment of its implementation. Furthermore, we know that in the presence of several competing technologies, new technologies are not necessarily better than old ones. As Young has shown, new technologies are often less efficient than old ones while they are still in the process of being developed. The approach to technical change in terms of learning by doing we favour here allows us to overcome this difficulty. It brings the learning dimension associated with the choice of the new technology that is being implemented. Here, it is the period of learning, that is, of the accumulation of experience and improvement, that develops the potential productivity of a new technology. As we know, there are at least three ways to increase efficiency : either due to incremental innovations affecting the existing stock of capital and

resulting in a total efficiency of new capital goods, or due to the acquisition of skills related to the use of new capital goods, or finally due to the existence of spillover effects occurring when there are capital goods of differing ages (Gilchrist and Williams 1998) or different kinds of activities (Evans, Honkapohja and Romer 1998).

As well known, a formal and complete study of this type of learning mechanism has been provided by Sato and Usawa. More recently, Romer has shown that in the presence of large spillover and learning effects in the endogenous growth framework, the growth rate of per capita may well be unbounded upwards. While Greiner and Semmler, have focussed on the role of learning by doing in the existence of endogenous cyclical growth paths in a two sector growth model in which human capital is increased through learning by doing. It is shown that persistent endogenous growth cycles do exist when there is a bunching of investment at nearby dates. This clustering of investment is explained by " adjacent complementarity with respect to the capital stock.", that is, " increasing capital at time t_3 implies a reallocation of resources from distant dates t_1 to nearby ones." (Greiner, 1995 p. 594). This means that this property leads to a clustering of investment and an overshooting of the capital stock $K(t)$ with respect to its steady state value. Let us now consider an economy with an initial capital stock $K(0) < K^*$ (steady state value of K). Consequently, investment increases up to the point at which to reach the steady state is reached. However, adjacent complementarity at nearby dates eventually leads to a capital stock greater than K^* Now, since $K(t_n) > K^*$, investment must decrease. The same overshooting mechanism implies that a phase of under-investment should replace that of over-investment, leading to a cyclical growth path. Following the approach of Greiner and Semmler we consider that learning is closely related to firms' investment activities. Thus, we assume that the stock of knowledge $A(t)$ is given by :

$$A(t) = \int_{-\infty}^{t} e^{\gamma(s-t)} \, \big(k(s) \big) ds \tag{0.14}$$

where γ is a positive parameter, and Φ is a positive, increasing function of $k = \dfrac{I}{K}$. This

relation means that the contribution of relative investment, $k = \dfrac{I}{K}$, further back in time is smaller than the recent one to the formation of the present stock of knowledge. Thus, we assume that there exists a nonlinear relation between gross investment and the accumulation of knowledge (Feitchinger and Sorger 1988, Greiner 1995, Greiner and Semmler 1997, Greiner and Semmler 2001). Hence, the first derivative with respect to time of $A(t)$ in relation (0.14) gives the dynamics of learning we used in the first Kaldorian model :

$$\dot{A}(t) = \left(k(t)\right) - \gamma A(t) \tag{0.15}$$

This formulation is founded both on the Kaldorian tradition and on the more recent qualitative and empirical works devoted to the relation between learning mechanisms and investment within the endogenous growth framework. The presence of the negative term $-\gamma A(t)$ simply means that a fraction of knowledge becomes obsolete as time goes on. That is to say, investment increases the stock of knowledge by learning by doing, but at the same time new capital goods are also introduced in the economy which require the utilisation of new knowledge. (see Greiner and Semmler 2001 for an empirical investigation of this formulation).

Equations (0.13) and (0.15) form a dynamical system in $(k(t), A(t))$. The remainder of this sub-section focuses on the analysis of the local dynamic properties of this system.

We first characterise the stationary states of this system. The following result holds :

- From equation (0.13), we obtain the isokine $\dot{k} = 0$, $f(k) = A$, where,

$$f(k) = \frac{-k^2 + 2(r+\delta)k - 2r\delta - \delta^2}{a\eta},$$

- From equation (0.15), we obtain the isokine $\dot{A} = 0$, $h(k) = A$, where,

$$h(k) = \frac{(k)}{\gamma}$$

Hence, any steady state value of k, k^*, is a solution of the equation $f(k) - h(k) = 0$. Accordingly, the existence and multiplicity of stationary states result from the shape of the learning function.

Now, assume that $k, \gamma > 0, 0 < k < 2r + \delta$. Assume further that (k) satisfies the following properties

$$(k) > 0 \tag{0.16}$$

$$(0) \quad 0 \qquad\qquad (0.17)$$

$$(k) < \frac{\gamma r^2}{a\eta} \qquad\qquad (0.18)$$

then two positive stationary values of k, k_1^* and k_2^* exist.

The meaning of this condition in terms of the learning mechanism is the following. First, condition (0.16) means that knowledge increases with investment per capita. Second, condition (0.17) means that apart from learning by investing and doing, knowledge increase are also partly autonomous. Finally, condition (0.18) simply requires that the learning process should be bounded from above. As a result, the model has two stationary states, one low $\left(k_1^*, A_1^*\right)$, and the other, (k_2^*, A_2^*) high, with respect to the golden rule growth rate $r + \delta$.

The main findings concerning the dynamics of the model are summarised in the table below.

Dynamics in a neighbourhood of the steady states

k^*	$0 < k_1^* < r + \delta - \gamma$	$k_2^* > r + \delta$
Trace	+	0 or -
Det	+ or -	+
Dynamics	Saddle point or locally unstable	Indeterminacy or endogenous cycles

The low stationary state $\left(k_1^*, A_1^*\right)$ is either locally unstable (and consequently cannot satisfy the transversality conditions), or is a saddle point.

The characteristics of the learning curve and of the adjustment cost function imply that the high stationary state (k_2^*, A_2^*) does not exhibit the saddle path property. The model has two negative real roots or two imaginary roots with negative real parts. Thus, this stationary state is locally stable, that is all trajectories which start in a neighbourhood of this point converge to it monotonously or cyclically. In this case, the model exhibits local indeterminacy of growth paths (cf. Benhabib. and Farmer 1994, Benhabib and Perli 1994). That is, it exists a continuum of converging paths $\{k(t), A(t)\}$ because only the initial condition $A(0)$, i.e. the initial amount of knowledge in the economy is given, while the other one $k(0) = \dfrac{I(0)}{K(0)}$, is not

predetermined. Accordingly, the initial capital stock in the economy, $K(0)$, is predetermined. But the initial level of investment $I(0)$ can be chosen freely independently of fundamentals factors given for instance the role played by "animal sprits" in entrepreneurial investment behaviour. Because the model contains no explanation for which level of initial condition $I(0)$ will be chosen in combination with $K(0)$, growth is said to be indeterminate.

Moreover, assume that the model has two distinct complex eigenvalues in some neighbourhood of the high stationary states (k_2^*, A_2^*). In this case, it is possible to show that an admissible critical value of γ, $\tilde{\gamma}$, exists (cf. Greiner and Semmler 1997). Then, the model then two pure imaginary roots, and a Hopf bifurcation arises for this value. That is, the real part of the roots crosses the imaginary axis at a non-infinite speed. Accordingly, the economy exhibits endogenous cyclical growth paths.

Finally, we illustrate these findings with a numerical example.

We specify the learning function as follows :
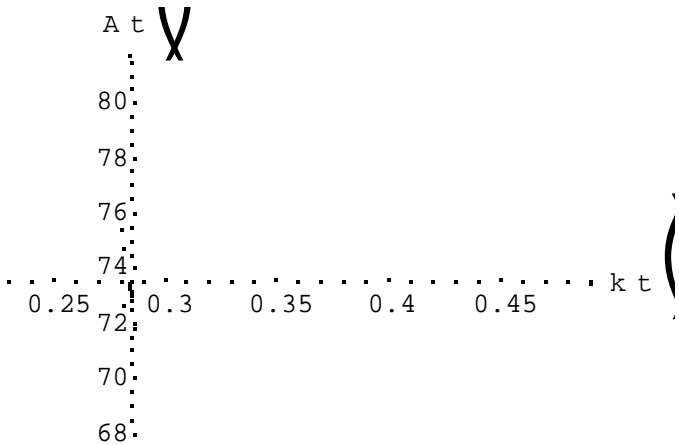
$$(k) = \mu_3 k^3 + \mu_2 k^2 + \mu_1 k + \mu_0$$

where, $\mu_3 = 0.55, \mu_2 = -0.1, \mu_1 = 5.5, \mu_0 = -0.1,$ and $\delta = 0.1,$ $a = 0.1,$ $r = 0.05$. Hence, we obtain the following diagram in (k, A) :
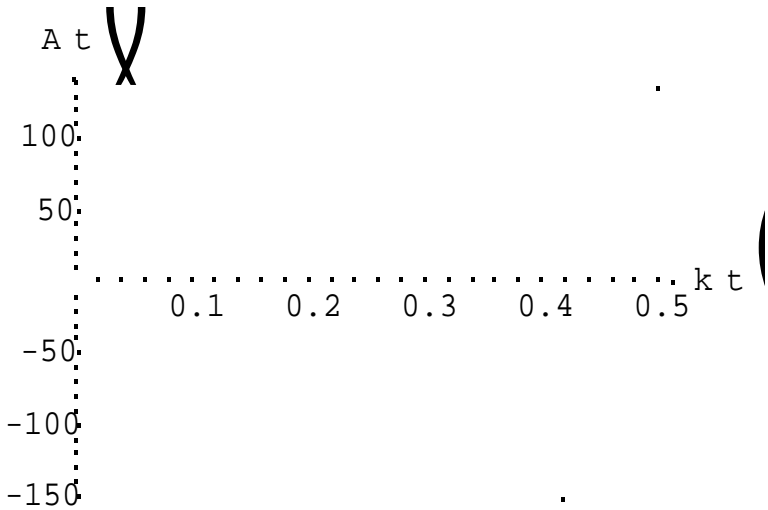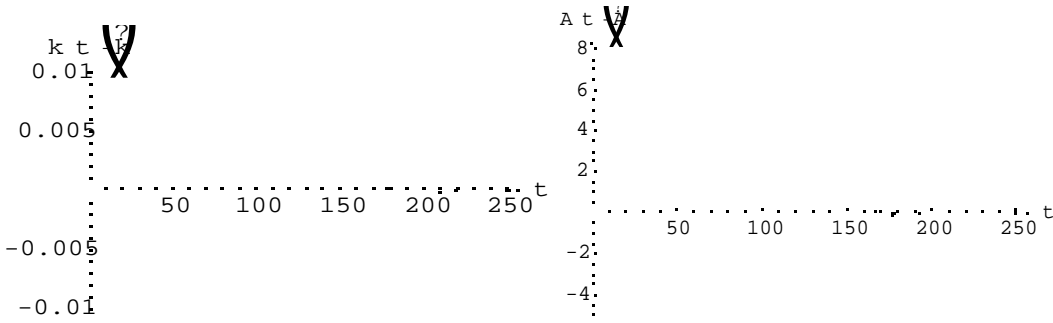

Consequently, it is easy to check that the model has two positive stationary states (k_1^*, A_1^*) and (k_2^*, A_2^*).

Example of local indeterminacy of the high stationary state (k_2^*, A_2^*) with cyclical growth,
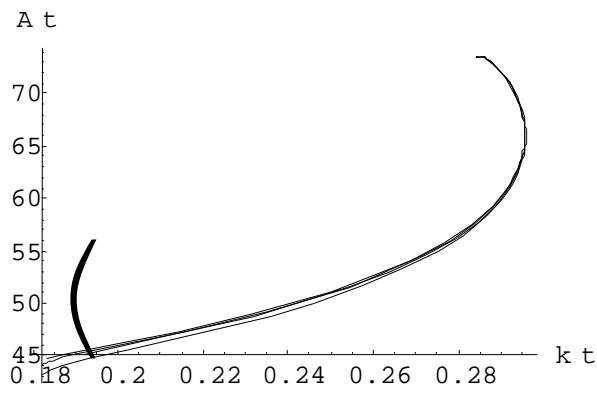
$$\gamma = 0.02$$
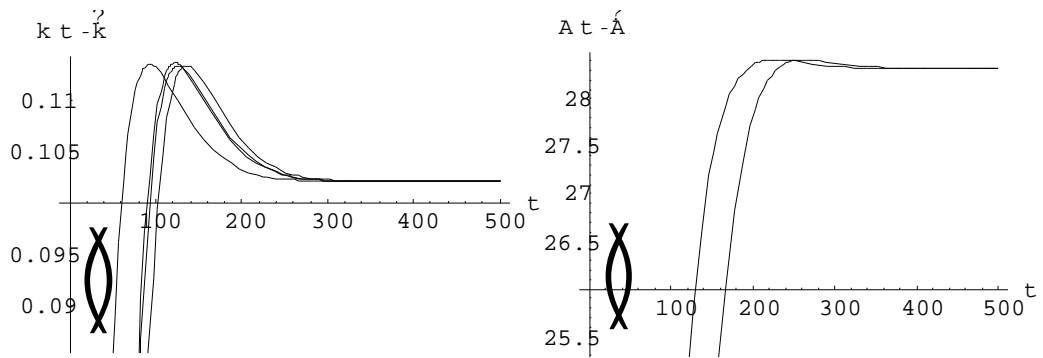
Phase diagram

Dynamics of k(t) and A(t)

Example of global indeterminacy of the high stationary state (k_2^*, A_2^*): initial conditions in a neighbourhood of the low equilibrium (k_1^*, A_1^*) locally unstable
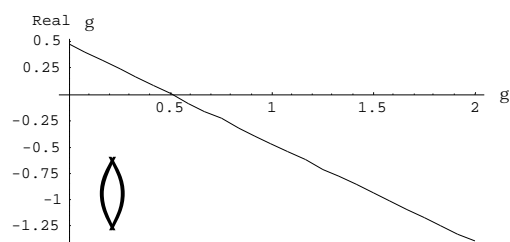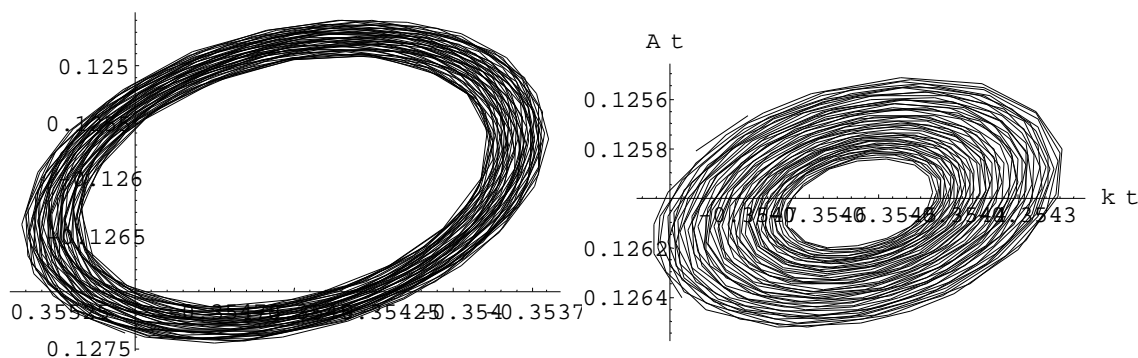

Phase diagram

Dynamics of k(t) and A(t)

Examples of endogenous cyclical growth paths in a neighbourhood of the high equilibrium (k_2^*, A_2^*) for $\tilde{\gamma} = .504528$

Real part of the eigenvalues as function of γ

Phase diagrams

This example indicates that growth cycles can emerge in the AK framework with learning for admissible values of the real interest rate. Only one admissible value of γ exists, which captures the weight of decreasing returns to the learning technology, $0 < \tilde{\gamma}$, such that the model has two pure imaginary roots and generates endogenous growth cycles paths. When γ is smaller than the critical value, the higher stationary state is indeterminate. Therefore, it has been shown that the mere introduction of a learning by investing mechanism, makes possible the existence of growth and cycles in the 'AK' framework with adjustment costs. Moreover, it appears that low decreasing returns to the learning technology go together with indeterminacy of the higher steady state.

In this section, our objective was to explore a Neo-Kaldorian approach to cyclical growth based on microeconomic foundations. Normally, models in this tradition favour a purely

macroeconomic treatment of technical change and of the effect of learning. References to behaviour of individuals or groups only provide an implicit foundation for the representation of the economy at the aggregate level. Second, this type of cyclical growth model is usually entirely expressed in real terms. However, it is well known that financial factors play a central role in the process of diffusion and development of new technology. The framework developed in this paper attempts to address these issues. First, the treatment of technical change through learning mechanisms is examined. We demonstrate that the shape of the learning curve plays a crucial role, by showing that the long-term dynamics of the economy is not, in the general case, independent of the short-term paths it follows. Second, the modelling of investment decisions and of their financing is achieved by introducing internal and external adjustment costs.

Concluding remarks

The results which were reached in this contribution show the analytical ability of the Kaldorian framework to cope with some of the complexities of capitalist economic dynamics. On the one hand, this framework permits to grasp one of the main aspects of the emergence and growth of knowledge, namely, the relation that learning by doing processes introduce between capital accumulation and technical progress. On the other hand, Kaldorian insights – considered from the standpoint of "old" as well as of "new" growth theory – appear to be sufficiently relevant to generate a form of economic dynamics which allows to avoid the dichotomy between BC and growth and to take into account a range of economic processes which better expresses the variety of the features of capitalist evolution. However, the models considered in the paper are developed in a usual macroeconomic framework which does not permit the analysis of structural change that Kaldor had in mind, especially in the last years of his life. Some further research is therefore to be done in this direction, avoiding, however, the duality of agriculture and industry (see Targetti, 1992 chapter 8). In the beginning of the second millenium, the development of the new technologies of information strongly contributes to undermine this duality, introducing new industrial asymetries but also new forms of knowledge and innovation which differ from learning by doing. These new realities might generate a new historical period in capitalist evolution. From this standpoint we must be able to discriminate, among Kaldorian analytical tools, those which are still relevant from those which are now useless. The contents of this programme for future research is not

surprising: one of the major aspects of Kaldor's reflections and scientific life was indeed the stress he put on the necessity of choosing flexible patterns of thought in order to use the appropriate tools for coping with the new historical realities of contemporary economic world.

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